Computational Capabilities of Recurrent Neural Networks

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Introduction

- We follow the so-called *mind-computer analogy* approach to cognitive science.
- We study the computational capabilities of basic models of recurrent neural networks.
- We show that recurrent neural networks provide a natural model of computation beyond the Turing limits.

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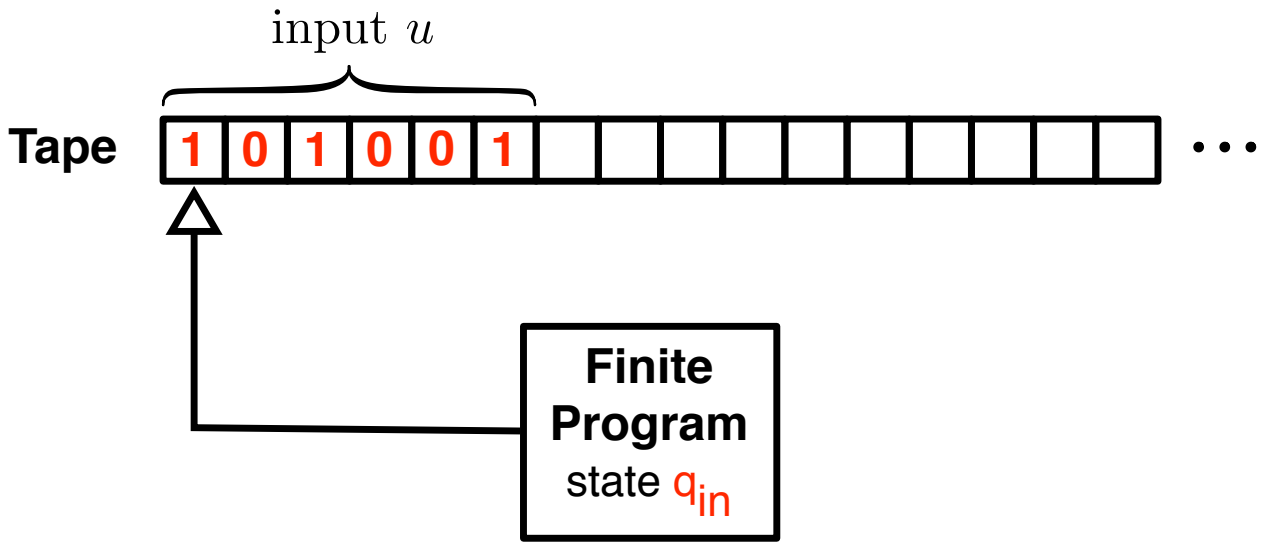

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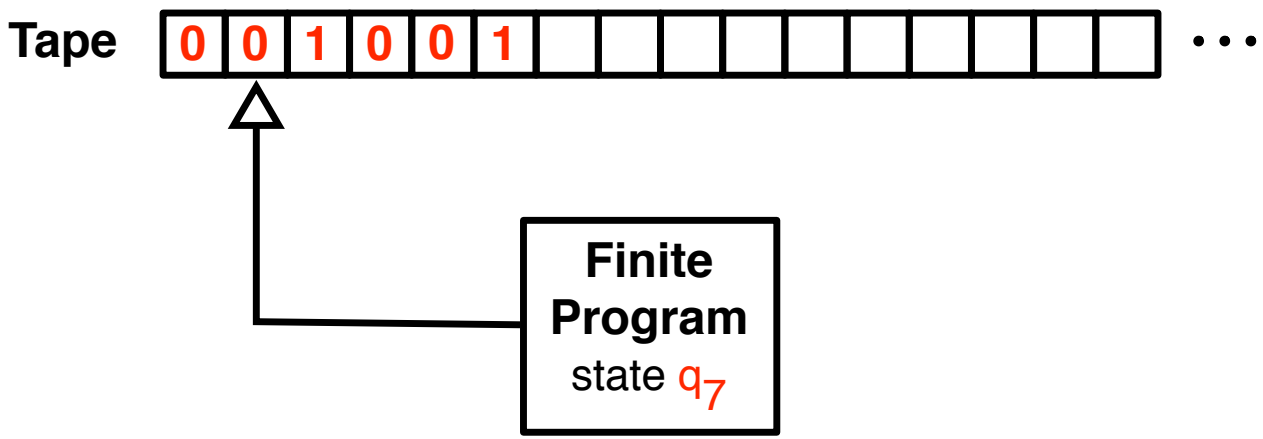

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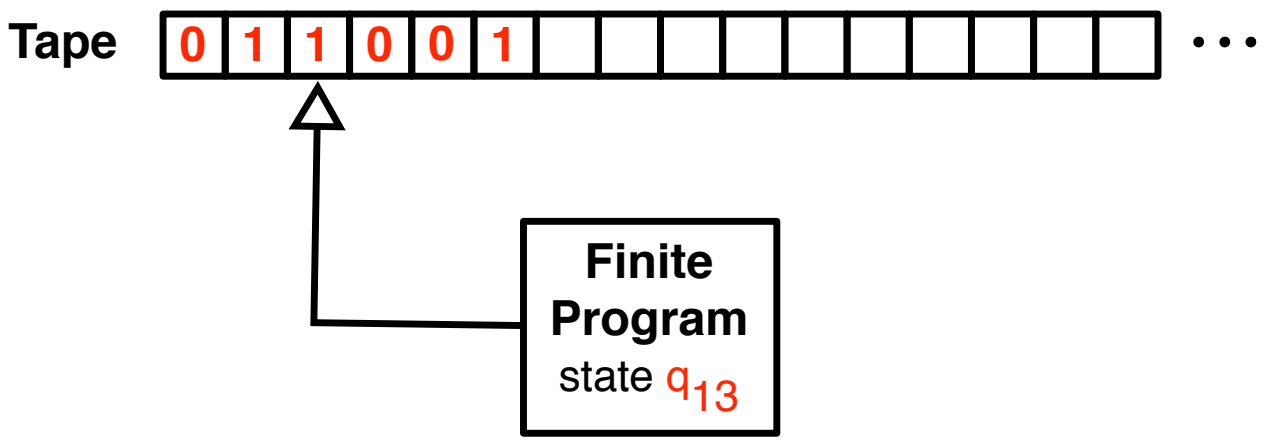

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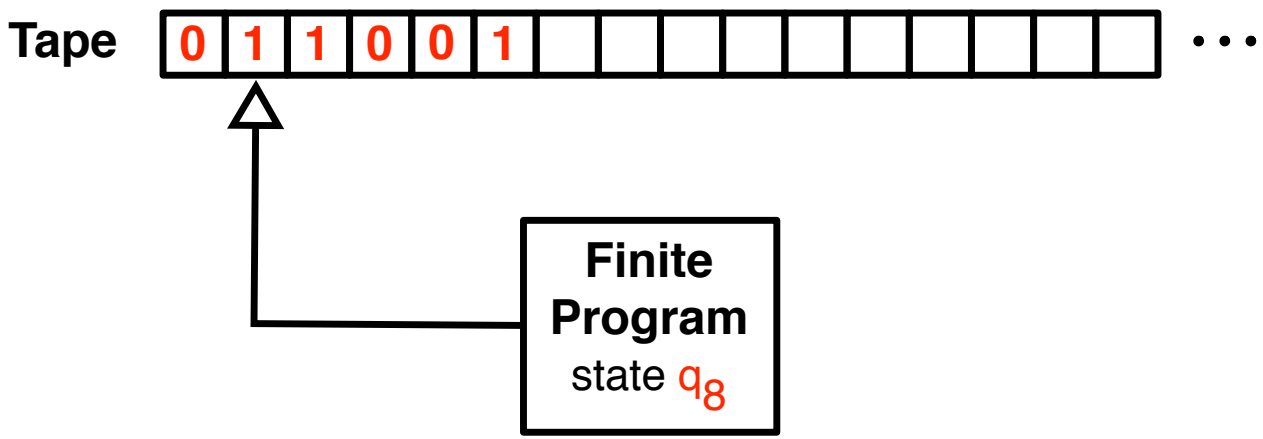

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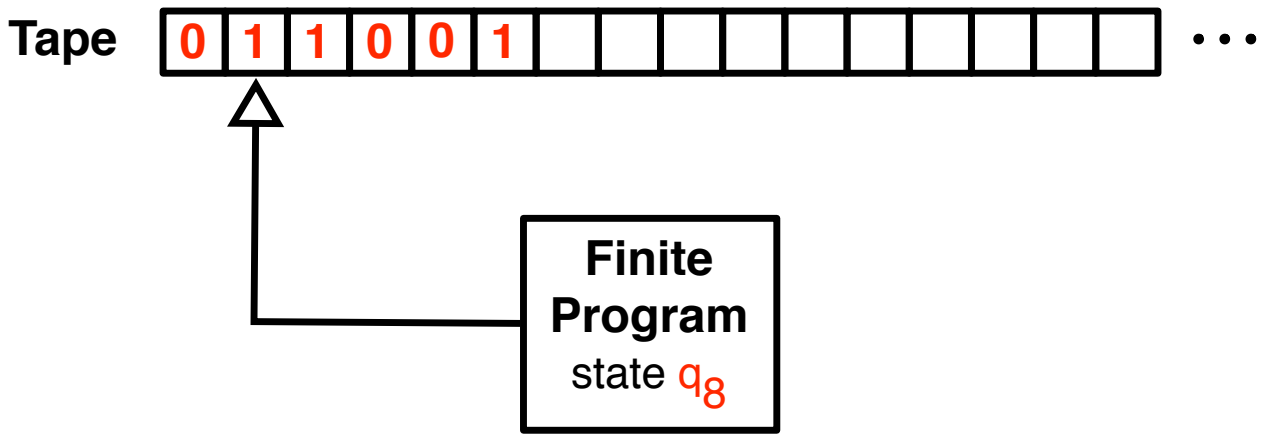

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The Church-Turing thesis

The Church-Turing Thesis states that the Turing machine model is capable of capturing all possible aspects of algorithmic computation.

> *A function is algorithmically computable if and only if it is computable by a Turing machine.*

Recurrent neural networks actually provide a natural model beyond the Turing limits.

