Attractor-Based Complexity of a Boolean Model of the Basal Ganglia-Thalamocortical Network

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Abstract—The attractor-based complexity of a Boolean neural network is a measure which refers to the ability of the network to perform more or less complicated classification tasks of its inputs via the manifestation of meaningful or spurious attractor dynamics. Here, we study the attractor-based complexity of a Boolean model of the basal ganglia-thalamocortical network. We show that the regulation of the interactive feedback is significantly involved in the maintenance of an optimal level of complexity. We also show that the complexity of the network depends sensitively on the values of its synaptic connections. These considerations support the general rationale that the synaptic plasticity and the interactive architecture play a crucial role in the computational and dynamical capabilities of biological neural networks.

I. INTRODUCTION

In theoretical neural computation, neural networks are generally considered as abstract computing systems and the issue of their computational capabilities is investigated from a theoretical computer scientist perspective [1]. As a consequence, the computational power of various neural models has been shown range from the finite state automata [2]–[4] up to the Turing [5], [6] or to the super-Turing levels [7]–[10]. More recently, the Turing and super-Turing capabilities of recurrent neural networks have been extended to alternative bio-inspired paradigms of computation, like reactive-system-based computation [11]–[16] (i.e., abstract systems working over infinite input streams) or interactive computation [17]–[21] (i.e., abstract systems performing sequential exchange of information with their environment).

In this context, based on biological considerations, Cabessa and Villa initiated the theoretical study of the expressive power of recurrent neural networks from the perspective of their attractor dynamics [14]. They introduced a novel attractorbased measure of complexity for Boolean recurrent neural networks, and studied the complexity of a Boolean model of the basal ganglia-thalamocortical network.

In the present paper, we provide a study of the attractorbased complexity of a more complex Boolean model of the basal ganglia-thalamocortical network, which takes into account the synaptic plasticity by neuromodulators as well as the bio-inspired interactivity of information processing [22]– [26]. We show that the regulation of the interactive feedback is significantly involved in the maintenance of an optimal level of complexity. We also show that the complexity of the network depends sensitively on the values of its synaptic connections. These considerations support the general rationale Alessandro E.P. Villa Department of Information Systems University of Lausanne CH-1015 Lausanne, Switzerland Email: alessandro.villa@unil.ch

that both synaptic plasticity and interactive architecture play a crucial role in the computational and dynamical capabilities of biological neural networks.

II. ATTRACTOR-BASED MEASURE OF COMPLEXITY

Cabessa and Villa proposed an attractor-based measure of complexity for Boolean recurrent neural networks [14]. More precisely, it was shown that Boolean recurrent neural networks provided with a classification of their attractors into meaningful and spurious types are computationally equivalent to Muller automata. The attractors of the Boolean network correspond precisely to the cycles in the graph of their corresponding Muller automaton, and, more generally, the possible evolutions of the Boolean network correspond precisely to the possible paths in the graph of their corresponding Muller automaton (cf. Example 1 below). According to this equivalence, the class of Boolean neural networks subjected to infinite input streams recognize precisely the class of so-called ω -regular neural languages. Consequently, the most refined topological classification of ω -regular languages [27] can be transposed from the automaton to the neural network context, and in turn, yields to some hierarchical classification of Boolean neural networks according to their attractor dynamics. This classification naturally induces an attractor-based measure of complexity for Boolean recurrent neural networks. In short, the complexity of a neural network corresponds to the length of a maximal sequence of cycles (C_0, \ldots, C_n) in its corresponding Muller automaton which satisfies the two following properties: firstly, C_i is included in C_{i+1} , for $i = 0, \ldots, n-1$; secondly, C_i and C_{i+1} have opposite meaningfulness, for $i = 0, \ldots, n-1$ (cf. Example 1) [11], [12], [14].

This complexity measure is therefore more related to the dynamics of the networks than to their topology, even if the two features are necessarily related. It notably refers to the ability of the networks to perform more or less complicated classification tasks via the manifestation of meaningful or spurious attractor dynamics [14]–[16].

The following paradigmatic example (taken from [14]) illustrates the computation of the attractor-based complexity of a Boolean neural network.

