

# ATTRACTOR-BASED COMPLEXITY OF BOOLEAN RECURRENT NEURAL NETWORKS

APPLICATION TO A SIMPLIFIED MODEL OF THE  
BASAL GANGLIA-THALAMOCORTICAL NETWORK

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Department of Mathematical Economics  
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# INTRODUCTION

- ▶ We introduce an attractor-based complexity measure for Boolean recurrent neural networks.
- ▶ The measure reflects the ability of the networks to discriminate between their input streams via the manifestation of attractor dynamics.
- ▶ We provide an application of this complexity measure to a simplified Boolean model of the basal ganglia-thalamocortical network.

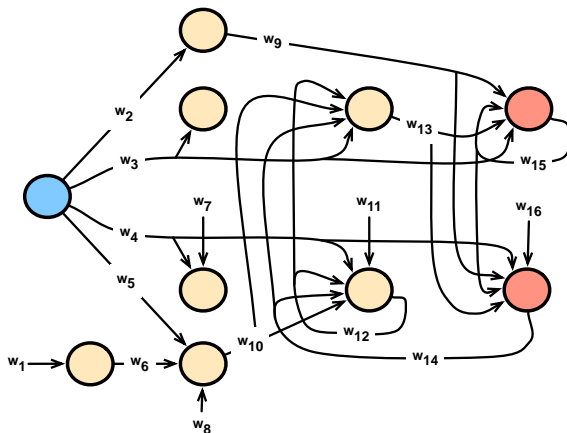
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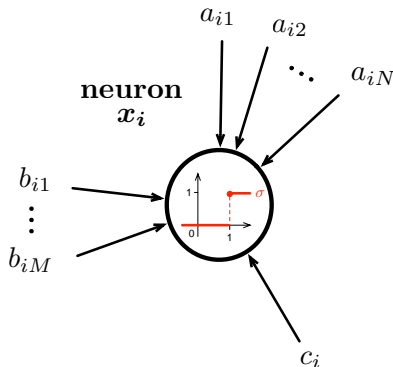
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# RECURRENT NEURAL NETWORK



# BOOLEAN RECURRENT NEURAL NETWORK

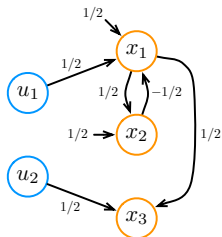


$$x_i(t+1) = \theta \left( \sum_{j=1}^N a_{ij} \cdot x_j(t) + \sum_{j=1}^M b_{ij} \cdot u_j(t) + c_i \right)$$

# FROM BOOLEAN NEURAL NETWORKS TO AUTOMATA

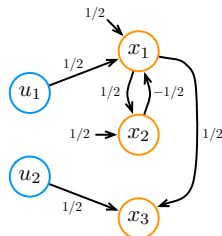
## Boolean Neural Network

## Automaton

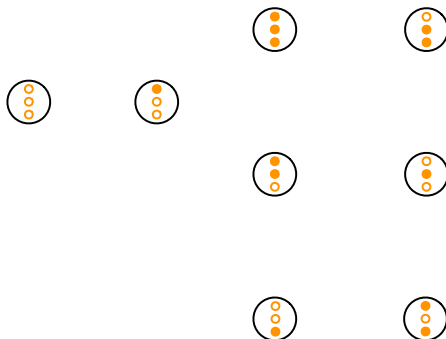


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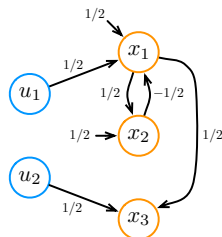
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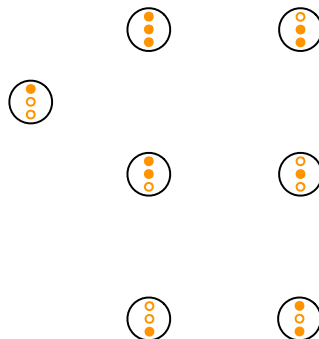