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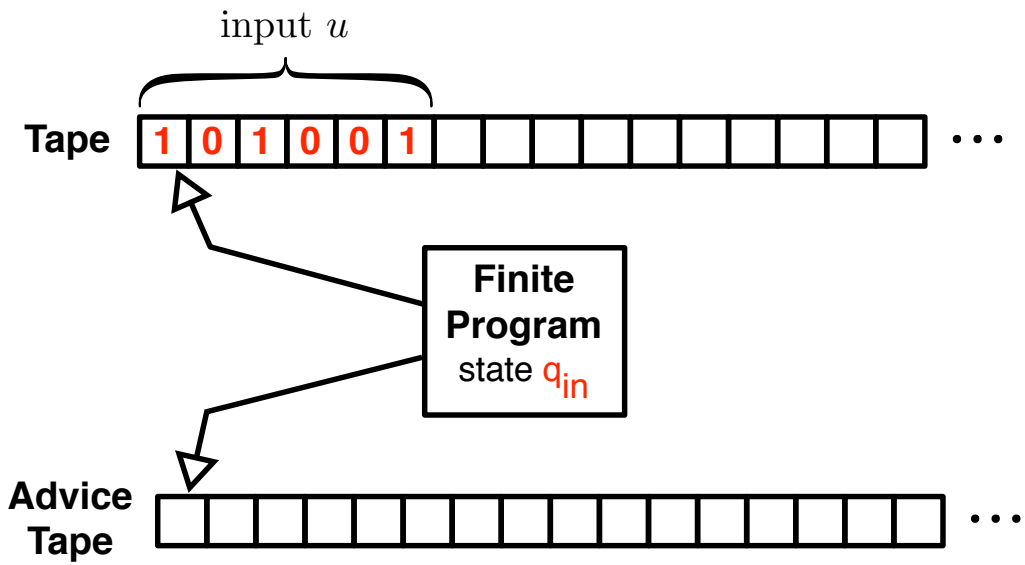

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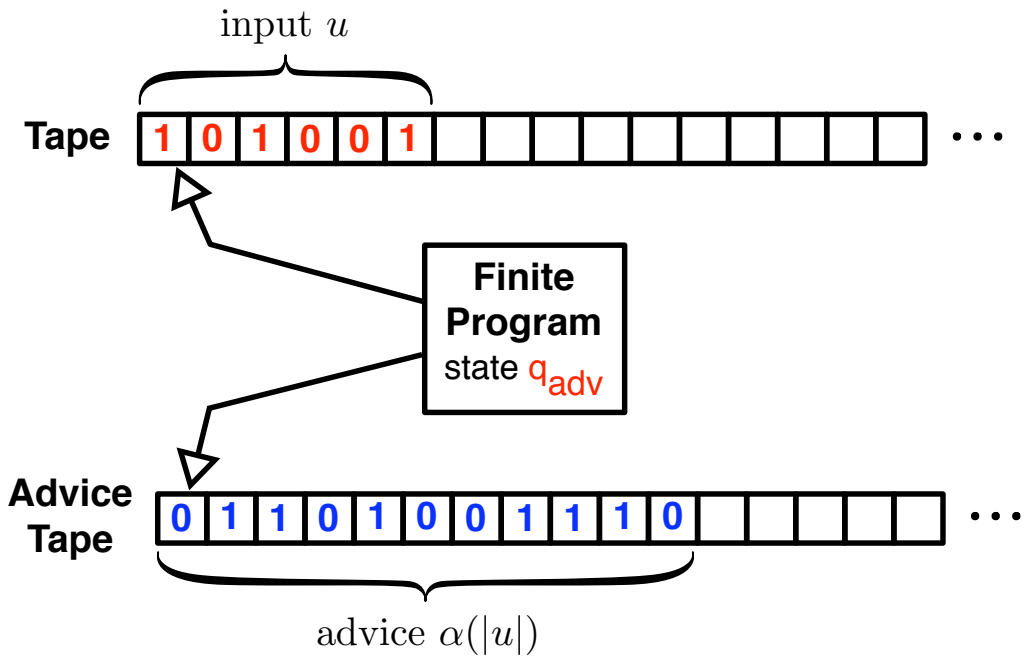

TM vs TM/A

The complexity class $\mathbf{P}$ ($\mathbf{PTIME}$) is the collections of all languages decidable in polynomial time by some TM.

The complexity class $\mathbf{P/poly}$ is the collections of all languages decidable in polynomial time by some TM/poly(A).

Lemma

TM/poly(A)s are strictly more powerful than TMs (super-Turing) already in polynomial time of computation, i.e. $\mathbf{P/poly} \supsetneq \mathbf{P}$.

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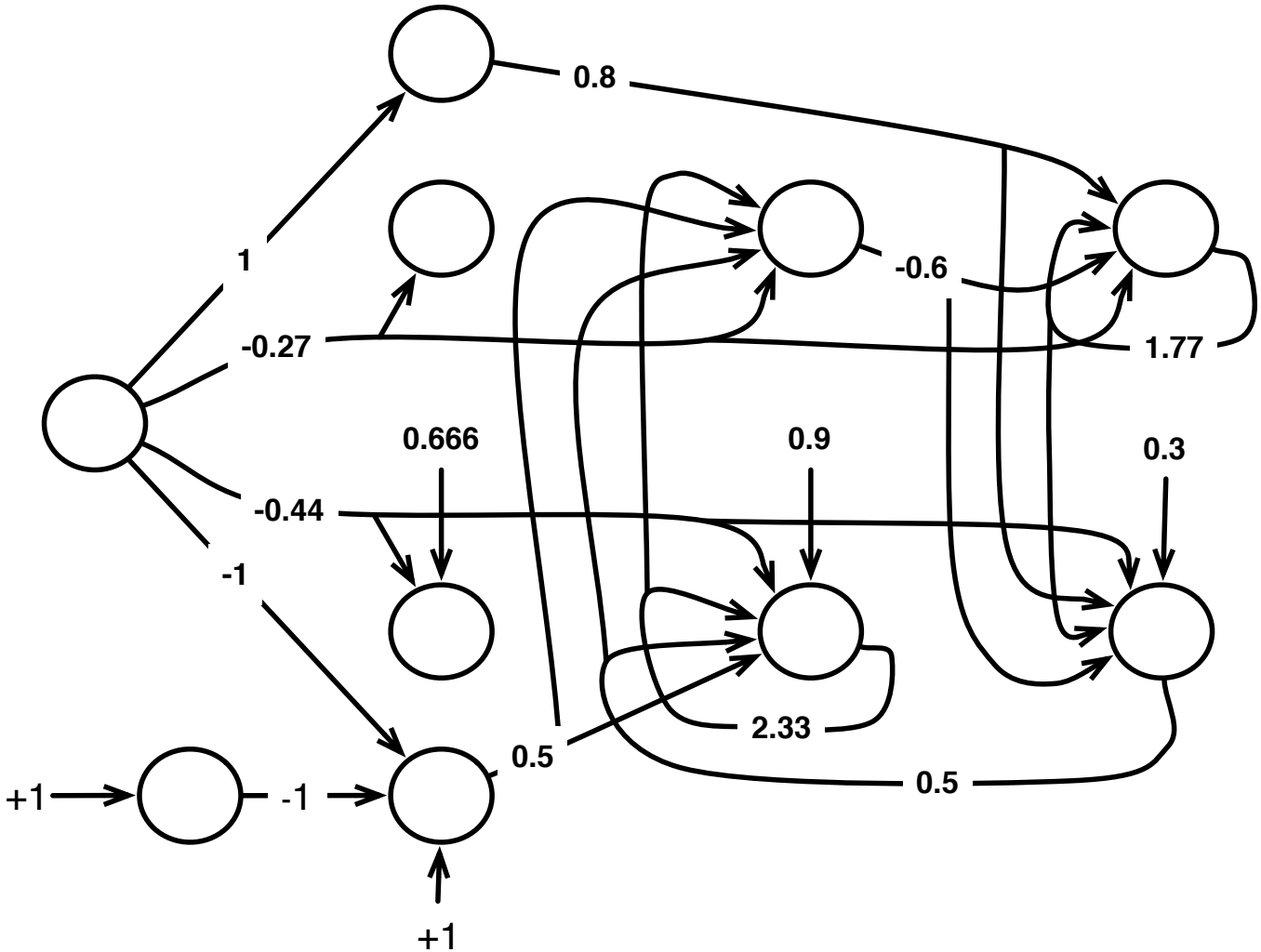
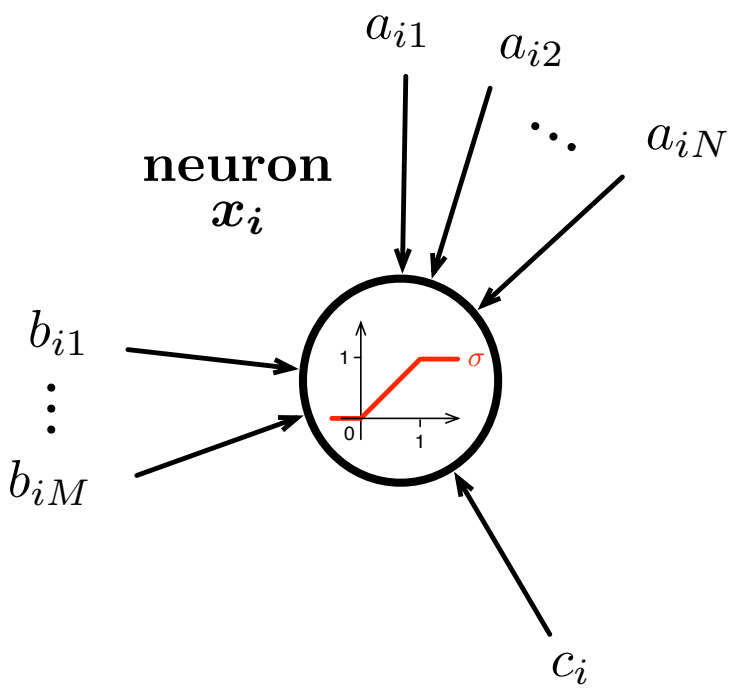
Recurrent neural networks



Dynamics: static synaptic weights



$$x_i(t+1) = \sigma\left(\sum_{j=1}^{N} a_{ij} \cdot x_j(t) + \sum_{j=1}^{M} b_{ij} \cdot u_j(t) + c_i\right)$$

Computational power

RNN[$\mathbb{Q}$]s: networks with static *rational* weights

RNN[$\mathbb{R}$]s: networks with static *real* (analog) weights

Theorem (Siegelmann & Sontag 94, 95)

- *RNN[$\mathbb{Q}$]s are Turing equivalent.*
- *RNN[$\mathbb{R}$]s are super-Turing, i.e.*
  RNN[$\mathbb{R}$]s are equivalent to TM/poly(A) in poly time.