Example 1. Consider the Boolean recurrent neural network \mathcal{N} given in Figure 1 below. The network has two input cells u_1 and u_2 as well as three internal cells x_1 , x_2 , and x_3 . The dynamics of the network is computed as follows: given the

activation values of the input and internal neurons $(u_j)_{j=1}^M$ and $(x_j)_{j=1}^N$ at time t, the activation values of the internal neurons x_i at time t+1 are updated by the following equation:

$$x_{i}(t+1) = \theta\left(\sum_{j=1}^{3} a_{ij} \cdot x_{j}(t) + \sum_{j=1}^{2} b_{ij} \cdot u_{j}(t) + c_{i}\right),$$

for $i = 1, 2, 3$ (1)

where the a_{ij} , b_{ij} , and c_i are the synaptic weights and bias of the network, and θ is the classical hard-threshold activation function defined by

$$\theta(x) = \begin{cases} 0 & \text{if } x < 1\\ 1 & \text{if } x \ge 1 \end{cases}$$

The activation states of the input units are represented by Boolean vectors of the form $(u_1(t), u_2(t))^T$, and hence, there are $2^2 = 4$ possible input states. The activation states of the internal cells are represented by Boolean vectors of the form $(x_1(t), x_2(t), x_3(t))^T$, and thus, there are $2^3 = 8$ possible states.

We recall that an *attractor* of Boolean neural network is a set of states into which the dynamics of the network could eventually become forever confined. Formally, an attractor of \mathcal{N} is a set of states $A = \{\vec{y_0}, \ldots, \vec{y_k}\} \subseteq \mathbb{B}^3$ such that, for some infinite input stream, the dynamics of \mathcal{N} visits infinitely often every state of A, and no other ones, from some time step onwards. For instance, the following infinite periodic input stream (where the " ω exponent" signifies that the pattern is repeated ad infinitum)

$$s = \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right]^{\omega}$$

induces via Equation 1 the corresponding infinite sequence of states of \mathcal{N}

Hence, the set of states $\{(0,0,0)^T, (1,0,0)^T, (0,1,1)^T\}$ is an attractor of \mathcal{N} , since the dynamics of the networks remains confined in that set of states from time step 1 onwards. In this example, we assume that the set of meaningful and spurious attractors of \mathcal{N} has been established by some criterion, and that the sole meaningful attractor of \mathcal{N} is $A = \{(0,0,0)^T, (1,0,0)^T, (0,1,1)^T\}$, all other ones being spurious.

The Muller automaton $\mathcal{A}_{\mathcal{N}}$ associated to network \mathcal{N} is illustrated in Figure 2. The nodes of the automaton are the states of \mathcal{N} , and there is an edge from node $\vec{s_i}$ to node $\vec{s_j}$ labelled by \vec{u} if and only if the network switches from state $\vec{s_i}$ to state $\vec{s_j}$ when it receives input \vec{u} . According to this construction, the *attractors* of \mathcal{N} correspond precisely to the *cycles* in the graph of $\mathcal{A}_{\mathcal{N}}$ [14]. For instance, the meaningful attractor $A = \{(0,0,0)^T, (1,0,0)^T, (0,1,1)^T\}$ corresponds to the cycle depicted in boldface in Figure 2. More generally, the possible *evolutions* of \mathcal{N} correspond precisely to the possible *paths* in the graph of $\mathcal{A}_{\mathcal{N}}$. According to these considerations, we define each cycle of $\mathcal{A}_{\mathcal{N}}$ as being meaningful or spurious if and only if it corresponds to some meaningful or spurious attractor of \mathcal{N} , respectively.

