An STDP Rule for the Improvement and Stabilization of the Attractor Dynamics of the Basal Ganglia-Thalamocortical Network

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Abstract. The basal ganglia-thalamocortical (BGT) network has been investigated for many years, in particular in relation to disorders of the motor system and of the sleep-waking cycle. Its attractor dynamics is related to significant aspects of processing and coding of information, the most important of which being associative memories. The consideration of a simplified Boolean model of the BGT network allows for an exhaustive analysis of its attractor dynamics. In this context, it has been shown that both global and local changes in the synaptic weights could strongly influence the attractor-based complexity of the network. We propose a novel adaptive spike-timing dependent plasticity (STDP) rule which allows the network to improve and stabilize its attractor complexity during its computational process. The rule is based on an adaptive learning rate which varies according to the attractor dynamics that the network continuously visits.

Keywords: Boolean recurrent neural networks · Learning
Attractors · STDP · Plasticity · Interactivity
Basal ganglia-thalamocortical circuit · Limbic system

1 Introduction

The basal ganglia-thalamocortical (BGT) network has been investigated for many years, in particular in relation to disorders of the motor system and of the sleep-waking cycle [8,11,13]. Its attractor dynamics is related to significant aspects of processing and coding of information, the most important of which being associative memories [2,10]. The consideration of a simplified Boolean model of the BGT network allows for a complete analysis of its attractor dynamics. Indeed, the attractors of the network correspond precisely to the cycles of

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its corresponding automaton, and therefore, can be computed explicitly and exhaustively.

It has been shown that local and global changes in the synaptic weights could strongly influence the attractor-based complexity of the BGT network. Moreover, modifications of the non-interactive and interactive weights can compensate and/or be combined to each other to drive the network into stable attractor dynamics of high complexity [4–6].

Based on these considerations, we propose a novel adaptive spike-timing dependent plasticity (STDP) rule which allows the BGT network to improve and stabilize its attractor complexity during its computational process. The rule is based on an adaptive learning rate which varies according to the attractor dynamics that the network continuously visits.

2 Boolean Model of the Basal Ganglia-Thalamocortical Network

The basal ganglia-thalamocortical (BGT) network is formed by several parallel and segregated circuits involving different areas of the cerebral cortex, striatum, pallidum, thalamus, subthalamic nucleus and midbrain [1,7]. A characteristic of the pathways of this network is a combination of "open" and "closed" loops, with ascending sensory afferences reaching the thalamus and the midbrain and descending motor efferences from the midbrain (the tectospinal tract) and the cortex (the corticospinal tract).

We consider a Boolean model of the BGT network where each brain area is modeled by a Boolean node. The Boolean model is formed by 9 nodes: the superior colliculus (SC), the thalamus (Thalamus), the thalamic reticular nucleus (NRT), the cerebral cortex (Cerebral Cortex), the striatopallidal and the striatonigral components of the striatum (Str-D1 and Str-D2), the subthalamic nucleus (STN), the external part of the pallidum (GPe), and the output nuclei of the basal ganglia formed by the GABAergic projection neurons of the intermediate part of the pallidum and of the substantia nigra pars reticulata (GPi/SNR). The closed-loop architecture of the network is implemented via feedback connections—or *interactive connections*—from the efferent output (OUT) to the input (IN). The network is illustrated in Fig. 1A and its weight matrix given in Table 1. This pattern of connectivity corresponds to the wealth of data reported in the literature [1,7].

The context of Boolean neural networks, although relatively simple, has the advantage of allowing for a complete analysis of the attractor dynamics of the networks. In fact, Boolean recurrent neural networks are known to be computationally equivalent to finite state automata [9,12], and the attractors of the networks correspond precisely to the cycles in the graphs of their corresponding automata [3]. The attractor dynamics can therefore be computed explicitly and exhaustively. The finite automaton associated to the BGT network of Fig. 1A is illustrated in Fig. 1B [3].

An *attractor-based measure of complexity* for the Boolean model of the BGT network has been introduced [3]. This complexity measure is related to the number of attractors of the network as well as to their classification into meaningful or spurious types. In the present study, we define the attractor-based complexity of the network to be its number of attractors. The BGT network of Fig. 1 with weights of Table 1 has an attractor complexity of 22.