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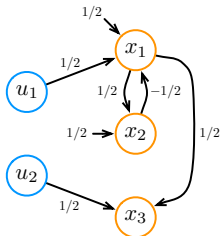


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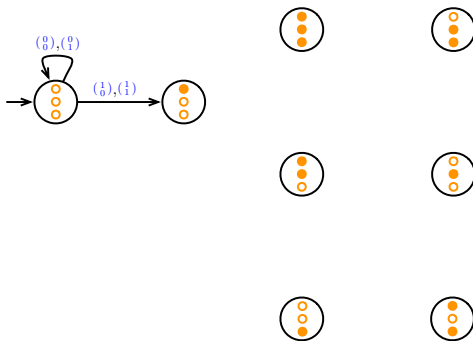


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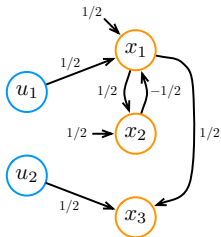


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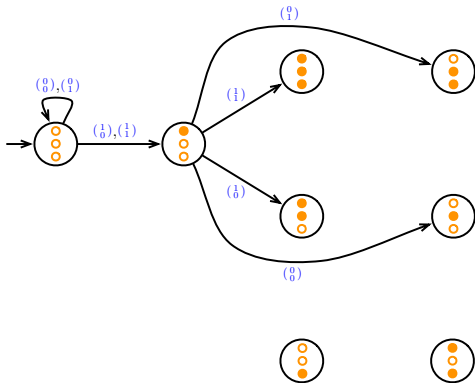


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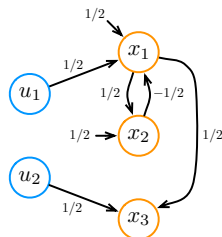


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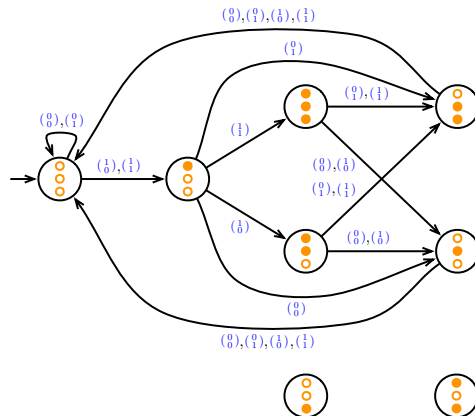


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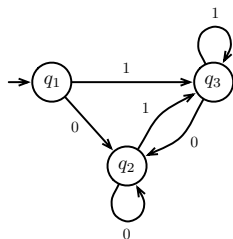


## Automaton



# FROM AUTOMATA TO BOOLEAN NEURAL NETWORKS

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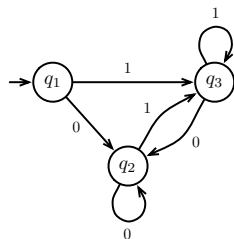


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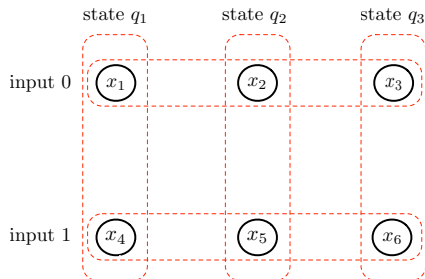


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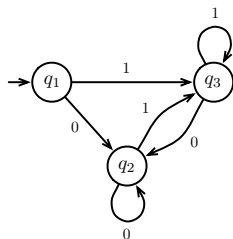

$$u_1$$

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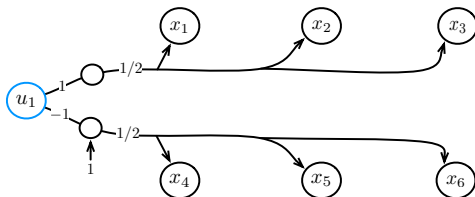