The attractor-based complexity of the network \mathcal{N} corresponds to the length minus one of a maximal sequence of cycles (C_0, \ldots, C_n) in the graph of corresponding Muller automaton $\mathcal{A}_{\mathcal{N}}$ which satisfies the two following properties [11], [12], [14]:

1. C_i is included in C_{i+1} , for i = 0, ..., n - 1;

2. C_i and C_{i+1} have opposite meaningfulness, for $i = 0, \ldots, n-1$.

(we take the length of the sequence minus one instead of the length in order to count the number of alternations of cycles rather than the number of cycles). In this case, the attractorbased complexity of \mathcal{N} is 3-1=2, since one has the maximal sequence of cycles (C_0, C_1, C_2) given by

$$C_{0} = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$
$$C_{1} = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$
$$C_{2} = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

This sequence of cycles satisfies the above conditions, since $C_0 \subseteq C_1 \subseteq C_2$, and C_0 is a spurious cycles (since it corresponds to some spurious attractor of \mathcal{N}), C_1 is a meaningful one (since it corresponds to the sole meaningful attractor of \mathcal{N}), and C_2 is a spurious one again (since it corresponds to some spurious attractor of \mathcal{N}). Note that every cycle containing C_2 is also spurious, since it corresponds by definition to some spurious attractor of \mathcal{N} , and hence, the sequence (C_0, C_1, C_2) is indeed maximal according to the second condition (the successive alternations between meaningful and spurious cycles).



Fig. 1. A simple first-order Boolean recurrent neural network with two input cells u_1 and u_2 and three internal cells x_1 , x_2 and x_3 .

III. BOOLEAN MODEL OF THE BASAL GANGLIA-THALAMOCORTICAL NETWORK

We consider one of the main systems of the brain which is involved in information processing: the basal gangliathalamocortical network. This network has been investigated for many years, in particular in relation to disorders of the motor system and of the sleep-waking cycle, see for instance [28]– [39]. More generally, we assume that the encoding of a large amount of the information treated by the brain is performed



Fig. 2. The Muller automaton $\mathcal{A}_{\mathcal{N}}$ associated to the Boolean recurrent neural network \mathcal{N} . The nodes of $\mathcal{A}_{\mathcal{N}}$ are the states of \mathcal{N} , and there is an edge from node $\vec{s_i}$ to node $\vec{s_j}$ labelled by \vec{u} if and only if the network \mathcal{N} switches from state $\vec{s_i}$ to state $\vec{s_j}$ when it receives input \vec{u} . According to this construction, the cycles in the graph of $\mathcal{A}_{\mathcal{N}}$ correspond precisely to the attractors of \mathcal{N} . The boldface cycle corresponds to the sole meaningful attractor of \mathcal{N} . Every other cycle corresponds to some spurious attractor of \mathcal{N} .

by recurrent patterns of activity circulating in the information transmitting system of this network. For this reason, we focus our attention on the complexity of the dynamics that may emerge from that system. We consider a Boolean recurrent neural network model of the information transmitting system of the basal ganglia-thalamocortical network, and study the attractor-based complexity of this network.

The basal ganglia-thalamocortical network is formed by several parallel and segregated circuits involving different areas of the cerebral cortex, striatum, pallidum, thalamus, subthalamic nucleus and midbrain [40]–[49]. A characteristic of all the circuits of the basal ganglia-thalamocortical network is a combination of "open" and "closed" loops with ascending sensory afferences reaching the thalamus and the midbrain, and with descending motor efferences from the midbrain (the tectospinal tract) and the cortex (the corticospinal tract). The pattern of connectivity corresponds to the wealth of data reported in the literature [40]–[49].

We assume that each brain area is formed by a neural network and that the network of brain areas corresponding to the basal ganglia-thalamocortical network can be modeled by a Boolean neural network formed by 9 nodes: superior colliculus (SC), Thalamus, thalamic reticular nucleus (NRT), Cerebral Cortex, the two functional parts (striatopallidal and the striatonigral components) of the striatum (Str), the subthalamic nucleus (STN), the external part of the pallidum (GPe), and the output nuclei of the basal ganglia formed by the GABAergic projection neurons of the intermediate part of the pallidum and of the substantia nigra pars reticulata (GPi/SNR).