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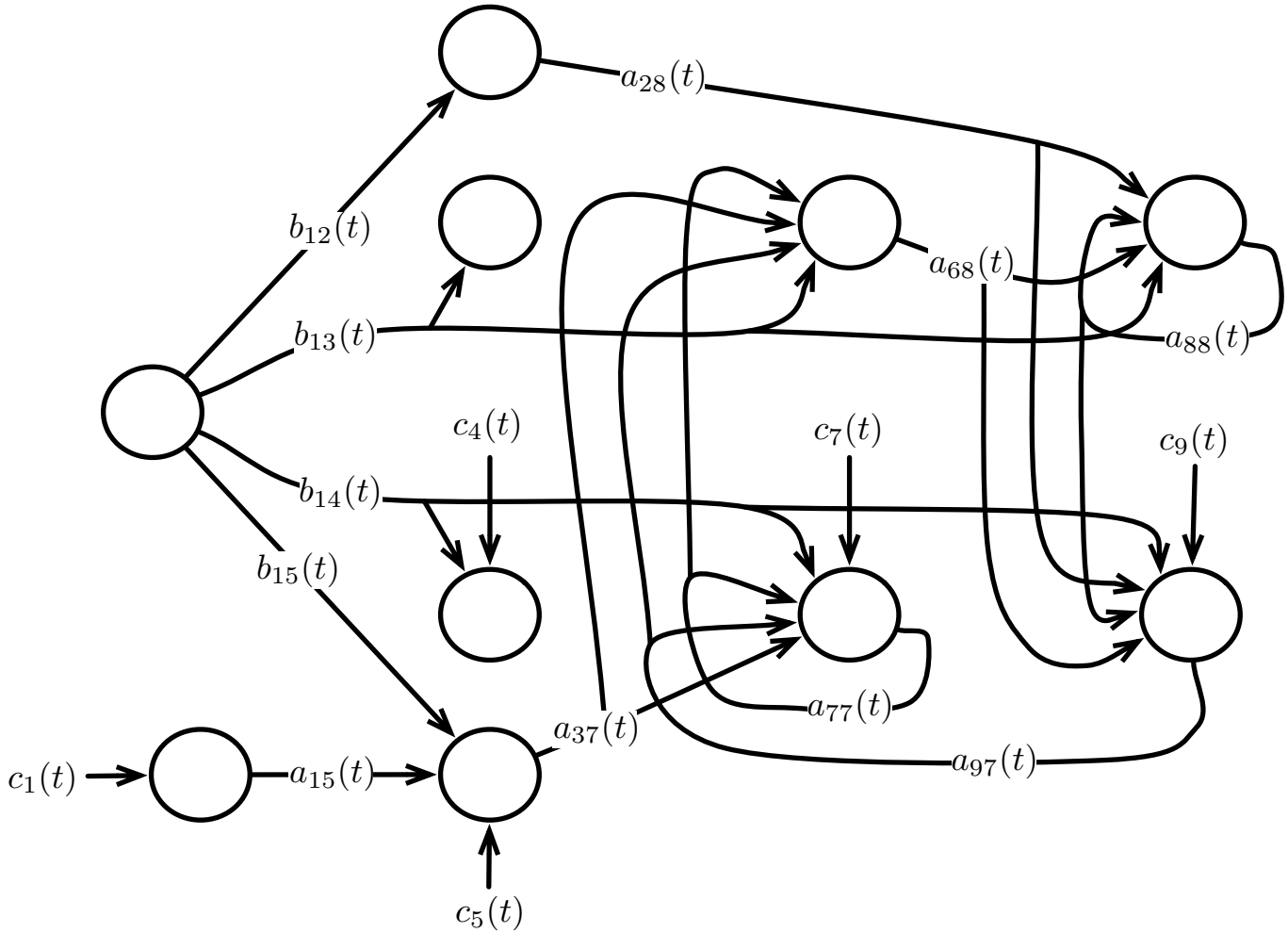
Evolving recurrent neural networks



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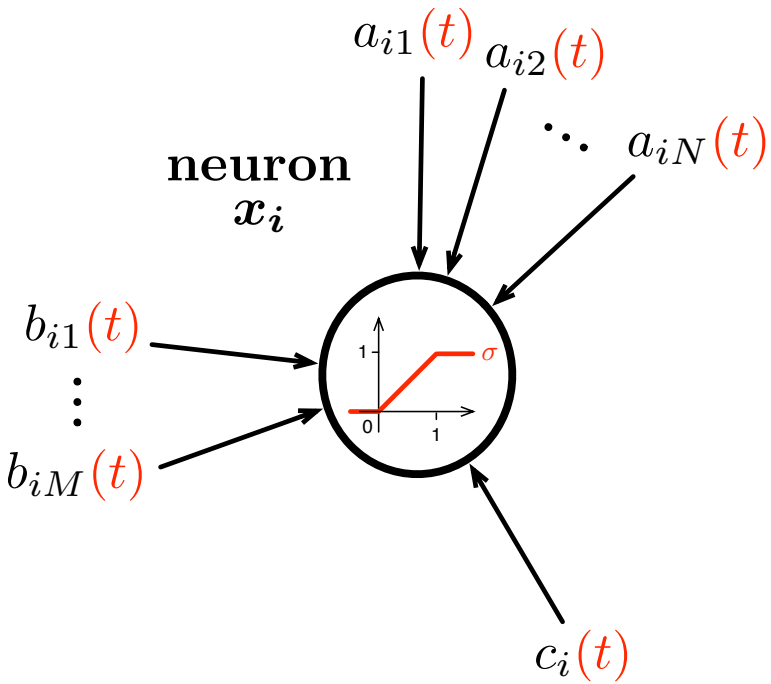

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Summary of the results

	Static	Evolving
$\mathbb{Q}$	Turing Siegelmann & Sontag 95	Super-Turing Cabessa & Siegelmann 11
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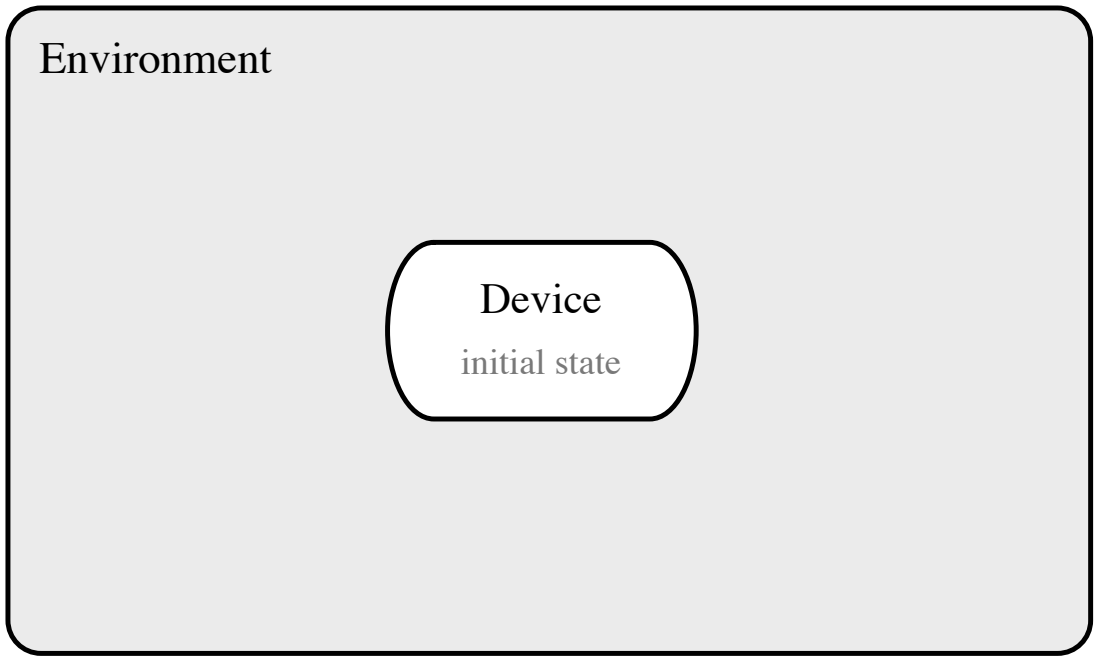
Classical computation



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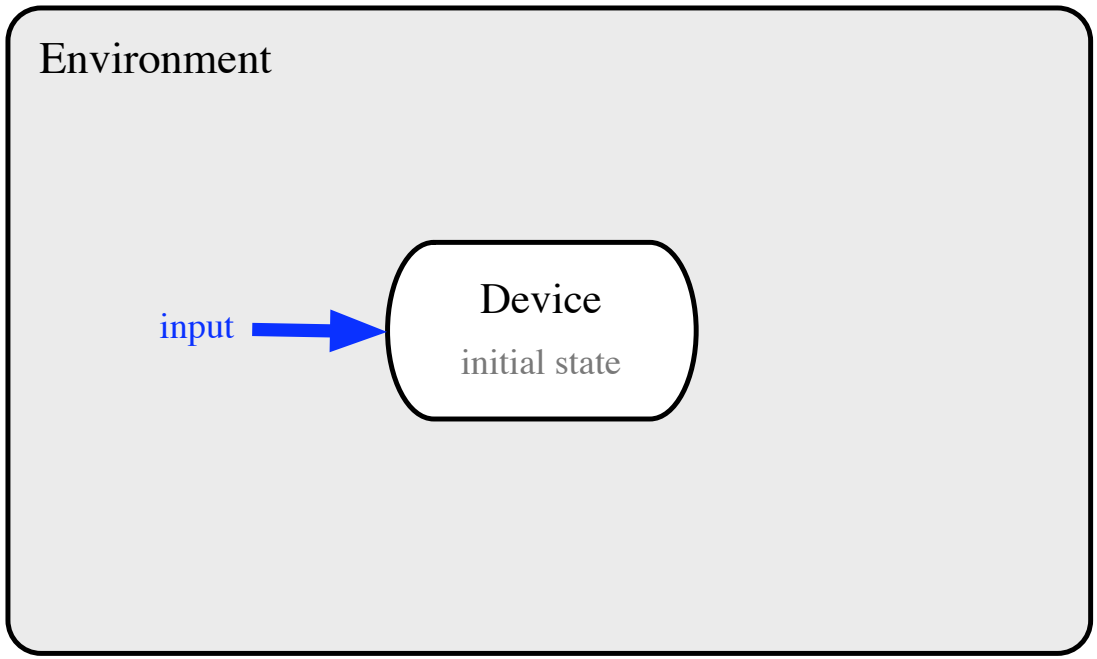