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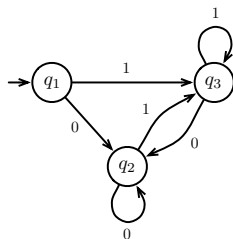
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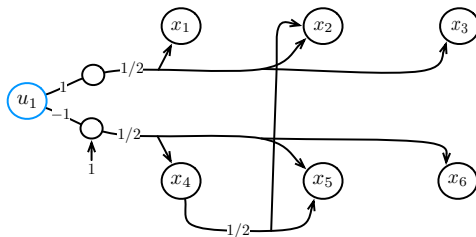


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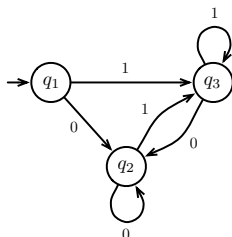


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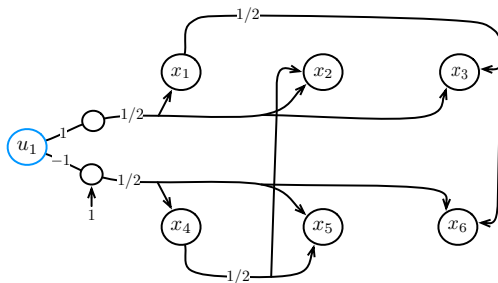


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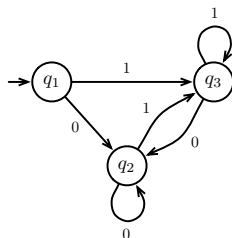


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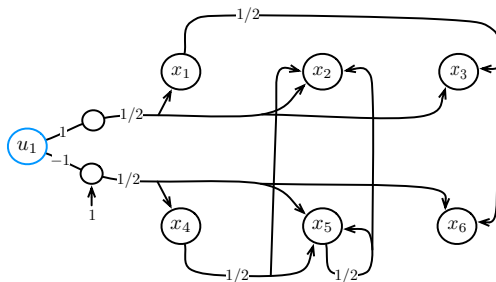


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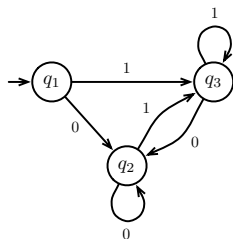


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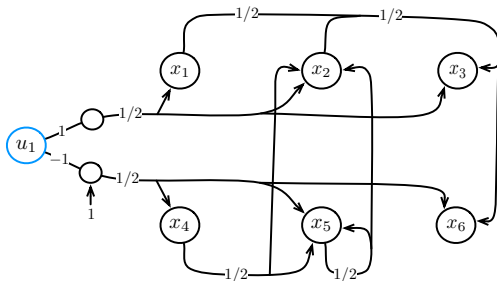


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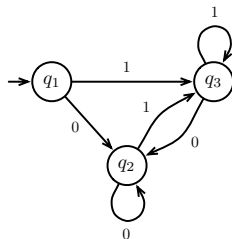


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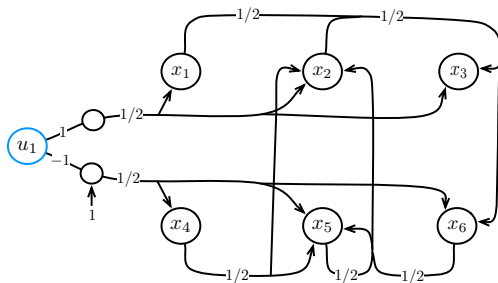


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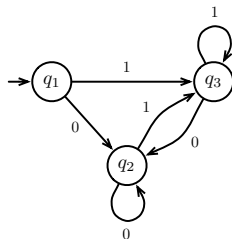


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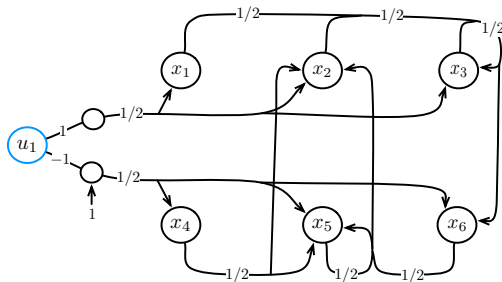