We consider the ascending sensory pathway (IN), that reaches SC and the Thalamus. SC sends a projection outside of the system (OUT), to the motor system. Part of this outgoing activity can be recurrently transmitted to the system via the interactive¹ connection int₁. The thalamus sends excitatory connections within the system via the thalamo-pallidal, thalamo-striatal and thalamo-cortical projections. Notice that STN receives also an excitatory projection from the Thalamus. NRT receives excitatory collateral projections from both the thalamo-cortical and cortico-thalamic projections. In turn, NRT sends an inhibitory projection to the Thalamus. The Cerebral Cortex receives also an excitatory input from STN and sends corticofugal projections to the basal ganglia (striatum and STN), to the thalamus, to the midbrain and to the motor system (OUT). Part of the activity sent out to the motor system can be recurrently transmitted to the system via the interactive connection int₂. The only excitatory nucleus of the basal ganglia is STN, that sends projections to the Cerebral Cortex, to GPe and to GPi/SNR. In the striatum (Str) the striatopallidal neurons send inhibitory projections to GPe and the striatonigral neurons send inhibitory projections to GPi/SNR, via the so-called "direct" pathway. The pallidum (GPe) plays a paramount role because it is an inhibitory nucleus, with reciprocal connections back to the striatum and to STN and a downstream inhibitory projection to GPi/SNR via the so-called "indirect" pathway. It is interesting to notice the presence of inhibitory projections that inhibit the inhibitory nuclei within the basal ganglia, thus leading to a kind of "facilitation", but also inhibitory projections that inhibit RTN, that is a major nucleus in regulating the activity of the thalamus. We emphasize the importance of the recurrent connections int₁ and int₂, for they allow to capture the interactive dynamics of the basal ganglia-thalamocortical network, which is assumed to be significantly involved in the processing of information. Our Boolean model of the basal ganglia-thalamocortical network is illustrated in Figure 3 and its connectivity patterns given in Table I.

IV. RESULTS

We study the attractor-based complexity of the Boolean basal ganglia-thalamocortical network as a function of small perturbations of its constitutive connections (i.e., those not associated with any interactivity) as well as larger variations of its interactive connections (int₁ and int₂). Overall, we notice that the regulation of the interactive feedback plays a crucial role in the maintenance of an optimal level of complexity. There is always an optimal region for the interactive weights outside of which the complexity of the network significantly decreases. We also show that the network's complexity depends sensitively on the values of its constitutive connections. Small perturbations of these weighted connections can highly decrease, increase or subtly modify the overall complexity of the network.

More precisely, fist of all, we preformed 1681 simulations to compute the attractor-based complexity of the network as a function of its two interactive weights int_1 and int_2 , where these latter are varying from -2 to 2 by steps of 0.1. The results are reported in Figure 4.

We notice the existence of an optimal region for the values of int_1 and int_2 where the complexity of the network takes a maximal value of 6. Around this optimal region, the complexity significantly falls to 0, 1, 2 or 3. Note that this optimal region is 'continuous', in the sense of forming a well defined block without holes, as opposed to smaller discontinuous blocks of complexity 6 that would be disseminated across the map. Hence, in the 'middle' of this optimal region, for instance around the point $(int_1, int_2) = (-0.5, 0.5)$, the

¹Here, an *interactive connection* refers to some connection that permits an exchange of information between the network and its environment.



Fig. 3. Boolean model of the basal ganglia-thalamocortical network. The network is constituted of 9 different interconnected brain areas, each one represented by a single node in the Boolean neural network model: superior colliculus (SC), Thalamus, thalamic reticular nucleus (NRT), Cerebral Cortex, the striatopallidal and the striatonigral components of the striatum (Str), the subthalamic nucleus (STN), the external part of the pallidum (GPe), and the output nuclei of the basal ganglia formed by the GABAergic projection neurons of the intermediate part of the pallidum and of the substantia nigra pars reticulata (GPi/SNR). We consider also the inputs (IN) from the ascending sensory pathway and the motor outputs (OUT). The excitatory pathways are labeled in blue and the inhibitory ones in orange. Part of the motor outputs can be recurrently transmitted to the system via the two interactive connections int₁ and int₂.

interactive weights int₁ and int₂ would be able to freely vary in a relatively consequent neighborhood without compromizing the optimal complexity of the network. In other words, in the 'middle' of this optimal region, the network's complexity turns out to be relatively robust to some potential changes of the interactive connections in every possible direction. Moreover, note that the point (int₁, int₂) = (0,0), which corresponds to the case where there is no interactivity at all (since both weights are set to 0), is indeed located inside the optimal region, but at the very border of it. Consequently, the absence of interactivity corresponds to some highly unstable situation in the neighborhood of which the network might have its complexity significantly decreased. This observation supports the idea that a certain non-null suitable amount of interactivity is necessary for the obtention of an optimal complexity.

