Recurrent Neural Networks: A Natural Model of Computation Beyond the Turing Limits

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6 October 2012

Introduction

- We follow the so-called *mind-computer analogy* approach to cognitive science.
- We study the computational capabilities of basic models of recurrent neural networks.
- We show that recurrent neural networks provide a natural model of computation beyond the Turing limits.

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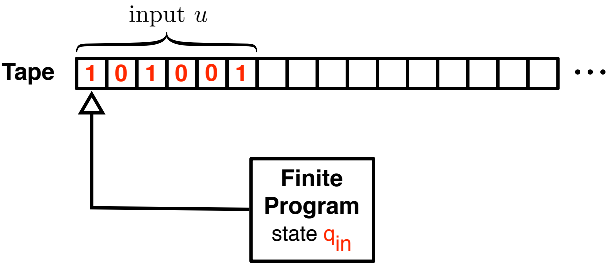

- input u is *accepted* by $\mathcal{M}$ if $\mathcal{M}(u)$ reaches the state q_{acc}
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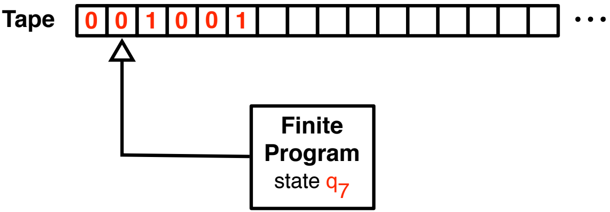

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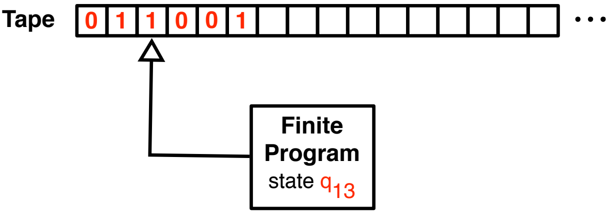

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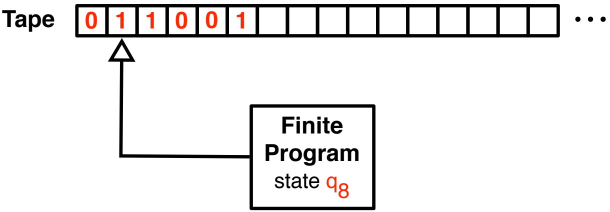

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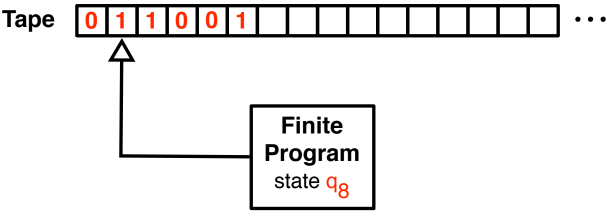

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The Church-Turing thesis

The Church-Turing Thesis states that the Turing machine model is capable of capturing all possible aspects of algorithmic computation.

> *A function is algorithmically computable if and only if it is computable by a Turing machine.*

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Turing machine with advice

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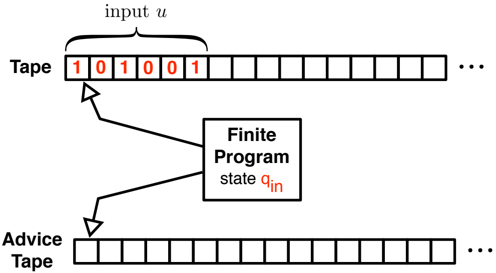

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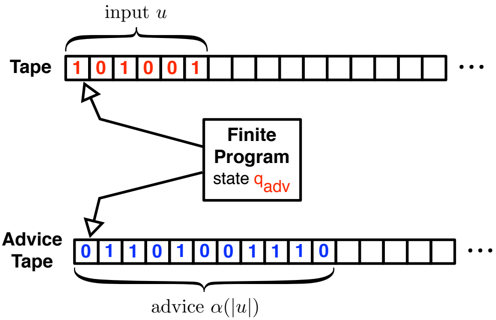

TM vs TM/A

The complexity class $\mathbf{P}$ ($\mathbf{PTIME}$) is the collections of all languages decidable in polynomial time by some TM.

The complexity class $\mathbf{P/poly}$ is the collections of all languages decidable in polynomial time by some TM/poly(A).

Lemma

TM/poly(A)s are strictly more powerful than TMs (super-Turing) already in polynomial time of computation, i.e. $\mathbf{P/poly} \supsetneq \mathbf{P}$.

