Attractor Dynamics Driven by Interactivity in Boolean Recurrent Neural Networks

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Abstract. We study the attractor dynamics of a Boolean model of the basal ganglia-thalamocortical network as a function of its interactive synaptic connections and global threshold. We show that the regulation of the interactive feedback and global threshold are significantly involved in the maintenance and robustness of the attractor basin. These results support the hypothesis that, beyond mere structural architecture, global plasticity and interactivity play a crucial role in the computational and dynamical capabilities of biological neural networks.

1 Introduction

Experimental studies suggest that spatiotemporal patterns of discharges, i.e., ordered and precise interspike interval relationships [1-3], as well as specific attractor dynamics [4,5] are likely to be significantly involved in the processing and coding of information in the brain. The association between attractor dynamics and spatiotemporal patterns has been demonstrated in nonlinear dynamical systems [6] and in simulations of large scale neuronal networks [7], thus suggesting that spatiotemporal patterns might be considered as witnesses of underlying attractor dynamics – which itself would be a key feature of neural coding.

On the basis of these bioinspired considerations, we study the attractor dynamics of a Boolean model of the basal ganglia-thalamocortical network [8]. We investigate the richness of the attractor dynamics of this network as a function of its interactive synaptic connections – which are assumed to be significantly involved in the crucial exchange of information between the network and its environment – as well as of its global threshold – which represents a global notion of plasticity [9–14]. We show that the regulation of the interactive feedback and global threshold are significantly involved in the maintenance and robustness of optimal attractor potentialities. It is noteworthy that experimental evidence of a context-dependent modifiable central feedback to projection neurons has been reported in the invertebrate neural circuit [15].

2 Boolean Recurrent Neural Networks

It has early been observed that Boolean recurrent neural networks are computationally equivalent to finite state automata [16,17]. More precisely, recurrent neural networks composed of McCulloch and Piits's cells [18] can simulate and be simulated by finite state automata. The translation from a Boolean networks to a corresponding finite automaton is illustrated in Fig. 1. The converse translation is not illustrated here.

Boolean Neural Network





Fig. 1. Translation from a given Boolean neural network \mathcal{N} to a corresponding finite automaton \mathcal{A} . The nodes of \mathcal{A} are the different states of \mathcal{N} (represented as colored triple dots that depict the three internal quiet or firing cells of \mathcal{N}). There is an edge from node s to node s' labelled by x in \mathcal{A} if and only if the network \mathcal{N} moves from state s to s' when receiving input x. (Color figure online)

According to the construction of Fig. 1, the possible dynamics of a given Boolean network correspond precisely to the possible paths in the graph of its associated automaton. Hence, the attractors of the Boolean network – i.e., the cyclic dynamics – correspond exactly to the cycles of the automaton. Consequently, in order to compute the attractors of a Boolean network, it suffices to construct its corresponding automaton and then list all the cycles of this automaton. Note that in this context, whenever the dynamics of Boolean networks is falling into some periodic attractor, the activity of the network units is necessarily characterized by some associated recurrent spatiotemporal pattern of discharges, as illustrated in Fig. 2.

This theoretical framework is illustrated by a simulation of a network formed by interconnected thalamocortical modules of spiking units described elsewhere [19]. This model accounts for a first order dynamics of the membrane



Fig. 2. In a Boolean neural networks, the attractor dynamics of the internal cells are the precise phenomenon that underly the emergence of spatiotemporal patterns of discharges. In fact, the raster plot of internal cells involved in some periodic attractor dynamics corresponds precisely to some spatiotemporal pattern of discharge.

potential characterized by a kinetic constant and for global excitability of the circuit. These parameters are controlled by the modulatory inputs that act differentially on the capacitance and resistance of the cell membrane. Monoamines and acetylcholine may regulate properties of voltage-sensitive ion channels [20] through the action of cellular second messengers. These mechanisms affect the shape of the postsynaptic potentials – i.e., the half-width of the decay – without modifying the membrane resistance which is related to the membrane potential. Modulatory projections from the brainstem may also affect the overall excitability of the thalamocortical network in relation to arousal, sleep-waking activity, and their role in modulation of sensory processes has been recognized long time ago [21, 22].

In the absence of background activity and noisy inputs, all the dynamics is deterministic, such that when an input pattern of activity is provided at the beginning of the simulation, the network activity stabilizes either to an extinction of the activity – no more units are firing – or to a cyclic pattern of activity – an attractor dynamics – which in turn induces a corresponding spatiotemporal



Fig. 3. Examples of raster displays showing repeating spatiotemporal patterns. The rows of the rasters correspond to each unit of a circuit composed of two coupled thalamocortical modules activation pattern. At time 1 the cells 5, 6, 8 and 25 were initially set active. The time constant of the membrane potential was fixed at 2.92 ms. Global excitability parameter was set at a lower level ep = -31 in panel (a) then in panel (b) where ep = -29. The spatiotemporal pattern started to repeat at time 34 with a cycle duration of 24 time steps and at time 50 with a cycle duration of 16 for panel (a) and panel (b), respectively.

pattern of discharges. The period of the attractor and the specificities of the associated spatiotemporal pattern may change greatly to tiny differences in the values of the two dynamical parameters for the same initial pattern of activation as illustrated by Fig. 3. Notice that with the same initial stimulation and same membrane dynamics, a change in the global excitability parameter may also lead to the extinction of the activity.