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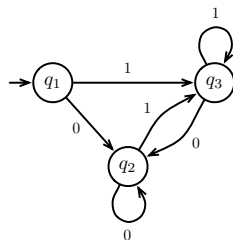


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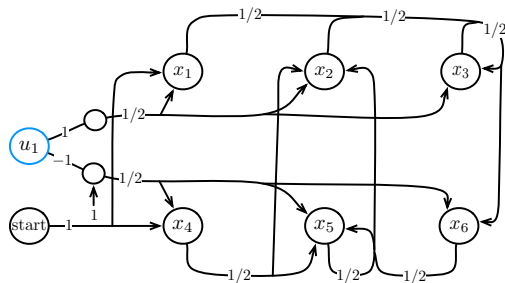


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# EQUIVALENCE BETWEEN BOOLEAN NEURAL NETWORKS AND AUTOMATA

## THEOREM (MINSKY 67)

*"It is evident that each neural network of the kind we have been considering is a finite-state machine."*

*"[...] It is interesting and even surprising that there is a converse to this. Every finite-state machine is equivalent to, and can be "simulated" by, some neural net."*



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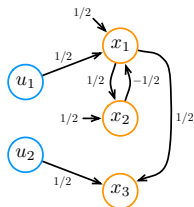
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# ATTRACTORS AND CYCLES

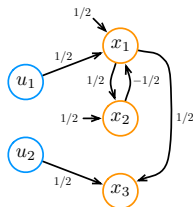
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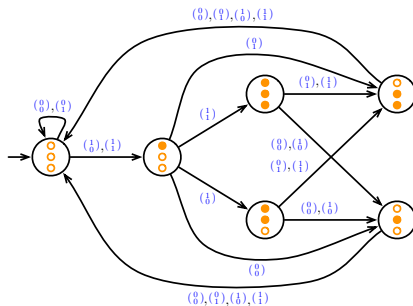


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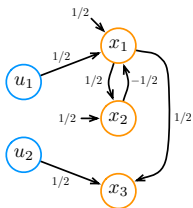


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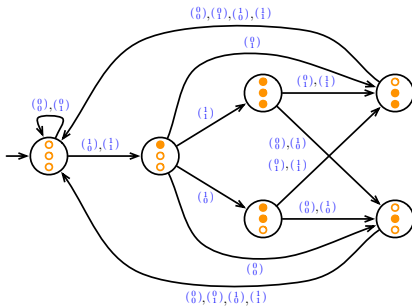


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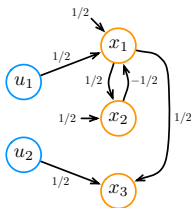
Input stream



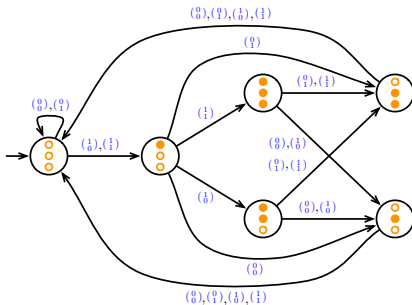


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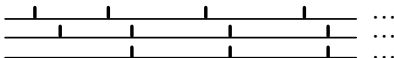
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Sequence of states



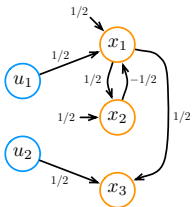
Raster plot



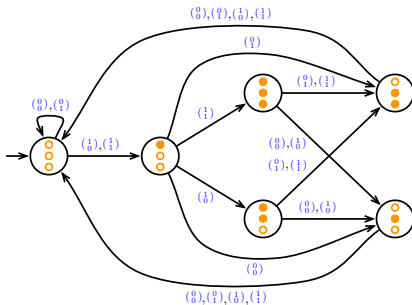
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## Boolean Neural Network