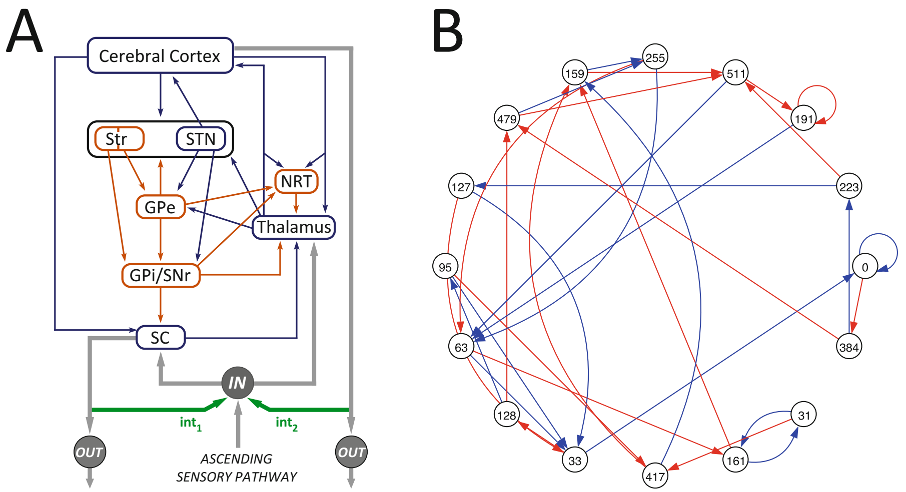

Fig. 1. A. Simplified Boolean model of the BGT network. Each brain area is represented by a single Boolean unit. The network is formed by 9 Boolean nodes: SC, Thalamus, NRT, Cerebral Cortex, Str-D1, Str-D2, STN, GPe, GPi/SNR. The inputs from the ascending sensory pathway (IN) is also a Boolean unit and the efferent outputs (OUT) are coming out of the cerebral cortex and superior colliculus. The excitatory and inhibitory pathways are labeled in blue and orange, respectively. The interactive connections int_1 and int_2 implement the closed-loop architecture. **B.** Finite automaton associated to the Boolean model of the BGT network. Each node of the automaton is a Boolean state of the network. There is a blue or red transition from node i to node j if and only if the network switches from state i to state j when receiving input 0 or 1, respectively. The attractors of the network correspond to the cycles in the automaton.

3 Adaptive STDP Rule

We introduce an adaptive spike-timing dependent plasticity (STDP) rule aimed at improving and stabilizing the attractor-based complexity of the BGT network during its computational process. This STDP rule modifies the connection strengths of the network not only as a function of the timing between the activations of the pre- and post-synaptic neurons, but also as a function of the attractors encountered throughout the computation.

Table 1. Adjacency matrix of the Boolean model of the BGT network of Fig. 1A.

Source		Target Node #									
Node #	Name	0	1	2	3	4	5	6	7	8	9
0	IN	·	1	1	·	·	·	·	·	·	·
1	SC	int_1	·	1	·	·	·	·	·	·	·
2	Thalamus	·	·	·	1	·	1	1	1	1	1
3	NRT	·	·	−1	·	·	·	·	·	·	·
4	GPi/SNr	·	−1	−1	−1	·	·	·	·	·	·
5	STN	·	·	·	·	2	·	2	·	·	2
6	GPe	·	·	·	−1/2	−1/2	−1/2	·	−1/2	−1/2	·
7	Str-D2	·	·	·	·	·	·	−1	·	·	·
8	Str-D1	·	·	·	·	−1/2	·	−1/2	·	·	·
9	C. Cortex	int_2	1/2	1	1/2	·	1/2	·	1/2	1/2	·

Formally, we consider the following *adaptive STDP rule* bounded by a definite weight interval $I = [I_1, I_2]$:

$$a_{ij}(t+1) = \begin{cases} I_1 & \text{if } a_{ij}(t+1) < I_1 \\ R & \text{if } I_1 \leq a_{ij}(t+1) \leq I_2 \\ I_2 & \text{if } a_{ij}(t+1) > I_2 \end{cases}$$

with

$$R = a_{ij}(t) + \lambda(t)\big[x_i(t+1)x_j(t) - C(x_i(t)x_j(t+1))\big] \tag{1}$$

and where $x_i(t)$ and $x_j(t)$ are the activation values of cells x_i and x_j at time t, $a_{ij}(t)$ is the synaptic weight from x_j to x_i at time t, C is a constant modulating the weight decrease (with default value equal to 1), and $\lambda(t)$ is the adaptive learning rate whose evolution is described below.