3 Boolean Model of the Basal Ganglia-Thalamocortical Network

We assume that the encoding of a large amount of the information treated by the basal ganglia-thalamocortical network is performed by recurrent patterns of activity circulating in the information transmitting system of this network.



Fig. 4. (a) Simple Boolean model of the basal ganglia-thalamocortical network and (b) its adjacency matrix. Each brain area is represented by a single node in the Boolean neural network model: superior colliculus (SC), Thalamus, thalamic reticular nucleus (NRT), Cerebral Cortex, the striatopallidal and the striatonigral components of the striatum (Str), the subthalamic nucleus (STN), the external part of the pallidum (GPe), and the output nuclei of the basal ganglia formed by the GABAergic projection neurons of the intermediate part of the pallidum and of the substantia nigra pars reticulata (GPi/SNR). We consider also the inputs (IN) from the ascending sensory pathway and the motor outputs (OUT). The excitatory pathways are labeled in blue and the inhibitory ones in orange. Part of the motor outputs are recurrently connected via the interactive connections int₁ and int₂. (Color figure online)

We extend our simplified model of the basal ganglia-thalamocortical network [8] in order to include *interactive connections*, enabling a feedback of information from the network activity to combine with the external inputs, see Fig. 4. We study the attractor dynamics of this network as a function of its interactive connections int_1 and int_2 and of its global excitability.

4 Results

We study the attractor dynamics of our simplified model of the basal gangliathalamocortical network, as a function of perturbations of its interactive connections (int₁ and int₂) and global threshold (θ). Overall, we notice that the regulation of the interactive feedback plays a crucial role in the maintenance of an optimal attractor-based level of complexity. There is always an optimal region for the interactive weights outside of which the number of attractors of the network significantly decreases. We also show that the network's attractor dynamics depends sensitively on the value of its global threshold. Small perturbations of the threshold significantly affect the attractor dynamics of the network.

More precisely, for each of the four threshold values $\theta = 0.4$, $\theta = 0.6$, $\theta = 0.8$ and $\theta = 1.0$, we preformed 1681 simulations to compute the number of basic attractors¹ of the network as a function of its two interactive weights int₁ and int₂,

¹ The basic attractors of a Boolean network are given by the basic cycles of its corresponding automaton, i.e., the cycles that do not visit the same vertex twice.



Fig. 5. Number of basic attractors of the network as a function of the interactive weights int_1 and int_2 , and for different values of the global threshold θ of the cells. Four patterns of variation are observed and reported in the subfigures (a)–(d). The green point corresponds to no interactivity. (Color figure online)

where these latter are varying from -2 to 2 by steps of 0.1. The results are reported in Fig. 5. In each case, we notice the existence of an optimal region for the values of int₁ and int₂ where the number of attractors takes maximal values of 22 (in cases (b), (c), and (d)) or 25 (in case (a)). Around this optimal region, the number of attractors was much lower. This optimal region is 'continuous', in the sense of forming a well defined block without holes, as opposed to smaller discontinuous blocks that would be disseminated across the map. Hence, in the 'center' of this optimal region, the interactive weights int₁ and int₂ can vary in a relatively consequent neighborhood without compromizing the attractor dynamics of the network.

Furthermore, we notice that the variation of the threshold θ affects significantly the attractors dynamics of the network. A higher excitability, i.e. a lower threshold ($\theta = 0.4$), favors the emergence of richer attractor dynamics in the optimal region (25 attractors in case (a) as opposed to 22 in the three other cases). However, this optimal region is surrounded by regions of lower complexities than in the other cases. Hence, an increase of the excitability (i.e. lowering of the threshold) acts as a "polarization" of the attractor dynamics: it increases the complexity of the optimal region and lowers the complexity of its neighbourhood.

5 Discussion

We have considered a simplified Boolean model of the basal ganglia-thalamocortical network, and provided new evidence of the effects that the global excitability and "interactivity" have on its dynamical properties. The interactivity is expresses in the form of a feedback informational loop, where the network's output together with the external environment produce a combined stream of information which is re-entered into the input layer of the network. This information can be assumed to represent precise contextual and explicit information recorded by the primary ascending (i.e. *lemniscal*) sensory channels via a thalamic relay [23]. The sensory information is also reaching modulatory centers in the brainstem and hypothalamus that may exert their modulatory influence by changing the global excitability of the network [24].

More generally, our results show that the interactive connections and global excitability of Boolean neural networks play a significant role in the maintenance and robustness of their attractor-based complexity. The networks are considered as dynamical systems operating in a range of control parameters. A global change in their excitability combined with selected interactively-generated input patterns will induce their dynamics to evolve into specific attractor dynamics, and in turn, into repeating spatiotemporal firing patterns. Those patterns should not be considered as high-order Morse codes, but rather as co-representations of contextual information, including a certain "central arousal" modulated by dopaminergic [25], cholinergic [26] and serotoninergic [27] pathways.