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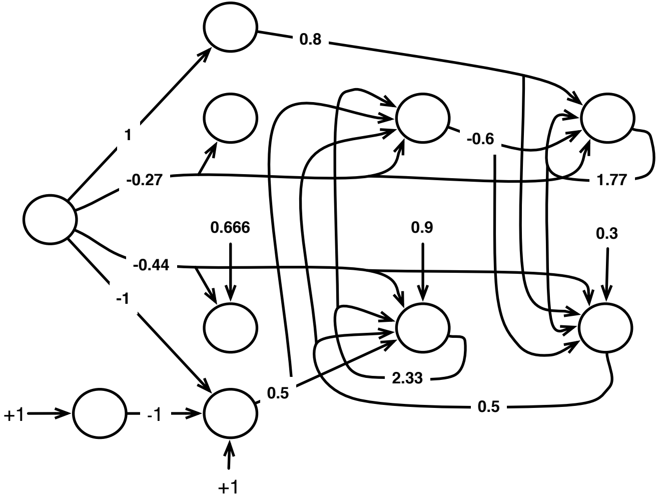
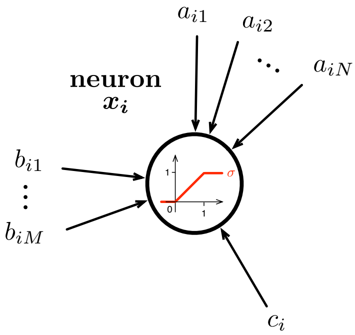
Recurrent neural networks



Dynamics: static synaptic weights



$$x_i(t+1) = \sigma\left(\sum_{j=1}^{N} a_{ij}\cdot x_j(t) + \sum_{j=1}^{M} b_{ij}\cdot u_j(t) + c_i\right)$$

Computational power

Two models of RNNs are of specific interest:

RNN[$\mathbb{Q}$]s: networks with static *rational* weights

RNN[$\mathbb{R}$]s: networks with static *real* weights

Theorem (Siegelmann & Sontag 94, 95)

- *RNN[$\mathbb{Q}$]s are Turing equivalent.*
- *RNN[$\mathbb{R}$]s are super-Turing, i.e.*
  RNN[$\mathbb{R}$]s are equivalent to TM/poly(A) in poly time.

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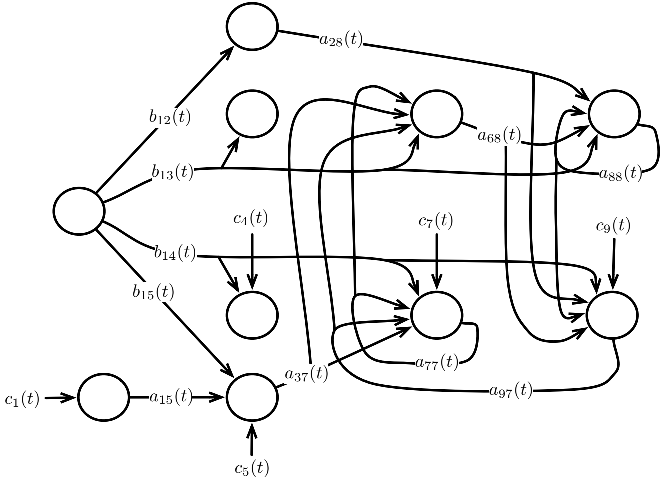
Evolving recurrent neural networks



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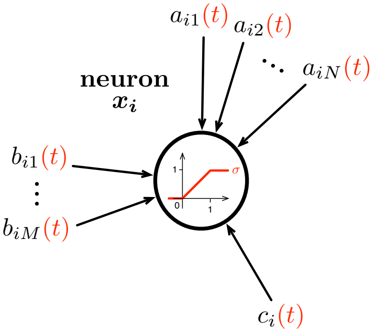

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Summary of the results

	Static	Evolving
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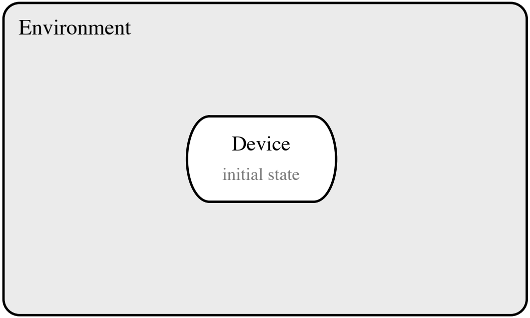
$\mathbb{R}$	Super-Turing Siegelmann & Sontag 94	Super-Turing Cabessa & Siegelmann 11

Classical computation



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"[...] no longer fully corresponds to the current notion of computing in modern systems." (Van Leeuwen & Wiedermann 2008)

