On Super-Turing Neural Computation

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Abstract In this paper, we provide a historical survey of the most significant results concerning the computational power of neural models. We distinguish three important periods: first, the early works from McCulloch and Pitts, Kleene, and Minky, where the computational equivalence between Boolean recurrent neural networks and finite state automata is established. Secondly, the two breakthroughs by Siegelmann and Sontag showing the Turing universality of rational-weighted neural networks, and the super-Turing capabilities of analog recurrent neural networks. Thirdly, the recent results by Cabessa, Siegelmann and Villa revealing the super-Turing computational potentialities of interactive and evolving recurrent neural networks.

Keywords Neural computation • Recurrent neural networks • Finite automata • Turing machines • Turing machines with advice • super-Turing

1 The Early Works

In theoretical neuroscience, understanding the computational and dynamical capabilities of biological neural networks is an issue of central importance. In this context, much interest has been focused on comparing the computational powers of diverse theoretical neural models with those of abstract computing devices.

This comparative approach was initiated by McCulloch and Pitts who proposed a modelisation of the nervous system as a finite interconnection of threshold logic units [19]. For the first time, neural networks were considered as discrete abstract machines, and the issue of their computational capabilities investigated from the automata-theoretic perspective. In this context, Kleene and Minsky proved that recurrent neural networks made up of threshold activation units were computationally equivalent to classical finite state automata [13, 20].

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Besides, in a seminal report entitled "Intelligent Machinery" [31], Turing brilliantly introduced many concepts which have later become central in the field of neural computation. For instance, Turing foresaw the possibility of surpassing the capabilities of finite state machines and reaching Turing universality via neural networks called "B-type unorganised machines". The networks consisted of a general interconnection of NAND neurons, and the consideration of infinitely many such cells could simulate the behaviour of a Turing machine. Moreover, Turing also introduced the key idea of "training" neural networks by considering the possibility of modifying the synaptic connections between the cells by means of what he called "connection-modifiers". Later, the Turing universality of infinite or heterogeneous neural networks has further been investigated in many directions, see for instance [8, 9, 11, 23]. These seminal works opened up the way to the theoretical computer scientist approach to neural computation. However, the purely discrete and mechanical approach under consideration quickly appeared too restrictive, far from the biological reality.

According to these considerations, von Neumann proposed another relevant approach to the issue of information processing in the brain from the hybrid perspective of digital and analog computation [22]. He considered that the non-linear character of the operations of the brain emerges from a combination of discrete and continuous mechanisms, and therefore envisioned neural computation as something strictly more powerful than abstract machines. Almost in the same time, Rosenblatt proposed the so-called "perceptron" as a more general computational neural model than the McCulloch-Pitts units [24]. The essential innovation consisted in the introduction of numerical synaptic weights and as well as a special interconnection pattern. This neural model gave rise to an algorithmic conception of "learning" achieved by adjusting the synaptic weights of the networks according to some specific task to be completed. This study is nowadays considered as foundational for the field of machine learning. The computational capabilities of the perceptron were further studied by Minsky and Papert [21].

2 Two Significant Breakthroughs

Later, Siegelmann and Sontag made two significant steps forward concerning the precise issue of the computational power of recurrent neural networks. Firstly, they focused their attention on the consideration of more realistic activation functions for the neurons and showed that by extending the activation functions of the cells from boolean to linear-sigmoid, the computational power of the neural networks would drastically increase from finite state automata up to Turing capabilities [28]. The Turing universality of neural networks was then generalised to a broader class of sigmoidal activation functions [12]. The computational equivalence between the so-called *rational recurrent neural networks* and the Turing machines has nowadays become standard result in the field.

Secondly and most importantly, following von Neumann considerations, they assumed that the variables appearing in the underlying chemical and physical phenomena could be modelled by continuous rather than discrete numbers, and therefore proposed a precise study of the computational power of recurrent neural networks from the perspective of analog computation [27]. They introduced the concept of an *analog recurrent neural network* as a classical linear-sigmoid neural net equipped with real- instead of rational-weighted synaptic connections. This analog information processing model turns out to be capable of capturing the non-linear dynamical properties that are most relevant to brain dynamics, such as rich chaotic behaviours [7, 25, 26, 29, 32]. In this context, they proved that analog recurrent neural networks are computationally equivalent to Turing machine with advice, hence capable of super-Turing computational capabilities from polynomial time of computation already. They further formulated the so-called Thesis of Analog Computation – an analogous to the Church-Turing thesis, but in the realm of analog computation – stating that no reasonable abstract analog device can be more powerful than first-order analog recurrent neural networks [26, 27].