The adaptive learning rate $\lambda(t)$ remains to be defined. Towards this purpose, given some constant $M > 0$, we let $n(t)$ be the number of attractors of the network at time t, and $n_{min}(t)$ and $n_{max}(t)$ be the minimal and maximal number of attractors that the network has encountered during the last M time steps:

$$\begin{aligned} n(t) &= \text{number of attractors of the network at time } t \\ n_{min}(t) &= \min\{n(t') : \max(0, t-M) < t' \leq t\} \\ n_{max}(t) &= \max\{n(t') : \max(0, t-M) < t' \leq t\}. \end{aligned} \tag{2}$$

The constant M is called the *memory* of the network. It corresponds to the time window during which the network "remembers" the minimum and maximum number of attractors that it has encountered.

The *adaptive learning rate* $\lambda(t)$ is then defined as the image of $n(t)$ by the linear interpolation between the two points $(n_{min}(t), \lambda_{max})$ and $(n_{max}(t), \lambda_{min})$, where $\lambda_{min}, \lambda_{max} \in \mathbb{R}$ are two bounds such that $\lambda_{min} < \lambda_{max}$. Formally,

$$\lambda(t) = \begin{cases} \lambda_{max} + \dfrac{(n(t) - n_{min}(t))(\lambda_{min} - \lambda_{max})}{n_{max}(t) - n_{min}(t)} & \text{if } n_{min}(t) \neq n_{max}(t) \\ \lambda_{max} & \text{otherwise.} \end{cases} \quad (3)$$

The computation of $\lambda(t)$ is illustrated in Fig. 2. The learning rate $\lambda(t)$ has to be understood as follows. If $n(t) = n_{min}(t)$ (resp. $n(t) = n_{max}(t)$), it means that the current number of attractors of the network is at a minimal (resp. maximal) level. In this case, $\lambda(t) = \lambda_{max}$ (resp. $\lambda(t) = \lambda_{min}$). This large (resp. low) learning rate will induce large (resp. low) variations of the synaptic weights (cf. Eq. 1) with the aim of destabilizing (resp. stabilizing) the network's current dynamics. If $n_{min}(t) < n(t) < n_{max}(t)$, then $\lambda_{max} > \lambda(t) > \lambda_{min}$ according to the linear interpolation. The closer $n(t)$ is to $n_{min}(t)$ (resp. to $n_{max}(t)$), the closer $\lambda(t)$ is to $\lambda_{max}(t)$ (resp. to $\lambda_{min}(t)$). If $n_{min}(t) = n_{max}(t)$, the network has settled into the same attractor dynamics during the M last steps. In this case, we set $\lambda(t) = \lambda_{max}$ with the aim of destabilizing the current dynamics.

Observe that, since $n_{min}(t)$ and $n_{max}(t)$ are functions of the memory M (cf. Eq. 2), then so is $\lambda(t)$ (cf. Eq. 3), and hence so is the STDP rule (cf. Eq. 1). Note also that if the network has no memory, i.e. $M = 1$, then $n_{min}(t) = n_{max}(t)$ (cf. Eq. 2), and thus $\lambda(t) = \lambda_{max}$ for all $t > 0$ (cf. Eq. 3), meaning that the network dynamics is driven by a *fixed-rate* STDP rule. By contrast, as soon as the network has a positive memory, i.e. $M > 1$, the learning rate $\lambda(t)$ becomes time dependent, meaning that the network dynamics is driven by an *adaptive* STDP rule. This *adaptive* feature is crucial towards the achievement of reaching a high and stable attractor-based complexity.