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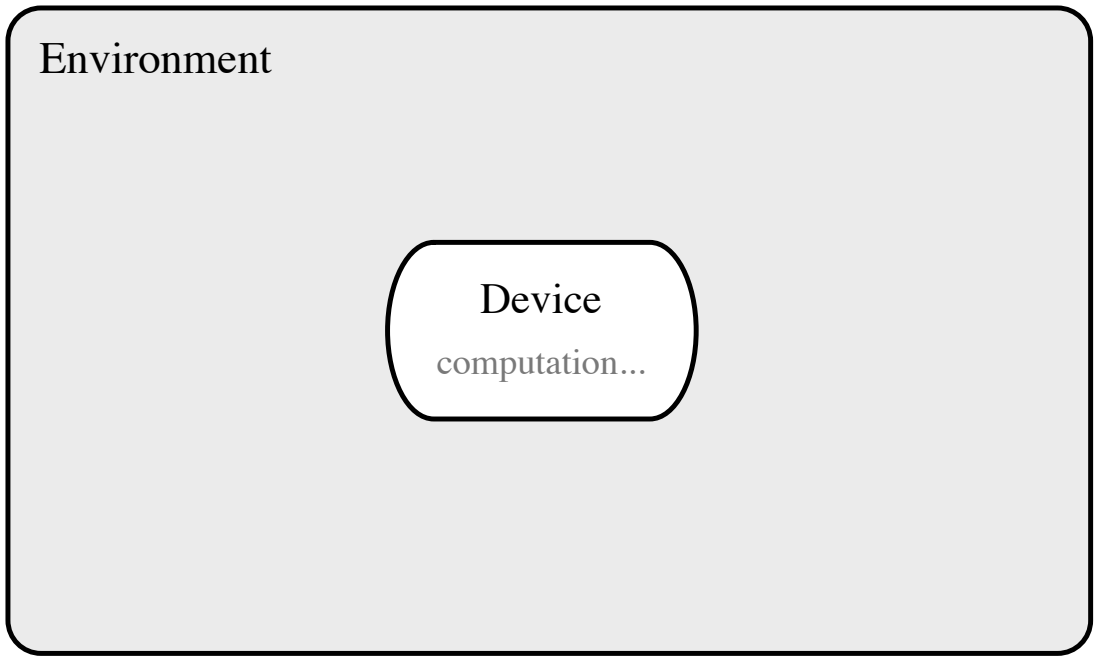

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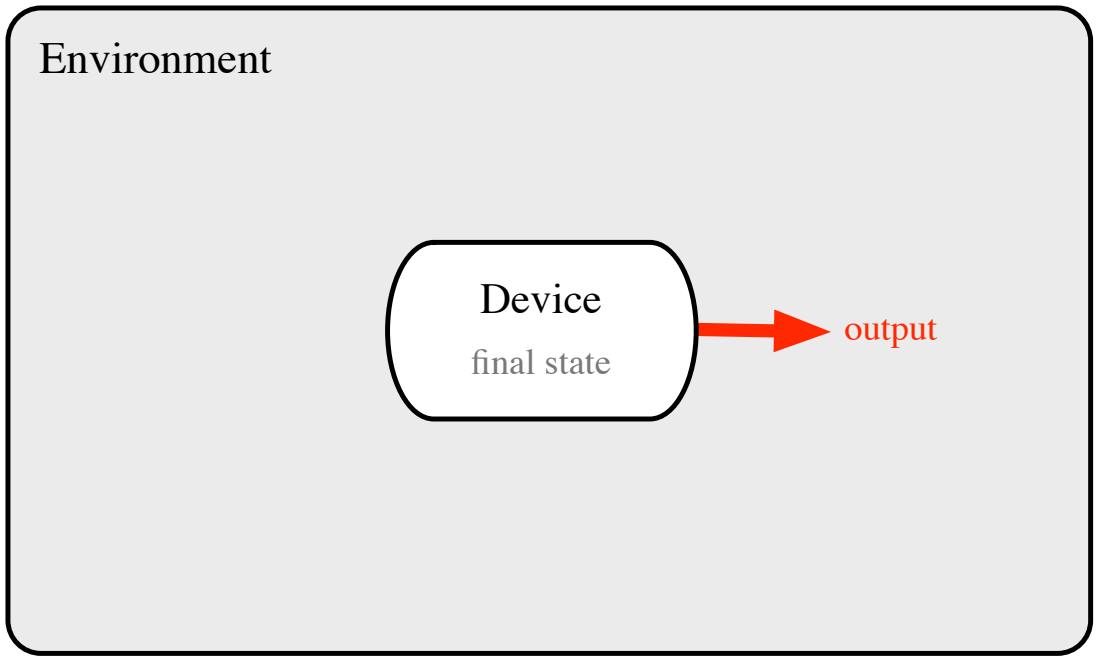

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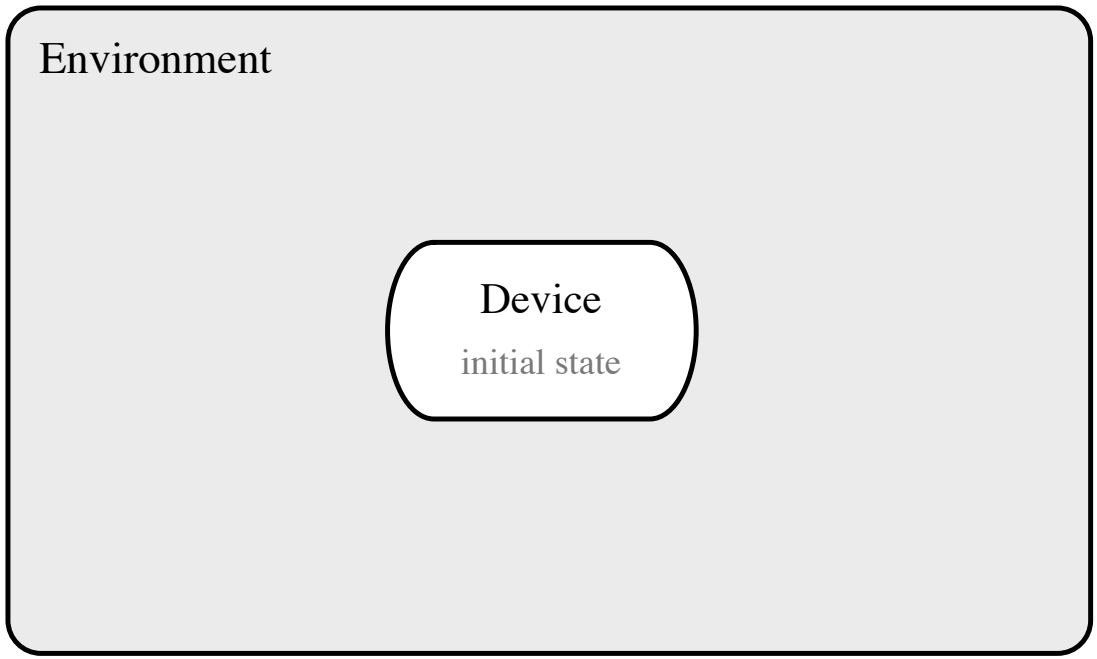

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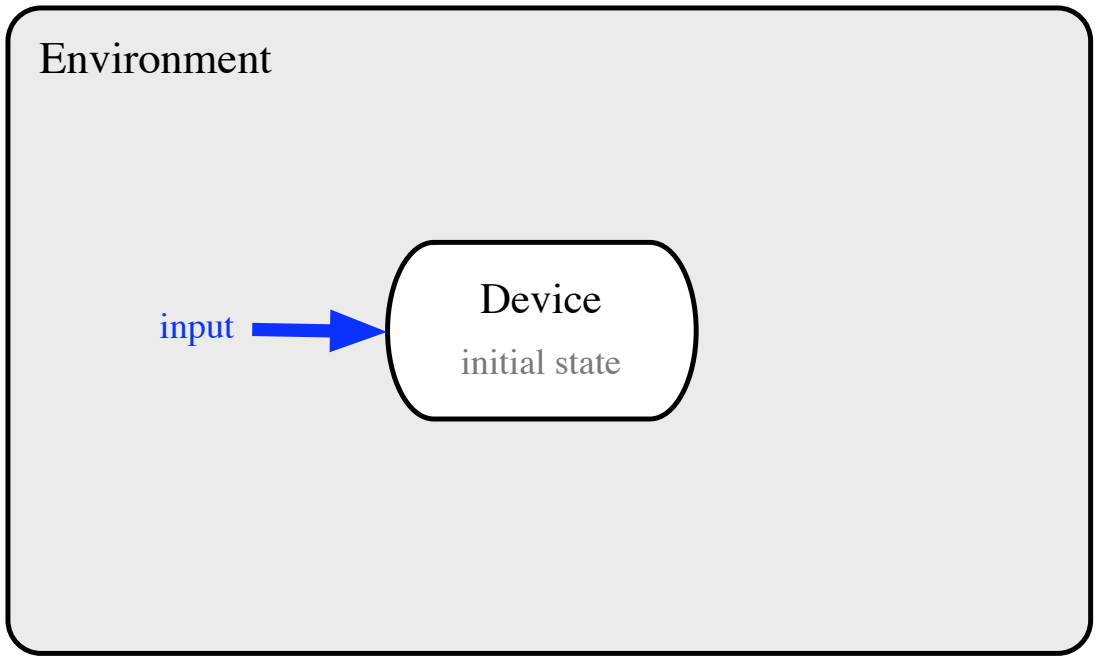

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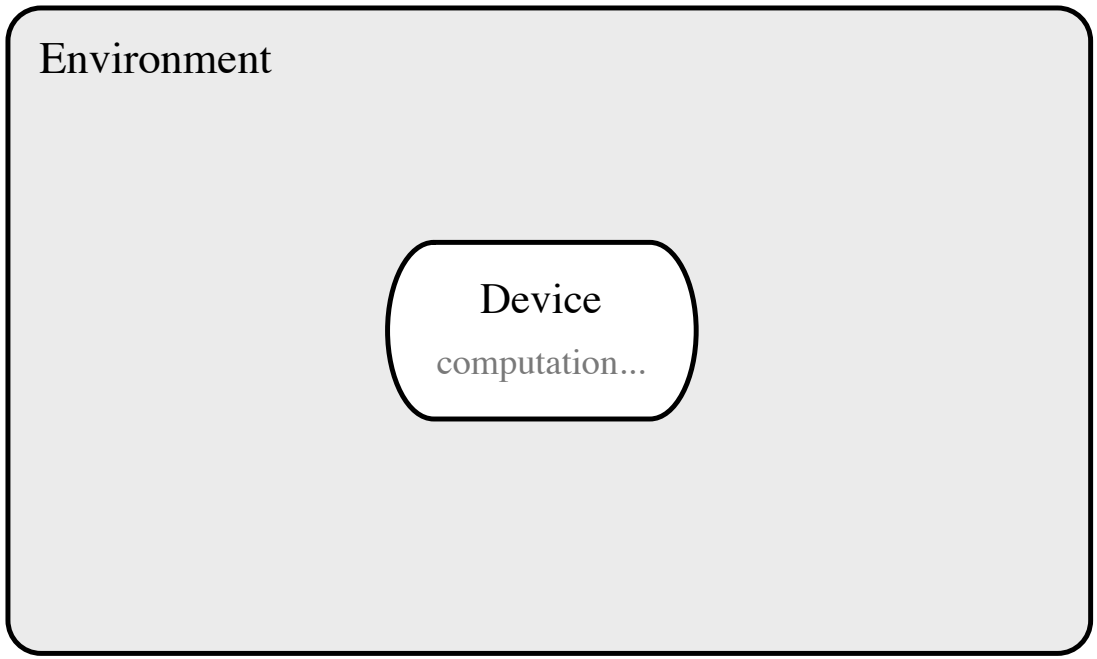