### ATTRACTOR



## Automaton



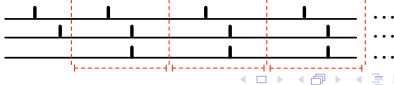
Input stream



Sequence of states



Raster plot

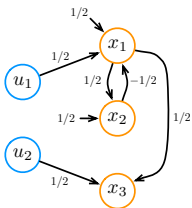




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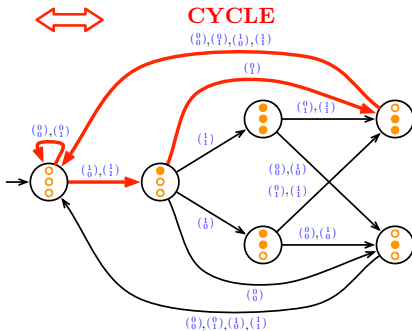
## Boolean Neural Network

### ATTRACTOR



## Automaton

### CYCLE



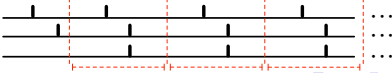
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# ATTRACTOR-BASED COMPLEXITY OF RNNs

- ▶ We assume that some aspect of the computational capabilities of recurrent neural networks are related to their attractor dynamics.
- ▶ We introduce attractor-based complexity measure inspired from automata theory.
- ▶ We assume that the attractors are classified into two categories: meaningful or spurious.
- ▶ The attractor-based complexity refers to the maximal number of alternations between meaningful and spurious attractors that are included one into the other.

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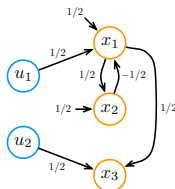
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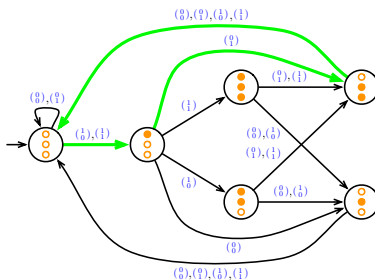
## Example:

- Assume that  $\mathcal{N}$  contains only one meaningful attractor; all others being spurious.
- Then, the attractor-based complexity of  $\mathcal{N}$  is 2. Maximal “growing” sequence of 2 alternations between spurious and meaningful attractors.

Boolean Neural Network



Automaton



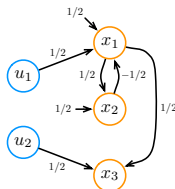


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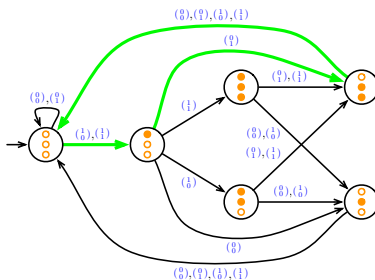
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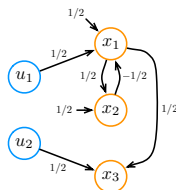


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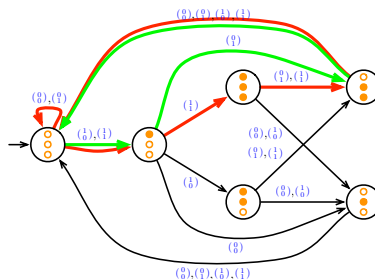
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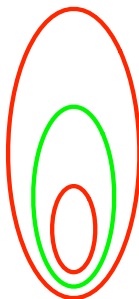
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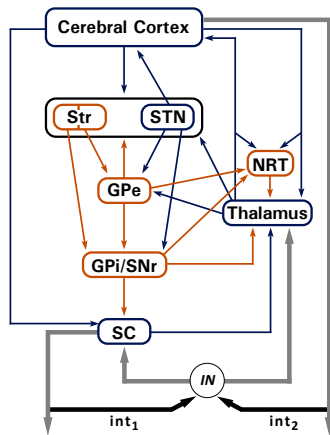
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# BOOLEAN MODEL OF THE BASAL GANGLIA-THALAMOCORTICAL NETWORK