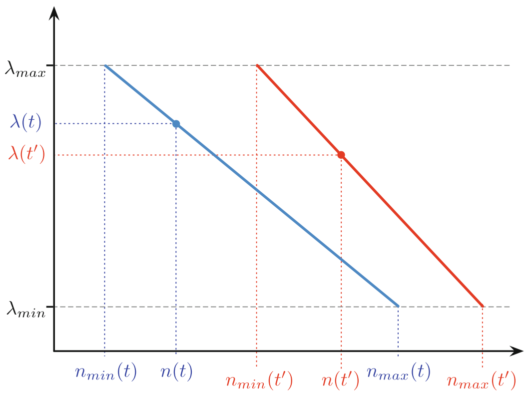

Fig. 2. Computation of the adaptive learning rate $\lambda(.)$ at two different time steps t (blue construction) and t' (red construction). The rate $\lambda(.)$ is defined as the image of $n(.)$ by the linear interpolation between the two points $(n_{min}(.), \lambda_{max})$ and $(n_{max}(.), \lambda_{min})$. (Color figure online)

4 Results

We now study the effect of the adaptive STDP rule on the attractor-based complexity of the BGT network. For this purpose, we implemented the adaptive STDP rule of Eq. 1 for the Boolean BGT network of Fig. 1. The learning interval of each weight a_{ij} of Table 1 was set to $I_{ij} = [a_{ij} - 0.025; a_{ij} + 0.8]$. The bounds of the intervals I_{ij} were chosen on the basis of an empirical analysis. The minimal and maximal learning rates were set to $\lambda_{min} = 0.002$ and $\lambda_{max} = 0.12$. We then performed simulations where we first jittered (each weight of) the matrix of Table 1 by random uniform noise $\epsilon_{ij} \sim \mathcal{U}(-0.025, 0.8)$, and then submitted the network to a random input stream and recorded the variation of its attractor-based complexity throughout its computational process.

In order to emphasise the effect of the network memory on its attractor-based complexity, we performed 10 simulations (of 300 time steps each) where memory $M = 1$, 10 simulations where memory $M = 120$ and 10 simulations where memory $M = 240$. For each lot of 10 simulations, we used the same seed to ensure that the same random jittering and random input streams were considered at each time, and therefore, that the differences observed are entirely due to the variations M. The results are displayed in Fig. 3.

Recall that $M = 1$ means that the network has non memory and the STDP rule is fixed-rate rather than adaptive (cf. Sect. 3). In this case of $M = 1$ (black dotted trace), the attractor-based complexity is usually unstable, with sporadic peaks of higher intensities interspersed by plateaus of lower values. This situation is particularly manifest in simulations 1, 3, 4, 8. Simulations 2, 5. 9, 10 are less peaky, but still unstable. Simulations 6 and 7 are by contrast very stable, with long plateaus of 10 and 1 attractors, respectively. The highest peak of complexity is reached at the beginning of simulation 10, with 154 attractors (pay attention to the x-axis of simulation 10).

For $M = 120$ (blue dashed trace), the attractor complexity is clearly more stable, and in general, it doesn't get stuck into minimal values. Note that the length of the plateaus are of the same order as that of the memory, namely 120 time steps. In all simulations, the network is able to maintain a high complexity during a fairly long period of time. In simulations 2, 6, 7, 9, 10 however, the network also stabilizes into plateaus of low complexity. In simulations 1, 3, 8 (to some extent), 9, the complexity is constantly improving along the computation. Simulations 4 and 5 still alternate between stable and unstable behaviors. The highest complexity of 377 is reached in simulation 10, and it is maintained during exactly 120 time steps.

For $M = 240$ (red solid trace), the attractor complexity is even more stable, and it almost never gets stuck into minimal values. Here again, the length of the plateaus are of the same order as the memory length, namely 240 time steps. In all but the 9-th simulations, the network is able to stabilize in a complexity that is higher than for $M = 120$, and for a longer period of time. However, in simulations 6, 7, 9, 10, the network also stabilizes into plateaus of low complexity. Simulation 4 is the only one to still presents some instability, at its beginning. The highest complexity of 377 is reached in simulation 10, and it is maintained during 193 time steps until the end of the simulation (it but would have probably be maintained for a longer period of time if the simulation would have continued). Overall, we see that as M increases, the network becomes more and more able to stabilize into attractor-based complexities of high intensities.