3 Present and Future

But until the mid 1990s, the neural models involved in the study of the computational capabilities of recurrent neural networks have always been oversimplified, lacking many biological features which turn out to be essentially involved in the processing of information in the brain. In particular, the effects that various kinds of noise might have on the computational power of recurrent neural networks had not been considered. Moreover, the ability of neural networks to evolve over time has also been neglected in the models under consideration. Biological mechanisms like synaptic plasticity, cell birth and death, changes in connectivity, etc., – which are widely assumed to be of primary importance in the processing and encoding of information –, have yet not been taken into consideration in the study of the computational capabilities of neural networks.

Concerning noise, Maass and Orponen showed that general analog computational systems subjected to arbitrarily small amount of analog noise have their computational power reduced to that of finite automata or even less [17]. In particular, the presence of arbitrarily small amount of analog noise seriously reduces the capabilities of both rational- and real-weighted recurrent neural networks to those of finite automata, namely to the recognition of regular languages. Maass and Sontag then extended this result by showing that, in the presence of gaussian or other common analog noise distribution with sufficiently large support, recurrent neural networks have their computational reduced to even less than finite automata, namely to the recognition of definite languages [18]. These two results were further generalised to the broader classes of quasi-compact and weakly ergodic Markov computational systems, respectively [1].

Concerning the evolvability of neural networks, Cabessa and Siegelmann considered a more biologically oriented model where the synaptic weights, the connectivity pattern, and the number of neurons can evolve rather than stay static [3]. The so-called *evolving recurrent neural networks* were proven to be computationally equivalent to the analog neural networks, and hence capable of super-Turing computational power, regardless of whether their synaptic weights are rational or real. These results are important, showing that the power of evolution brings up additional potentialities to first-order recurrent neural networks and provides an alternative and equivalent way to the incorporation of the power of the continuum towards the achievement of super-Turing computational capabilities of neural networks. This feature is particularly interesting since certain analog assumptions in neural models have sometimes been argued to be too strong.

However, in this global line of thinking, the issue of the computational capabilities of neural networks has always been considered from the strict perspective of Turing-like classical computation [30]: a network is viewed as an abstract machine that receives a finite input stream from its environment, processes this input, and then provides a corresponding finite output stream as answer, without any consideration to the internal or external changes that might happen during the computation. But this classical computational approach is inherently restrictive, and has nowadays been argued to "no longer fully corresponds to the current notion of computing in modern systems" [16], especially when it refers to bio-inspired complex information processing systems [14, 16]. Indeed, in the brain (or in organic life in general), information is rather processed in an interactive way, where previous experience must affect the perception of future inputs, and where older memories may themselves change with response to new inputs. Hence, neural networks should rather be conceived as performing sequential interactions or communications with their environments, and be provided with memory that remains active throughout the whole computational process, rather than proceeding in a closed-box amnesic classical fashion. Accordingly, the computational power of recurrent neural networks should rather be conceived from the perspective of interactive computation [10].

Along these lines, Cabessa and Siegelmann studied the computational power of recurrent neural networks involved in a basic interactive computational paradigm [4]. They proved that the so-called *interactive recurrent neural networks* with rational and real synaptic weights are computationally equivalent to interactive Turing machines and interactive Turing machines with advice, respectively. These achievements provide a generalisation to the bio-inspired interactive computational context of the previous classical results by Siegelmann and Sontag [27, 28]. Besides, Cabessa and Villa also provided a study of the super-Turing computational capabilities of analog neural networks involved in another kind of reactive and memory active computational framework [5].

The last advances concerning the study of the computational power of recurrent neural networks were provided by Cabessa and Villa [2, 6]. They studied the computational potentialities of a recurrent neural model combining the two relevant features of evolvability and interactivity introduced in [3, 4], and showed that the so-called *interactive evolving recurrent neural networks* are capable of super-Turing computational potentialities, equivalent to interactive Turing machine with advice, irrespective of whether their synaptic weights are rational or real.

These results show that the consideration of evolving capabilities in a first-order interactive neural model provides the potentiality to break the Turing barrier, irrespective of whether the synaptic weights are rational or real. They support the extension of the Church-Turing Thesis to the context of interactive computation: "Any (non-uniform interactive) computation can be described in terms of interactive Turing machines with advice" [15]. As for the classical computational framework, the super-Turing computational capabilities can be achieved without the need of a framework based on the power of the continuum – in the case of interactive evolving recurrent neural networks with rational weights. This feature is particularly meaningful, since while the power of the continuum is a pure conceptualisation of the mind, the evolving capabilities of the networks are, by contrast, really observable in nature.

From a general perspective, we believe that such theoretical studies about the computational power of bio-inspired neural models might ultimately bring further insight to the understanding of the intrinsic natures of both biological as well as artificial intelligences. We also think that foundational approaches to alternative models of computation might in the long term not only lead to relevant theoretical considerations, but also to important practical applications. Similarly to the theoretical work from Turing which played a crucial role in the practical realisation of modern computers, further foundational considerations of alternative models of computation will certainly contribute to the emergence of novel computational technologies and computers, and step by step, open the way to the next computational era.

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