IN

input node

SC

superior colliculus

GPi/SNr

output nuclei of the basal ganglia  
formed by the GABAergic projection  
neurons of the intermediate part of  
the pallidum and of the substantia  
nigra pars reticulata

Thalamus

thalamus

GPe

external part of the pallidum

NRT

thalamic reticular nucleus

Str-D1

striatopallidal component  
of the striatum

Str-D2

striatonigral component  
of the striatum

STN

subthalamic nucleus

Cerebral Cortex

cerebral cortex

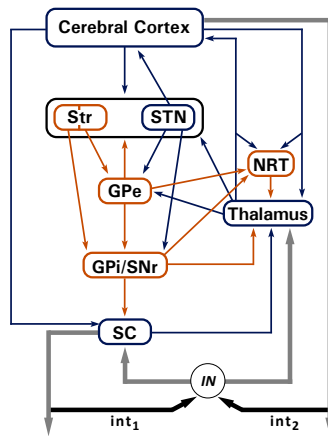
# BOOLEAN MODEL OF THE BASAL GANGLIA-THALAMOCORTICAL NETWORK

Source	Target (Node #)									
Node # (Name)	0	1	2	3	4	5	6	7	8	9
0 (IN)	.	1	1	.	.	.	.	.	.	.
1 (SC)	int <sub>1</sub>	.	1	.	.	.	.	.	.	.
2 (Thalamus)	.	.	.	1	.	1	1	1	1	1
3 (RTN)	.	.	-1	.	.	.	.	.	.	.
4 (GPi/SNr)	.	-1	-1	-1	.	.	.	.	.	.
5 (STN)	.	.	.	.	2	.	2	.	.	2
6 (GPe)	.	.	.	-1/2	-1/2	-1/2	.	-1/2	-1/2	.
7 (Str-D2)	.	.	.	.	.	.	-1	.	.	.
8 (Str-D1)	.	.	.	.	-1/2	.	-1/2	.	.	.
9 (CCortex)	int <sub>2</sub>	1/2	1/2	1/2	.	1/2	.	1/2	1/2	.

TABLE: Adjacency matrix

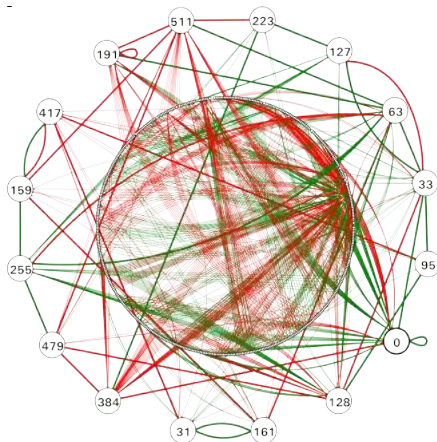
## CORRESPONDING AUTOMATON

- 9 activation nodes and 1 input node in the network  $\Rightarrow$  512 states and a binary alphabet for the automaton



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## Meaningfulness criterion

- ▶ A constitutive cycle – i.e., a basic attractor – is *spurious* if it is characterised either by active SC and quiet Thalamus at the same time step, or by a quiet GPi/SNr during the majority of the duration of the cycle.
- ▶ A constitutive cycle is *meaningful* otherwise.
- ▶ A non-constitutive cycle – i.e., a composed attractor – is considered *meaningful* if it contains a majority of meaningful constitutive cycles.
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# ATTRACTOR-BASED COMPLEXITY

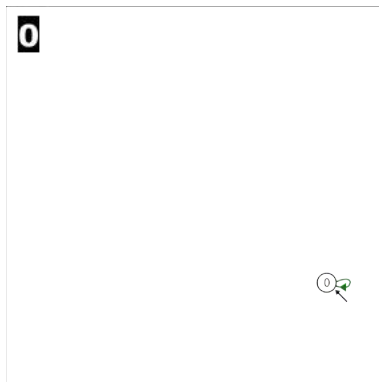
In the corresponding Muller automaton, we found a maximal “alternating tree of cycles” of length  $\omega^6$