It has been shown tiny decreases in the weights of the three specific connections (Thalamus, STN), (GPe, STN) and CCortex, STN) (from their original values of Table 1) drastically increases the number of attractors of the BGT network from 22 to 143 [5,6]. Therefore, it is rational to think that a targeted modification of these weights by the adaptive STDP rule might drive the network dynamics into a higher attractor complexity. This hypothesis is explored by implementing a larger decrease-update exclusively for those specific connection strengths. Formally, the value of constant $C = 5$ in Eq. (1) was set to 5 for these connections and kept to its default value of 1 for other connections. The effect of this *targeted adaptive STDP rule* on the attractor-based complexity of the network is illustrated in Fig. 4.

In this case, the attractor-based complexity of the network is indeed drastically higher by few orders of magnitude, but the stabilization process associated with the increase of M has deteriorated. For $M = 1$ (black dotted trace), the complexity is highly unstable, except in simulations 5, 7, 8, where the network gets trapped into a minimal complexity of 1. The highest complexity of 1170 attractors is reached at the beginning of simulation 9. For $M = 120$, the complexity is clearly more stable than for $M = 1$, but the stabilization is not as clear as it was for the previous case of Fig. 3. We less systematically see plateaus of stability that are of the same order as the memory length of 120 time steps. This situation nevertheless occurs in simulations 3 (two plateaus of 25 and 42 attractors of 120 time steps). in simulations 6 (two plateaus of 89 and 198 attractors of 120 and 121 time steps) and in simulation 8(two plateaus of 32 attractors of durations 123 and 129 time steps). The network also sometimes gets trapped into a minimal complexity of 1, like in simulations 9 and 10. The highest complexity of 1735 attractors is reached at the beginning of simulation 10 and is maintained during 11 time steps. For $M = 240$, the complexity is not significantly more stable than for $M = 120$, and this contrasts with the previous case of Fig. 3. However, except for simulation 1, the network is able to reach complexities that are always equal or higher than for $M = 120$. The network remains trapped into a minimal complexity of 1 in simulations 9 and 10. In simulation 6, the huge complexity of 6126 attractors is reached maintained during 17 time steps.

5 Conclusion

We have proposed a novel adaptive STDP rule which allows the BGT network to improve and stabilize its attractor-based complexity during its computational process. The rule is based on an adaptive learning rate which varies according to the attractor dynamics that the network continuously visits. We have shown that the stability of the attractor complexity tends to increase as the network's memory becomes larger. We have also shown that a targeted adaptive STDP