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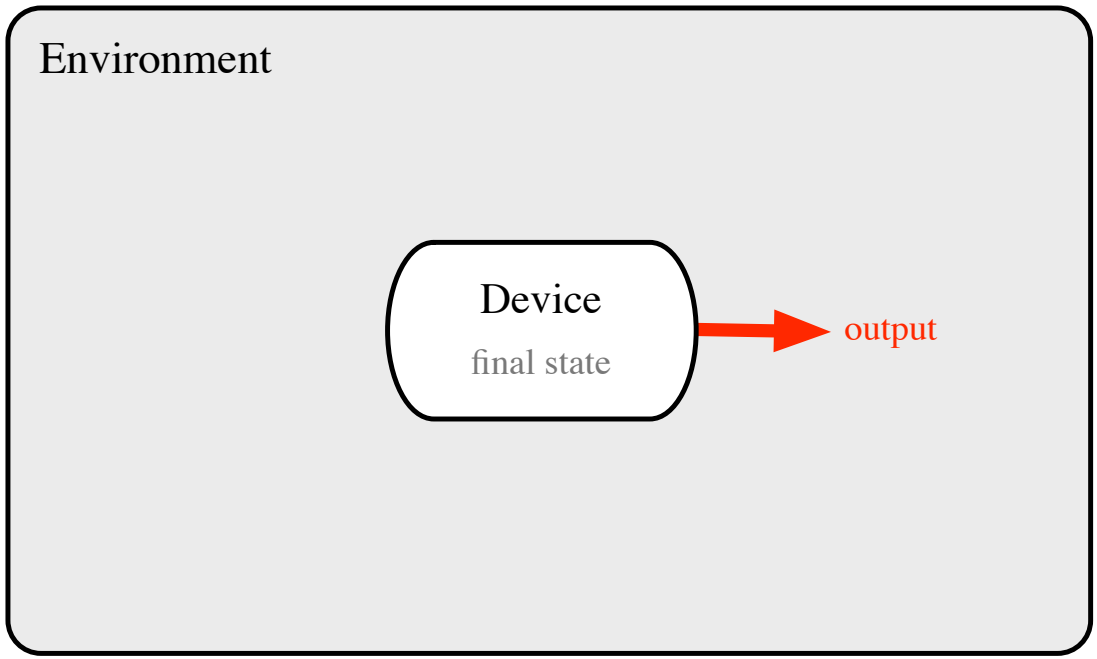

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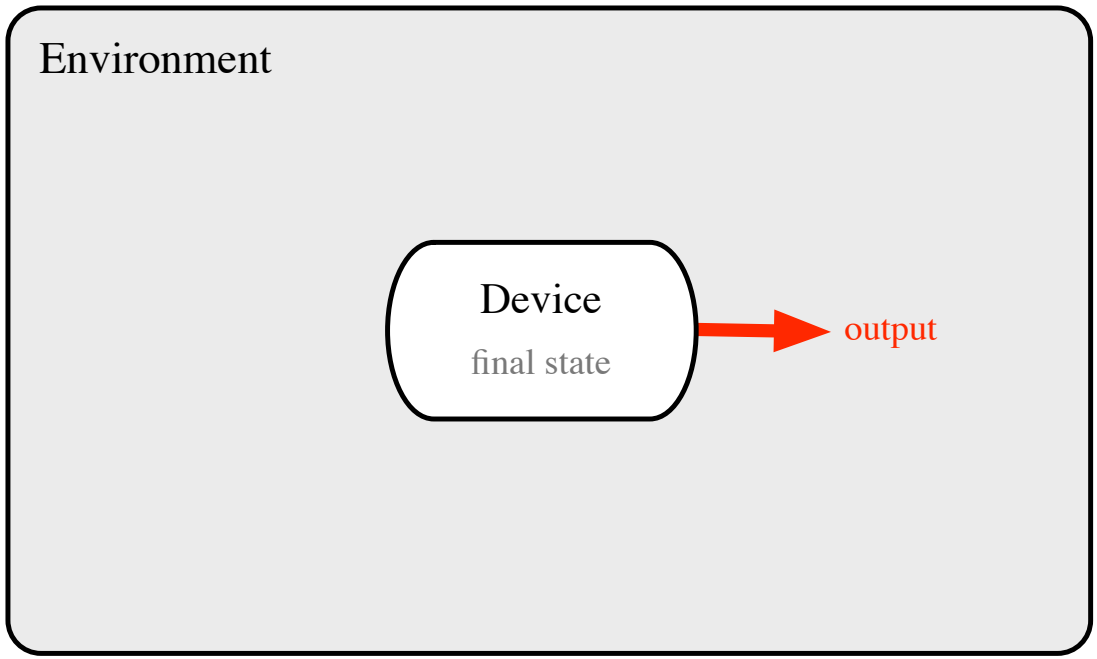

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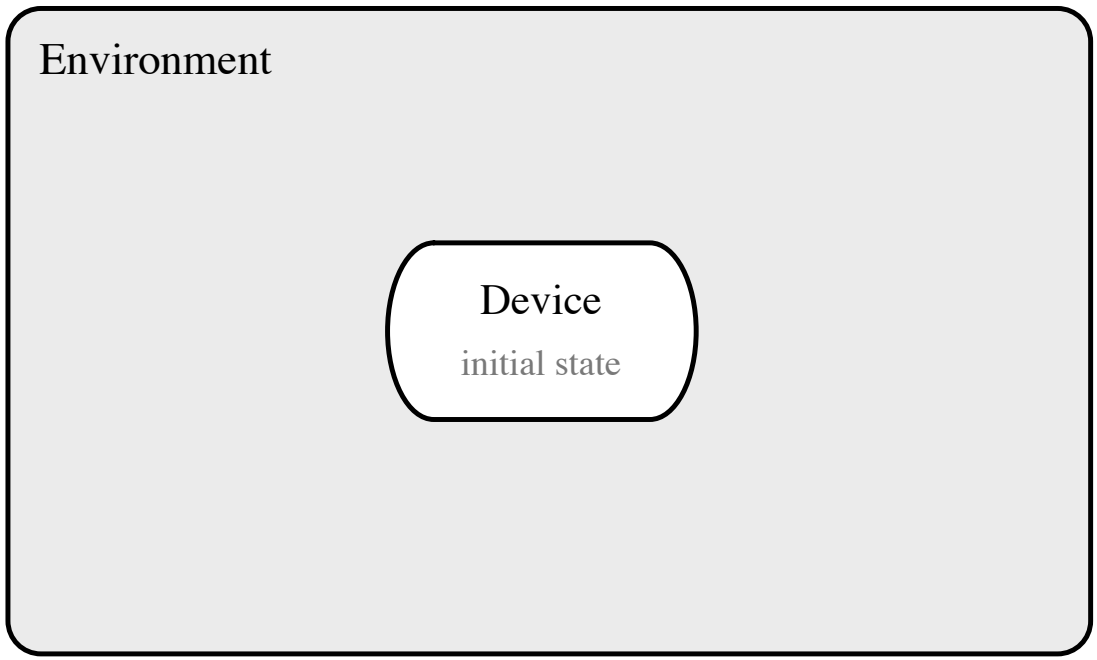

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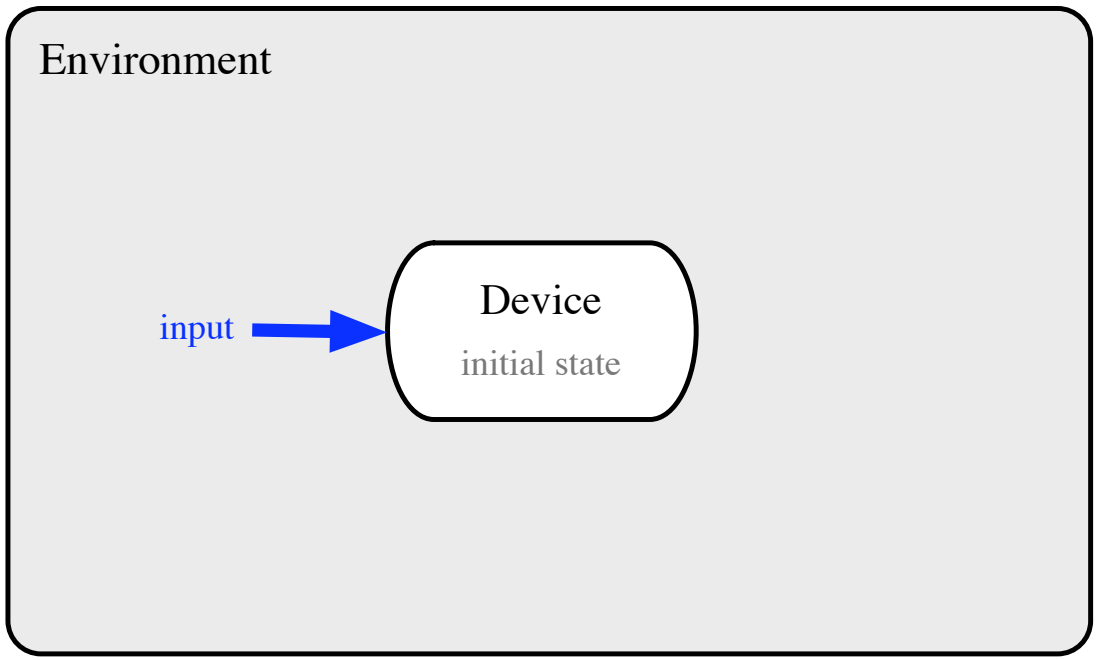