Overall, these considerations show that the values of the interactive connections play a significant role in the maintenance and robustness of an optimal level of complexity.



Fig. 4. Attractor-based complexity of the Boolean basal gangliathalamocortical network as a function of its two interactive weights int₁ and int₂. The weights int₁ and int₂ vary from -2 to 2 by steps of 0.1. The obtained complexities are represented by colored squares of dimension 0.1 × 0.1. The scale of colors is shown at the right side of the plot. We see the existence of an optimal region of complexity 6 surrounded by regions of complexities 0, 1, 2, and 3.

Secondly, we studied the variation of the attractor-based complexity of the network as a function of small perturbations of its constitutive weights. By slightly varying the weights of the networks by ± 0.2 , one could obtain various patterns of complexity with optimal regions ranging from values 0 to 9. For instance, Figure 5 illustrates a case where some small perturbation of the synaptic weights of the network induces a significant increase of its complexity to level 9.

We then studied the role of small perturbations of the constitutive weights in a more precise way. For each weight

Source Target Str-D1 CCortex IN Thalamus RTN GPi/SNr STN GPe Str-D2 Node # Name SC IN 0 SC int₁ Thalamus RTN -1 GPi/SNr -1 -1 -1 STN GPe -1/2 - 1/2 - 1/2 - 1/2 - 1/2 Str-D2 -1 -1/2 Str-D1 -1/2 CCortex 1/2 1/2 1/2 1/2 1/2 1/2 int₂

TABLE I. THE ADJANCENCY MATRIX OF THE BOOLEAN MODEL OF THE BASAL GANGLIA-THALAMOCORTICAL NETWORK.



Attractor-based complexity of the Boolean basal ganglia-Fig. 5. thalamocortical network as a function of its two interactive weights int1 and int₂. The constitutive weights of the network have been slightly modified as follows: (IN,SC) = 1.0, (IN,Thalamus) = 1.1, (SC,Thalamus) = 1.0, (Thalamus,RTN) = 1.0, (Thalamus,STN) = 1.1, (Thalamus,GPe) = 0.9, (Thalamus,Str-D2) = 1.1, (Thalamus,Str-D1) = 0.8, (Thalamus,CCortex) = 0.9, (RTN,Thalamus) = -1.0, (GPi/SNr,SC) = -0.9, (GPi/SNr,Thalamus) = -1.0, (GPi/SNr,RTN) = -0.9, (STN,GPi/SNr) = 2.1, (STN,GPe) = 2.1, (STN, CCortex) = 2.1, (GPe, RTN) = -0.6, (GPe, GPi/SNr) = -0.6,(GPe,GPi/STN) = -0.6, (GPe,Str-D2) = -0.6, (GPe,Str-D1) = -0.7, (Str-D2,GPe) = -1.0, (Str-D1,GPi/SNr) = -0.6, (Str-D1,GPe) = -0.5, (CCortex,SC) = 0.4, (CCortex, Thalamus) = 1.1, (CCortex, RTN) = 0.3, (CCortex, STN) = 0.5, (CCortex,Str-D2) = 0.3, (CCortex,Str-D1) = 0.3. The interactive weights int₁ and int₂ vary from -2 to 2 by steps of 0.1. We see the existence of an optimal region $[0; 0.9] \times [0; 0.9]$ of complexity 9 surrounded by regions of complexities 0, 1 and 2.

w, we perturbed it by ± 0.1 , and, in each case, performed 441 simulations to compute the attractor-based complexity of the corresponding network as a function of its two interactive weights int₁ and int₁. We focused on the heterogeneous region of $[-0.5; 1.5] \times [-0.5; 1.5]$ of Figure 4, where the network's complexity discloses the highest variability. The results are reported in Figure 6. The five following patterns of variation are observed:

1. A small perturbation of the weight has no or almost no consequence on the complexity of the network. This situation is depicted in Figure 6(a). In this case, the weight (Thalamus,GPe) varies from 0.9 to 1.1, and the complexity remains unchanged. The same situation has been observed for variations of 11 of the 30 weights.