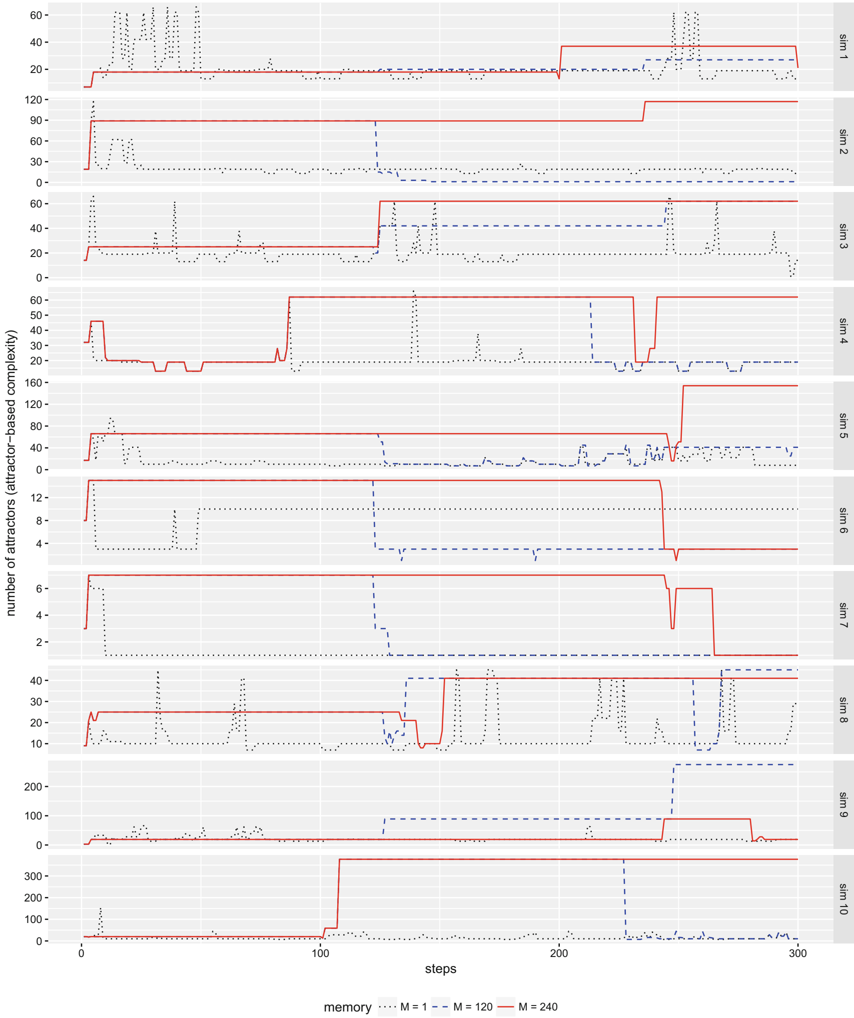

Fig. 3. Results of 10 simulations representing the variations of the attractor-based complexity of the BGT network over time. For each simulation, the weight matrix of the BGT network is initially randomly jittered. Then, the network is subjected to a random input stream and its attractor based complexity computed at each time step. The results for the network memory $M = 1, 120, 240$ are represented.

rule is able to drastically increase the complexity of the network, but at the price of a less stable attractor dynamics.

For future work, the relationship between the synaptic patterns and the attractor dynamics of neural networks is envisioned to be studied in more general architectures, beyond the case study represented by the Boolean BGT network.