- $\Rightarrow$  **Complexity of the Boolean model is 6.**

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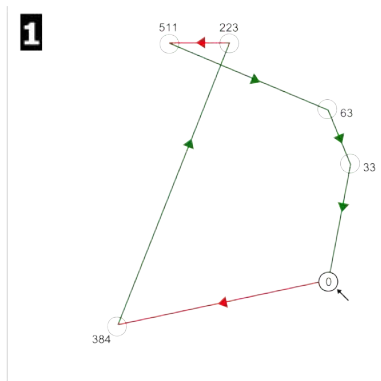
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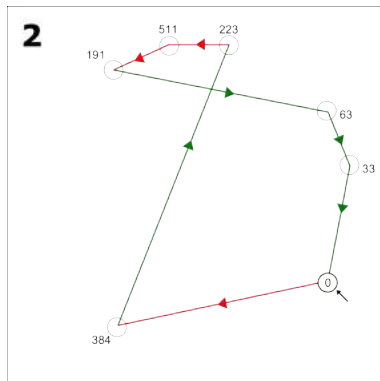




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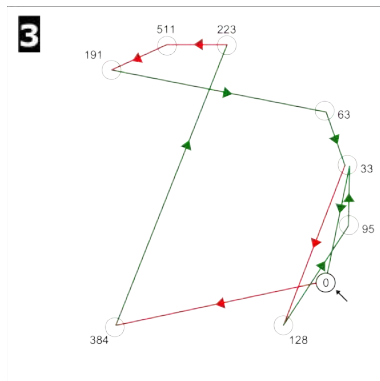
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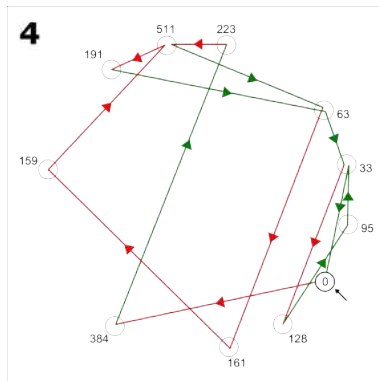
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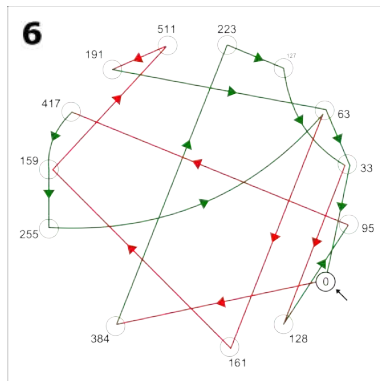




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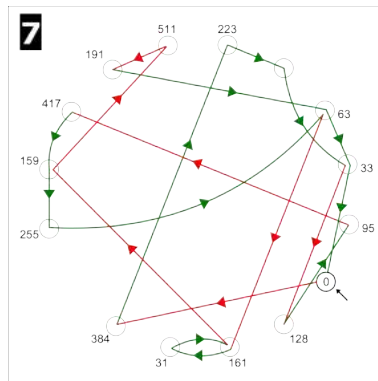
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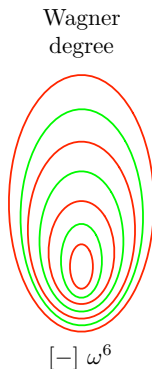
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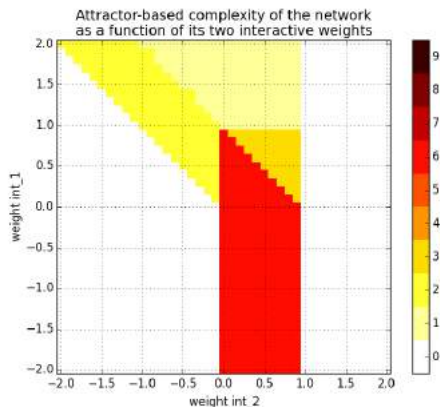
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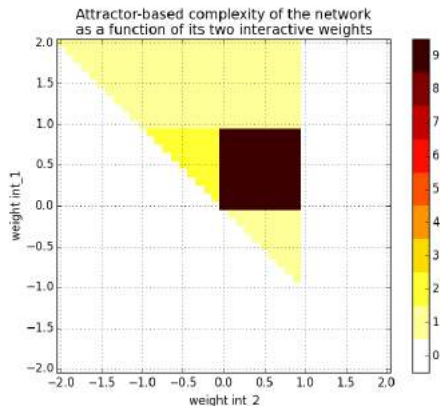
# ATTRACTOR-BASED COMPLEXITY