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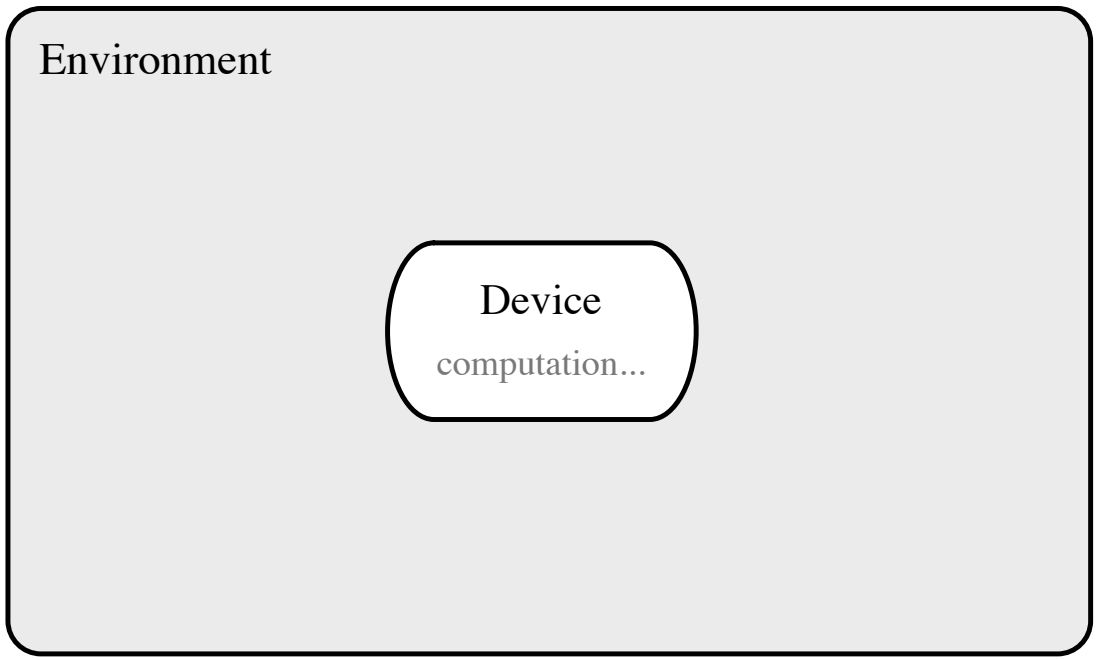

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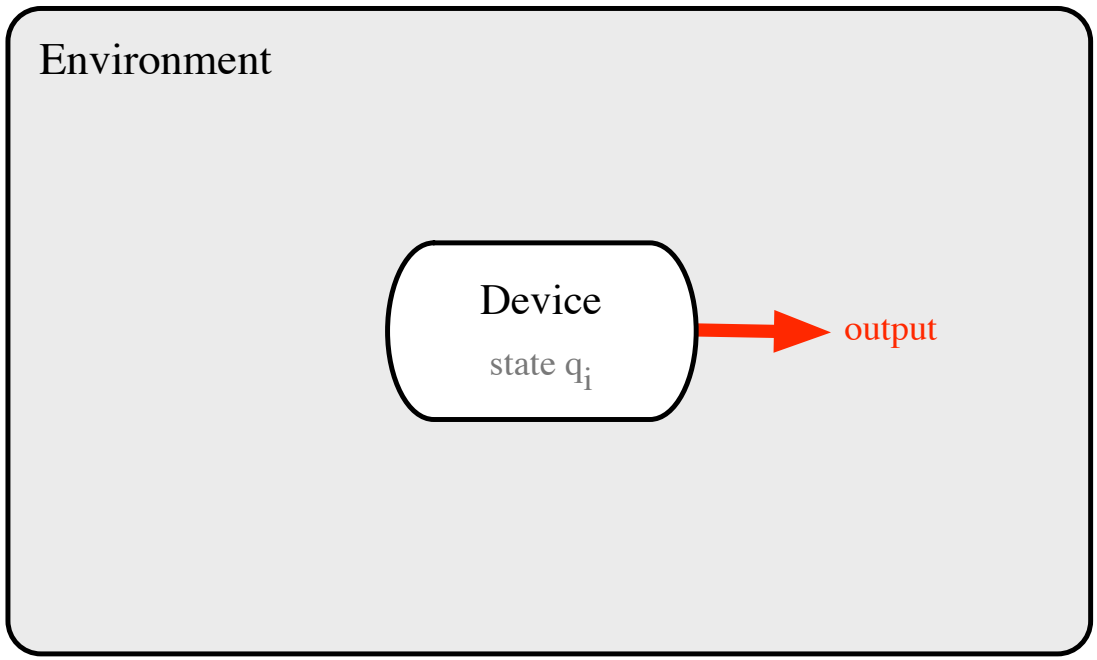

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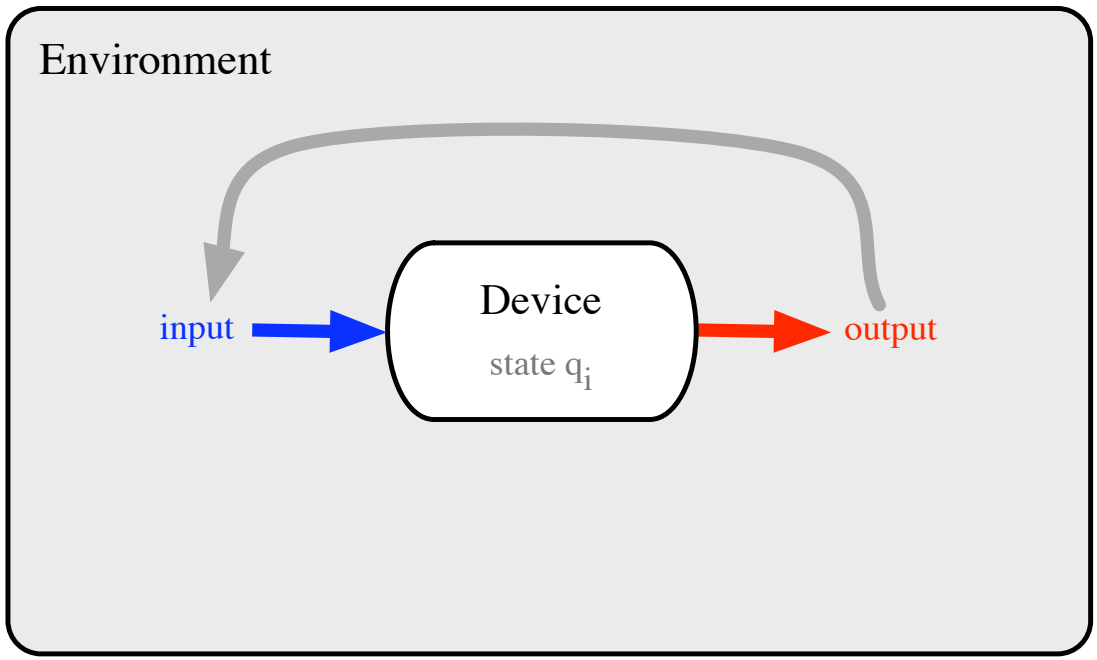

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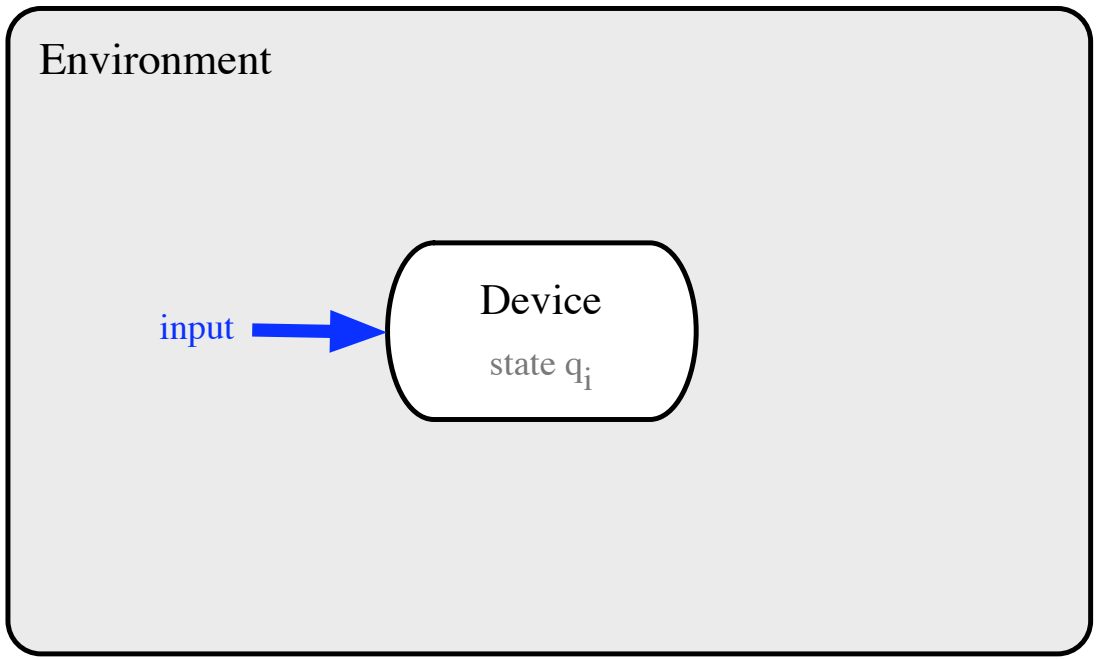

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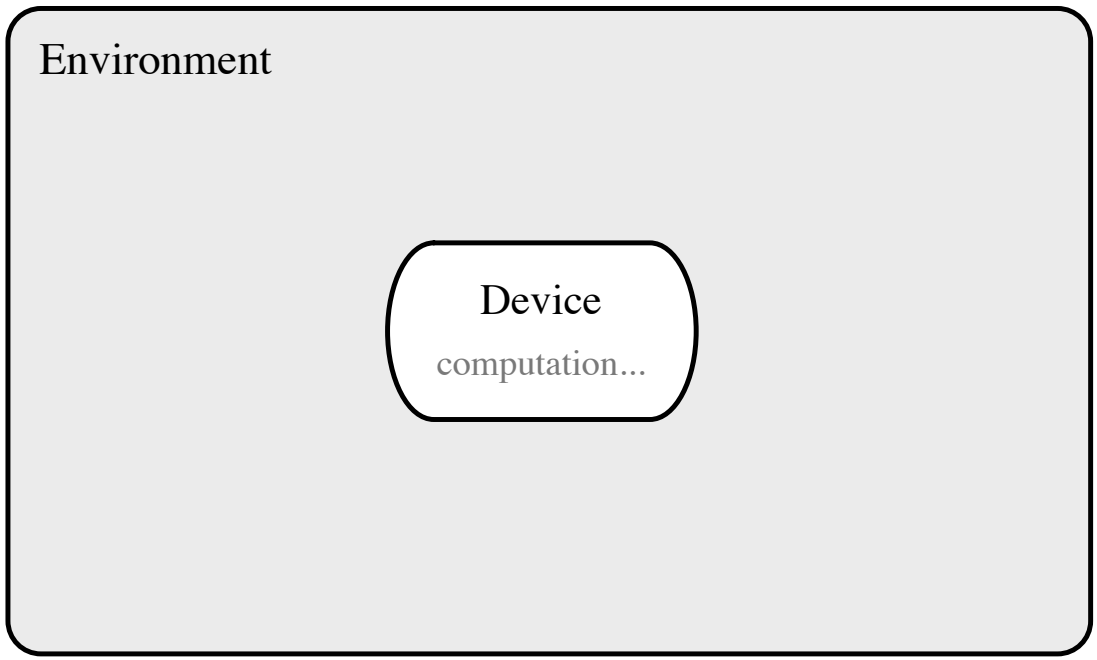