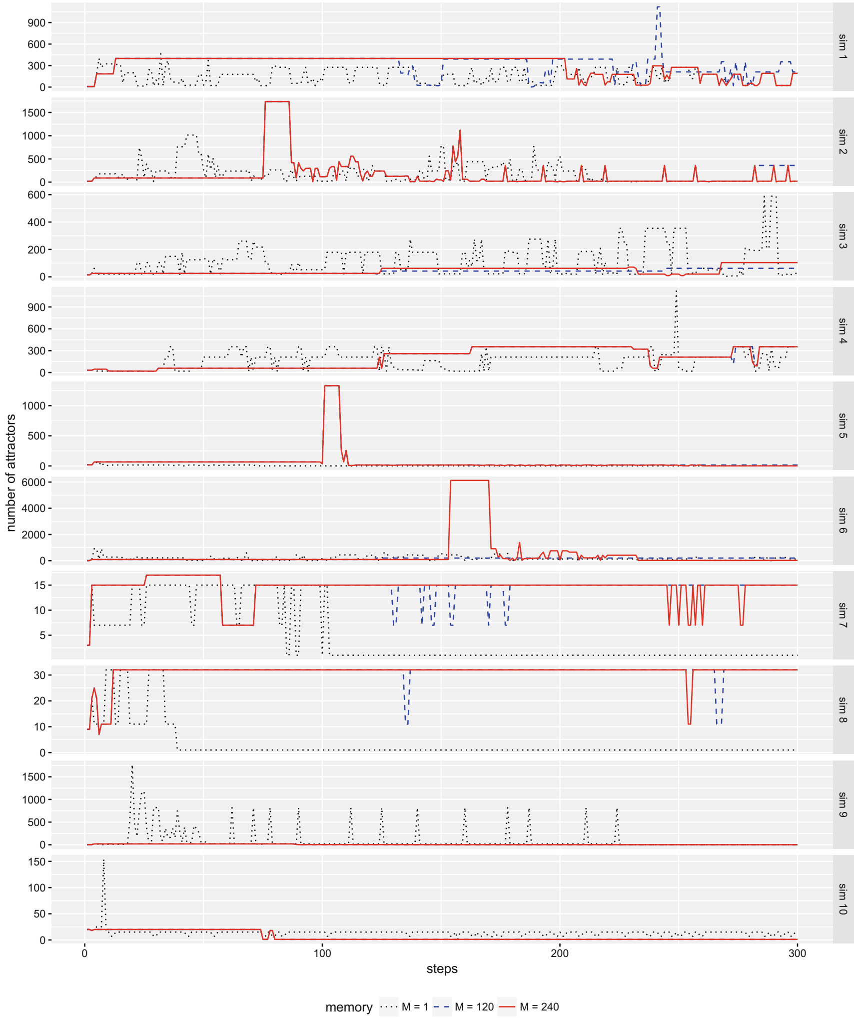

Fig. 4. Results of 10 simulations representing the variations of the attractor-based complexity of the BGT network over time. In this case, the network is subjected to a targeted adaptive STDP rule where constant $C = 5$ for the three weights (Thalamus, STN), (GPe, STN) and CCortex, STN) and $C = 1$ for all other weights (cf. Eq. 1). The results for the network memory $M = 1, 120, 240$ are represented.

References

1. Alexander, G.E., Crutcher, M.D.: Functional architecture of basal ganglia circuits: neural substrates of parallel processing. Trends Neurosci. **13**(7), 266–271 (1990)
2. Beiser, D.G., Hua, S.E., Houk, J.C.: Network models of the basal ganglia. Curr. Opin. Neurobiol. **7**(2), 185–90 (1997)

3. Cabessa, J., Villa, A.E.P.: An attractor-based complexity measurement for boolean recurrent neural networks. PLoS ONE **9**(4), e94204+ (2014)
4. Cabessa, J., Villa, A.E.P.: Attractor-based complexity of a boolean model of the basal ganglia-thalamocortical network. In: 2016 International Joint Conference on Neural Networks, IJCNN 2016, Vancouver, BC, Canada, 24–29 July 2016, pp. 4664–4671. IEEE (2016)
5. Cabessa, J., Villa, A.E.P.: Attractor dynamics driven by interactivity in boolean recurrent neural networks. In: Villa, A.E.P., Masulli, P., Pons Rivero, A.J. (eds.) ICANN 2016. LNCS, vol. 9886, pp. 115–122. Springer, Cham (2016). https://doi.org/10.1007/978-3-319-44778-0_14
6. Cabessa, J., Villa, A.E.P.: Interactive control of computational power in a model of the basal ganglia-thalamocortical circuit by a supervised attractor-based learning procedure. In: Lintas, A., Rovetta, S., Verschure, P.F.M.J., Villa, A.E.P. (eds.) ICANN 2017. LNCS, vol. 10613, pp. 334–342. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-68600-4_39
7. Hoover, J.E., Strick, P.L.: Multiple output channels in the basal ganglia. Science **259**(5096), 819–821 (1993)
8. Jones, B.E.: From waking to sleeping: neuronal and chemical substrates. Trends Pharmacol. Sci. **26**(11), 578–586 (2005)
9. Kleene, S.C.: Representation of events in nerve nets and finite automata. In: Shannon, C., McCarthy, J. (eds.) Automata Studies, pp. 3–41. Princeton University Press, Princeton, NJ (1956)
10. Lansner, A., Fransén, E., Sandberg, A.: Cell assembly dynamics in detailed and abstract attractor models of cortical associative memory. Theor. Biosci. **122**(1), 19–36 (2003)
11. Leblois, A., Boraud, T., Meissner, W., Bergman, H., Hansel, D.: Competition between feedback loops underlies normal and pathological dynamics in the basal ganglia. J. Neurosci. **26**(13), 3567–3583 (2006)
12. Minsky, M.L.: Computation: Finite and Infinite Machines. Prentice-Hall Inc., Englewood Cliffs (1967)
13. Nakahara, H., Amari Si, S., Hikosaka, O.: Self-organization in the basal ganglia with modulation of reinforcement signals. Neural Comput. **14**(4), 819–844 (2002)