The *interactive (or feedback) connections* play a significant role in the maintenance and robustness of an optimal level of complexity.

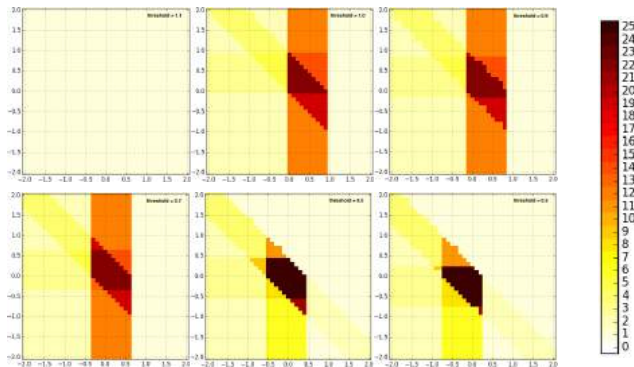


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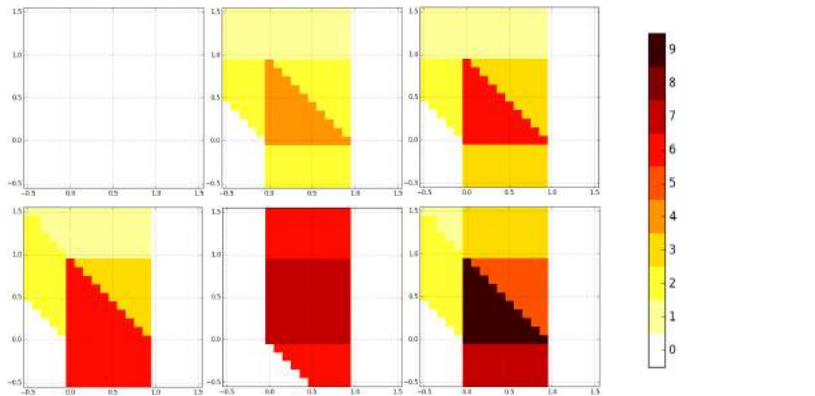
By slightly varying the weights of the networks by  $\pm 0.2$ , one could increase the optimal complexity from 6 to 9.

# NUMBER OF BASIC ATTRACTORS: GLOBAL THRESHOLD (OR WEIGHTS) MODIFICATIONS



Lowering the global threshold, i.e., potentiating the global synaptic level, increases the maximal numbers of attractors and improves the robustness of the “no interactivity” configuration.

# ATTRACTOR-BASED COMPLEXITY: LOCAL WEIGHTS MODIFICATIONS



Even single weight variations of  $\pm 0.1$  can significantly affect the complexity pattern.

# CONCLUSIONS

- ▶ The *number of attractors* and the *attractor-based complexity* might be relevant measures of the computational capabilities for Boolean recurrent neural networks.
- ▶ Global and local modifications of the it synaptic weights significantly affect the attractor complexity of the networks.
- ▶ The values of the *interactive connections* also play a significant role in the maintenance and robustness of an optimal level of attractor-based complexity.
- ▶ These considerations support the rationale that *synaptic plasticity* might be crucially involved in the computational capabilities of neural networks.

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