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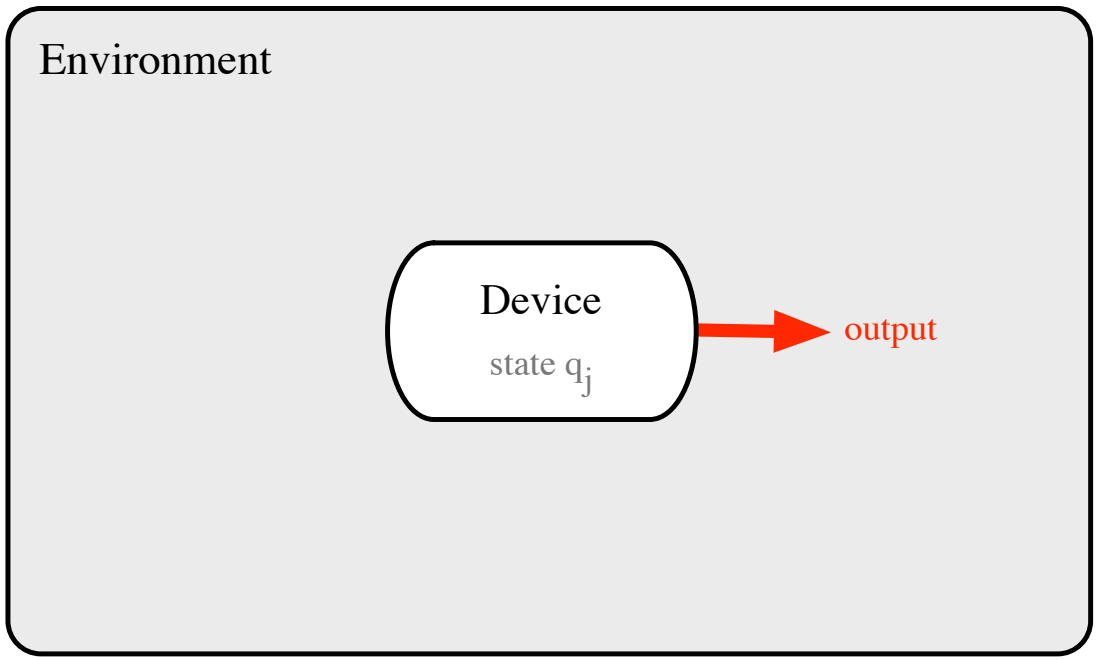

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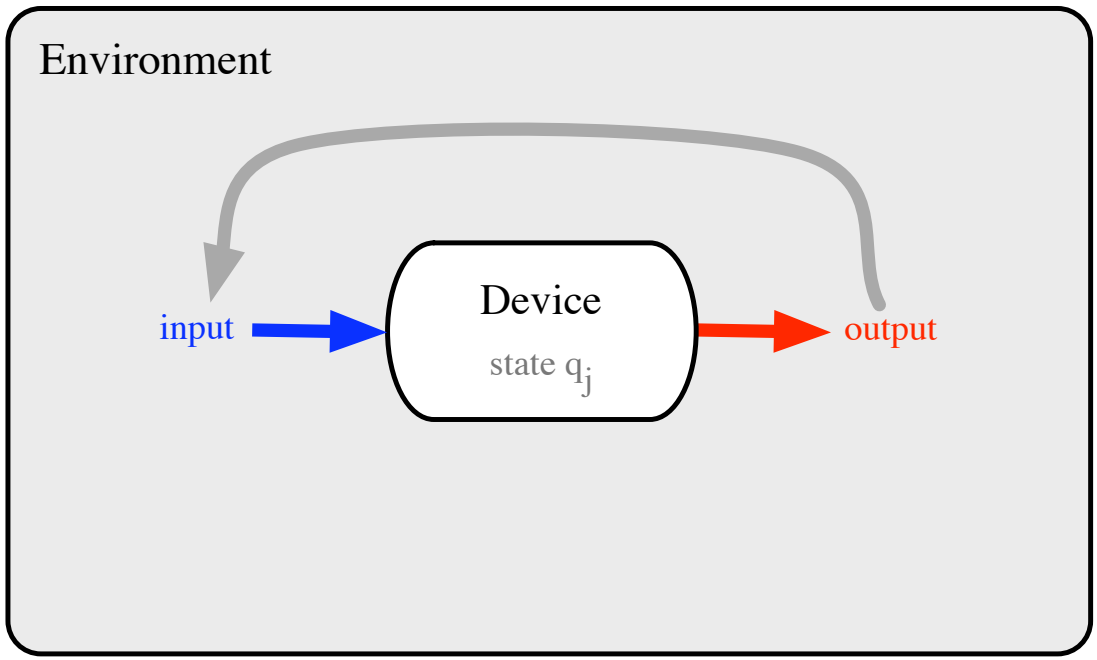

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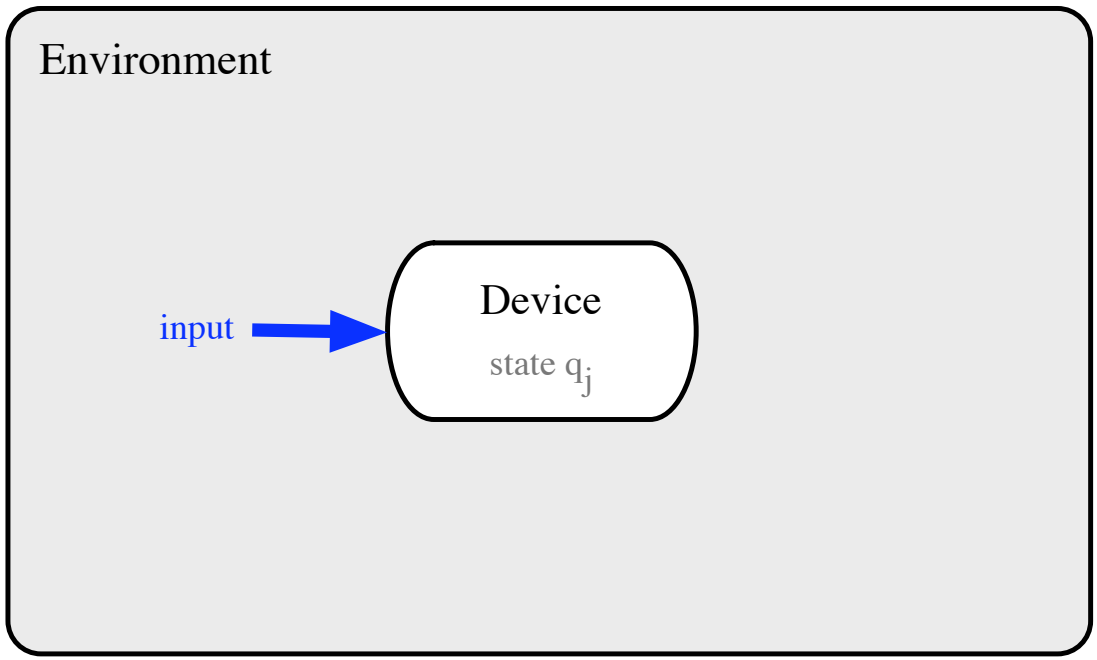

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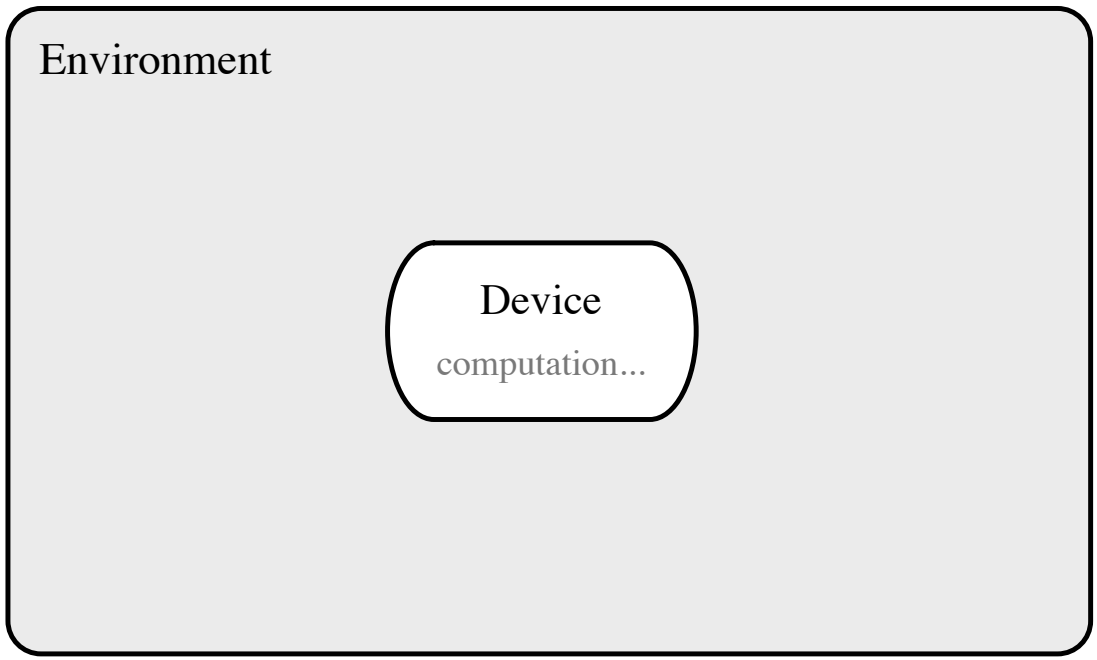