2. A small decrease of the weight induces a total extinction of the complexity of the network. This situation is depicted in Figure 6(b). In this case, when the weight (GPi/SNr,Thalamus) is decreased by 0.1, the global complexity of the network falls to zero. The same situation has been observed for variations of 9 of the 30 weights. It notably shows that certain weights' perturbations might have a drastic negative effect on the network's complexity.

3. A small decrease of the weight induces a global decrease of the complexity. This situation is depicted in Figure 6(c). It has been observed for the sole case of the weight (Thalamus,RTN),

and shows that this specific weight's perturbation might induce an overall negative modulation of the network's complexity.

4. A small decrease of the weight induces a global increase of the complexity. This situation is depicted in Figure 6(d). When the weight (GPe,Str-D2) is decreased by 0.1, the optimal complexity of the network turns out to be globally increased from 6 to 9. The same situation has been observed for variations of 4 of the 30 weights.

5. A small decrease of the weight induces an increase of the complexity over certain regions, and a decrease over others. This situation is depicted in Figure 6(e). In this case, when the weight (RTN,Thalamus) is decreased by 0.1, the complexity of the network is rather modified, yet neither globally decreased, nor globally increased. Part of the optimal region is increased from complexity 6 to 7, and other regions are decreased to complexity 0. The same situation has been observed for variations of 3 of the 30 weights. These situations show that certain weights' perturbations might have complex or perhaps more targeted effects on the network's complexity.

Note that the complexity pattern of the network is affected in cases of its original weights being decreased by 0.1; it always remains unchanged when the weights are increased by 0.1.

Overall, the results show that network's complexity depends very sensitively on the variation of its constitutive synaptic weights.

V. DISCUSSION

We studied the attractor-based complexity of a Boolean model of the basal ganglia-thalamocortical network as a function of large and small variations of its interactive and constitutive connections, respectively.

First of all, we noticed the existence of a 'continuous' optimal region for the interactive weights outside of which the complexity of the network significantly decreases. However, we didn't find any situation where the complexity of the network in the absence of interactivity (given by the points (0,0) in Figures 4, 5 and 6) would be increased by the addition of some interactive feedback. In fact, the total absence of interactivity always corresponds to unstable situations where the complexity of the network is indeed maximal, but absolutely not robust to potential variations of interactivity, even minor ones (the points (0,0) in Figures 4, 5 and 6 are always at the border of the optimal region). Consequently, the regulation of the interactive feedback plays an important role in the maintenance and in the robustness of an optimal level of complexity.

Secondly, we showed that the network's complexity depends sensitively on the values of its constitutive connections. Small perturbations of these weighted connections can highly decrease, increase or subtly modify the overall complexity of the network. Therefore, the mechanism of synaptic plasticity does highly influence the complexity of the network.

These considerations support the general rationale that both synaptic plasticity and interactive architectures do play a significant role in the computational and dynamical capabilities of neural networks, and hence, in the processing of information in biological neural networks.



(a) Attractor-based complexity of the Boolean basal gangliathalamocortical network as a function of its two interactive weights int₁ and int₂. The weights int₁ and int₂ now vary from -0.5 to 1.5by steps of 0.1, in order to focus on the more heterogeneous region of complexities. This is a zoom of Figure 4 on this specific region. The same situation is also obtained, for instance, when the weight (Thalamus,GPe) varies from 0.9 to 1.1, by steps of 0.1.



(b) The weight (GPi/SNr,Thalamus) is slightly decreased to -1.1. (c) The weight (Thalamus,RTN) is slightly decreased to 0.9. The complexity of the network totally falls down to zero



(d) The weight (GPe,Str-D2) is slightly decreased to -0.6. The (e) The weight (RTN,Thalamus) is slightly decreased to -1.1. complexity of the network is globally increased. The complexity of the network is subtly modified, neither globally decreased, nor globally increased.

Fig. 6. Attractor-based complexity of the network as a function of small perturbations of its synaptic weights. Five patterns of variation are observed and reported in the subfigures (a)-(e).

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