References

- Abeles, M., Gerstein, G.L.: Detecting spatiotemporal firing patterns among simultaneously recorded single neurons. J. Neurophysiol. 60(3), 909–924 (1988)
- 2. Villa, A.E.P., Abeles, M.: Evidence for spatiotemporal firing patterns within the auditory thalamus of the cat. Brain. Res. **509**(2), 325–327 (1990)
- Villa, A.E.P., Fuster, J.M.: Temporal correlates of information processing during visual short-term memory. Neuroreport 3, 113–116 (1992)
- Celletti, A., Villa, A.E.P.: Determination of chaotic attractors in the rat brain. J. Stat. Phys. 84(5), 1379–1385 (1996)
- Villa, A.E.P., Tetko, I.V., Celletti, A., Riehle, A.: Chaotic dynamics in the primate motor cortex depend on motor preparation in a reaction-time task. Curr. Psychol. Cogn. 17, 763–780 (1998)
- Asai, Y., Villa, A.E.P.: Integration and transmission of distributed deterministic neural activity in feed-forward networks. Brain. Res. 1434, 17–33 (2012)
- Iglesias, J., Villa, A.E.P.: Recurrent spatiotemporal firing patterns in large spiking neural networks with ontogenetic and epigenetic processes. J. Physiol. Paris 104(3– 4), 137–146 (2010)
- 8. Cabessa, J., Villa, A.E.P.: An attractor-based complexity measurement for Boolean recurrent neural networks. PLoS ONE **9**(4), e94204 (2014)
- McCormick, D.A., Bal, T.: Sleep and arousal: thalamocortical mechanisms. Annu. Rev. Neurosci. 20, 185–215 (1997)

- Terman, D., Rubin, J.E., Yew, A.C., Wilson, C.J.: Activity patterns in a model for the subthalamopallidal network of the basal ganglia. J. Neurosci. 22(7), 2963–2976 (2002)
- 11. Silkis, I.: A hypothetical role of cortico-basal ganglia-thalamocortical loops in visual processing. Biosystems **89**(1–3), 227–235 (2007)
- Spiga, S., Lintas, A., Diana, M.: Altered mesolimbic dopamine system in THC dependence. Current Neuropharmacol. 9(1), 200–204 (2011)
- Lintas, A.: Discharge properties of neurons recorded in the parvalbumin-positive (PV1) nucleus of the rat lateral hypothalamus. Neurosci. Lett. 571, 29–33 (2014)
- Guthrie, M., Leblois, A., Garenne, A., Boraud, T.: Interaction between cognitive and motor cortico-basal ganglia loops during decision making: a computational study. J. Neurophysiol. 109(12), 3025–3040 (2013)
- Blitz, D.M., Nusbaum, M.P.: Modulation of circuit feedback specifies motor circuit output. J. Neurosci. 32(27), 9182–9193 (2012)
- Kleene, S.C.: Representation of events in nerve nets and finite automata. In: Shannon, C., McCarthy, J. (eds.) Automata Studies, pp. 3–41. Princeton University Press, Princeton (1956)
- Minsky, M.L.: Computation: Finite and Infinite Machines. Prentice-Hall Inc., Englewood Cliffs (1967)
- McCulloch, W.S., Pitts, W.: A logical calculus of the ideas immanent in nervous activity. Bull. Math. Biophys. 5, 115–133 (1943)
- Villa, A.E.P., Tetko, I.V.: Spatio-temporal patterns of activity controlled by system parameters in a simulated thalamo-cortical neural network. In: Herrmann, H., Wolf, D., Poppel, E. (eds.) Supercomputing in Brain Research: From Tomography to Neural Networks, pp. 379–388. World Scientific, Singapore (1995)
- Levitan, I.B.: Modulation of ion channels in neurons and other cells. Annu. Rev. Neurosci. 11, 119–136 (1988)
- Foote, S.L., Morrison, J.H.: Extrathalamic modulation of cortical function. Annu. Rev. Neurosci. 10, 67–95 (1987)
- McCormick, D.A., Pape, H.C.: Noradrenergic and serotonergic modulation of a hyperpolarization-activated cation current in thalamic relay neurones. J. Physiol. 431, 319–342 (1990)
- Kandel, E.R., Schwartz, J.H., Jessell, T.M., Siegelbaum, S.A., Hudspeth, A.J.: Principles of Neural Science, 5th edn. McGraw-Hill, New York (2012)
- Saper, C.B., Lowell, B.B.: The hypothalamus. Curr. Biol. 24(23), R1111–R1116 (2014)
- Sesack, S.R., Grace, A.A.: Cortico-basal ganglia reward network: microcircuitry. Neuropsychopharmacology 35(1), 27–47 (2010)
- Villa, A.E.P., Lorenzana, V.M.B., Vantini, G.: Nerve growth factor modulates information processing in the auditory thalamus. Brain. Res. Bull. **39**(3), 139–147 (1996)
- Lopez-Garcia, J.A.: Serotonergic modulation of spinal sensory circuits. Curr. Top. Med. Chem. 6(18), 1987–1996 (2006)