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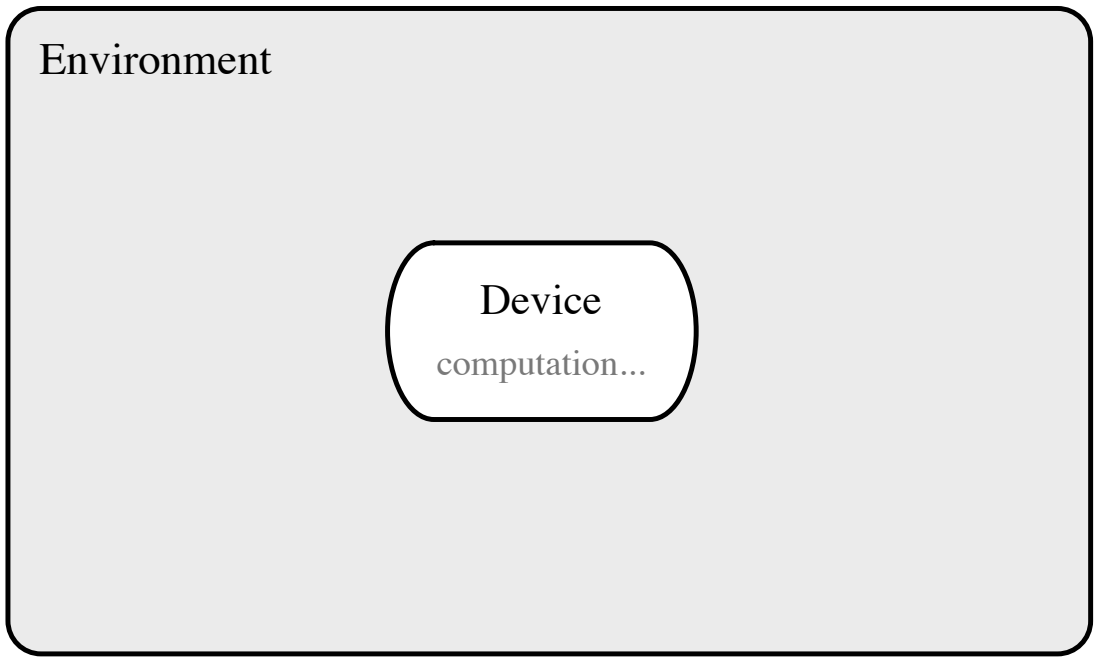

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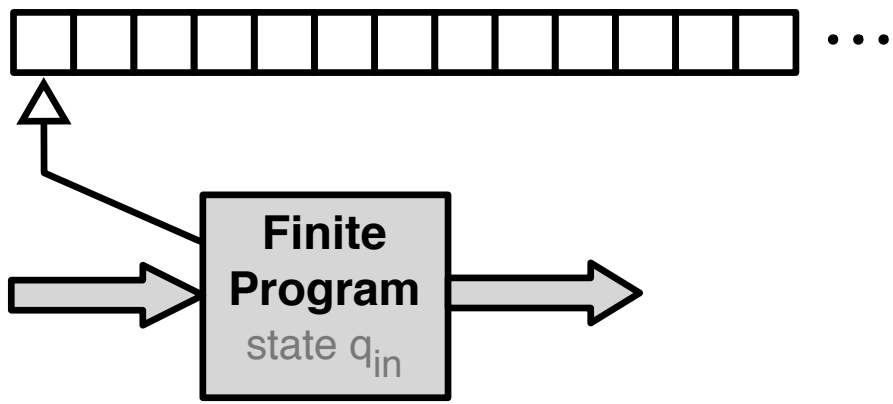

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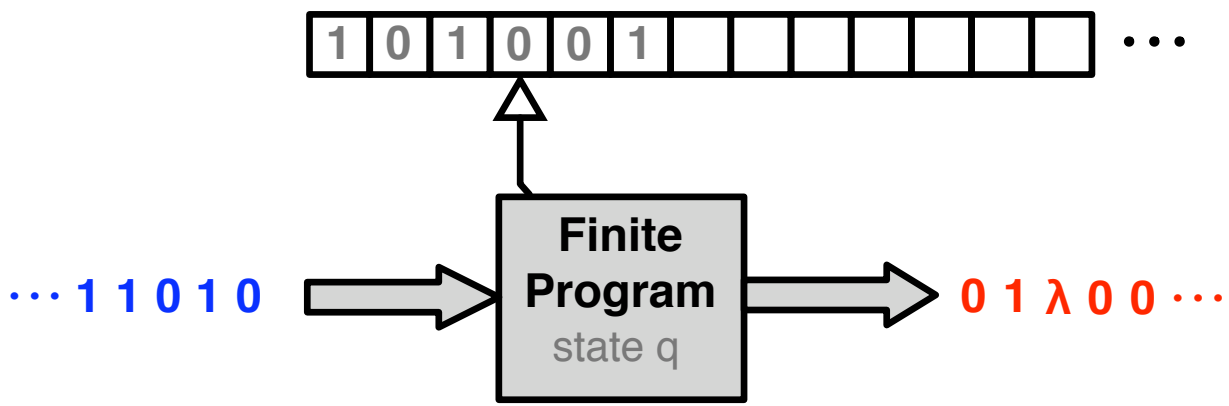

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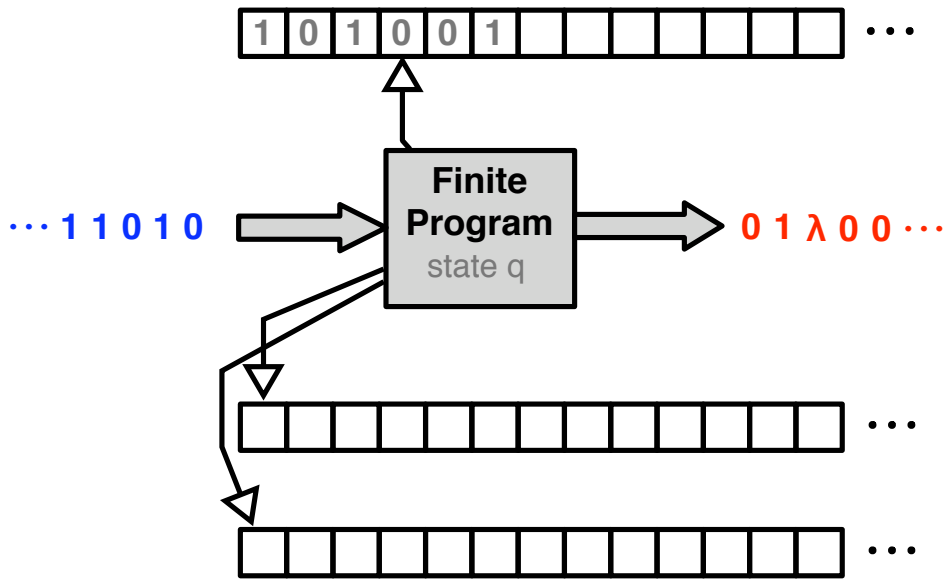

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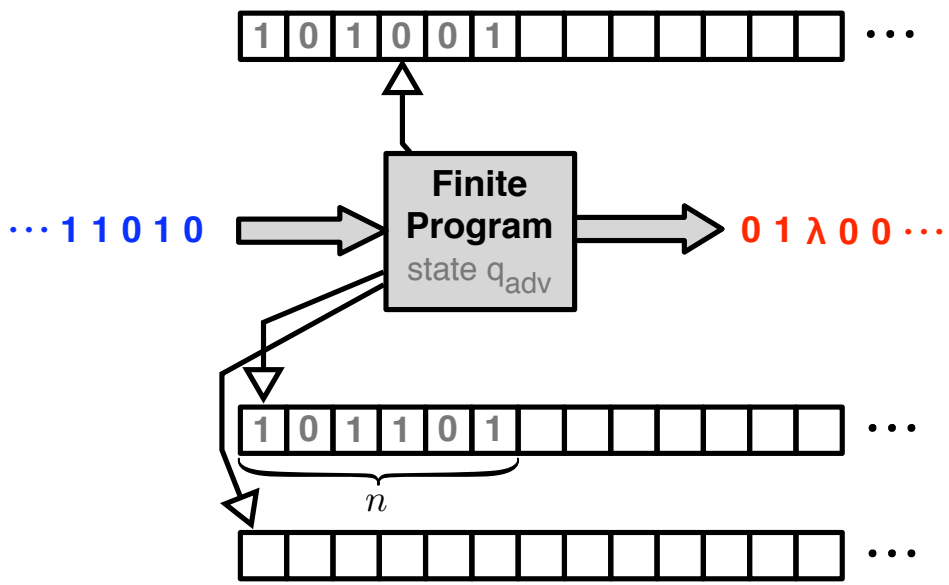

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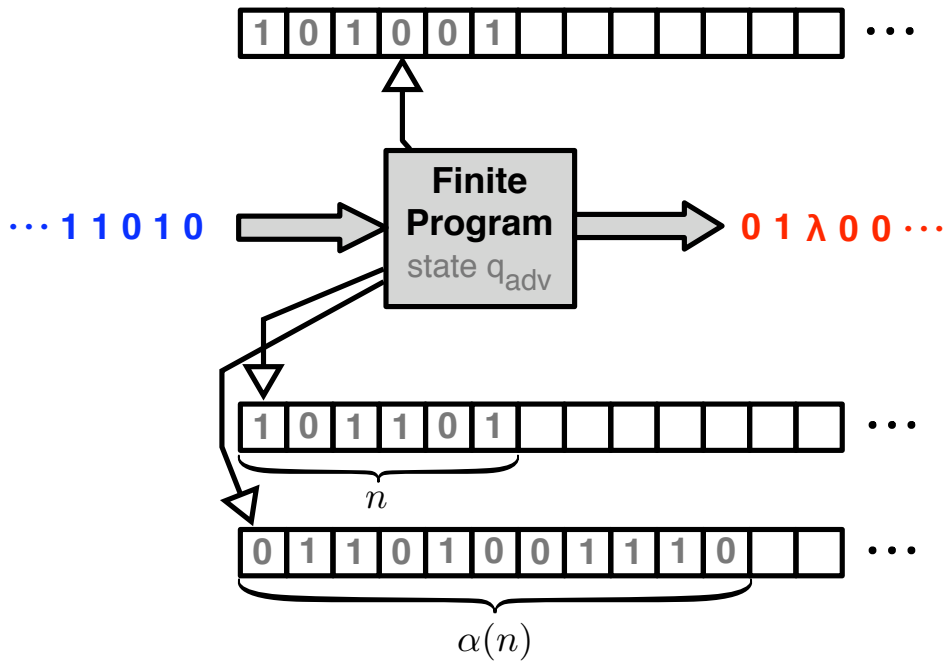

Lemma

Int-TM/As are strictly more powerful than int-TMs, i.e., they can compute strictly more ω-translations.

Computational power

The results concerning the computational power of RNNs in classical computation generalize to the interactive computational framework.

	Static	Evolving
$\mathbb{Q}$	Turing Cabessa & Siegelmann 12	Super-Turing Cabessa 12
$\mathbb{R}$	Super-Turing Cabessa & Siegelmann 12 Cabessa & Villa 12	Super-Turing Cabessa 12

Conclusions

- Recurrent neural networks provide a natural abstract model of computation beyond the Turing limits (Cabessa & Villa 12).
- *Architectural Evolution* represents an equivalent alternative to the *power of the continuum* towards the achievement of super-Turing capabilities.
- The results support the idea that *architectural evolution* might play a crucial role in the computational capabilities of biological neural networks.
- Future work: study the computational power of other kinds of architecturally evolving networks: other sigmoid functions, other learning rules, spiking networks, etc.

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