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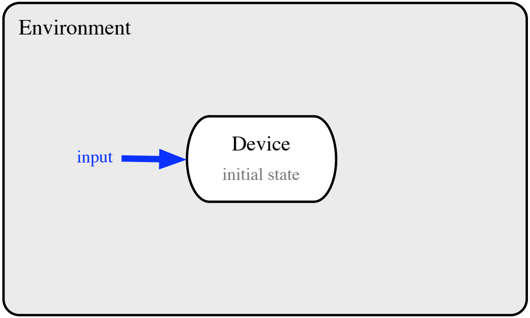

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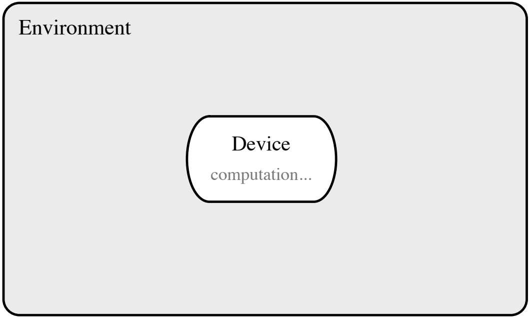

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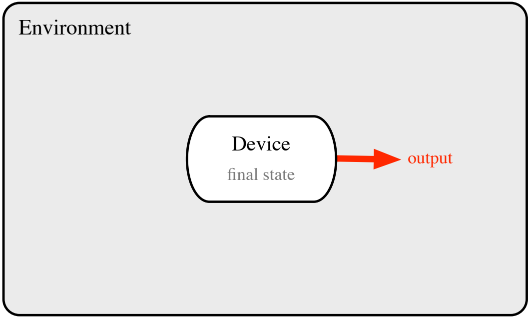

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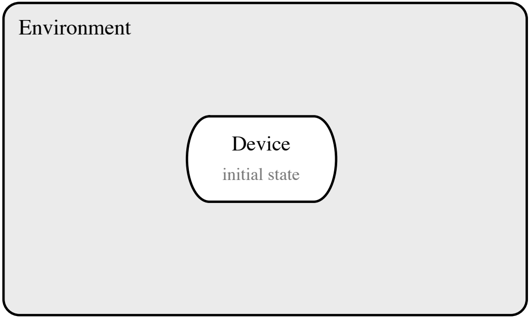

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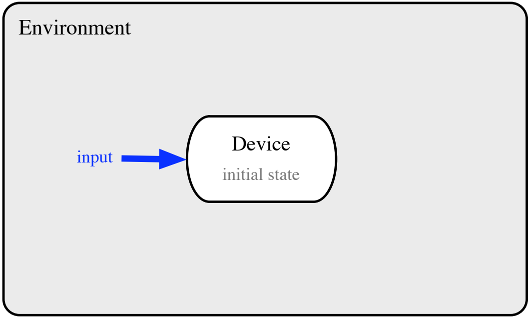

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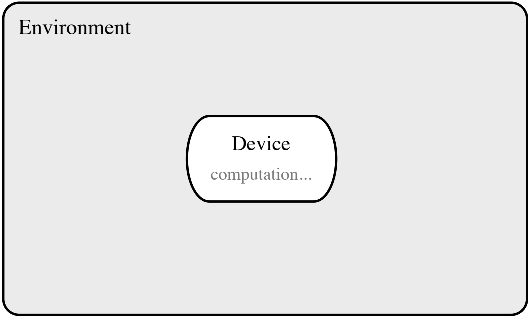

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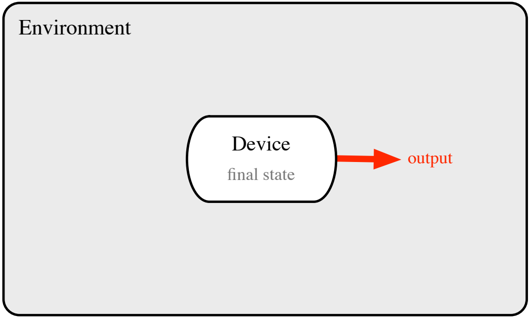

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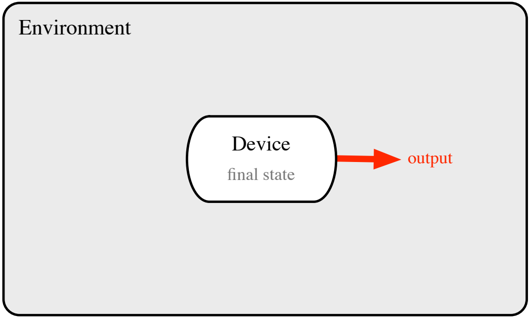

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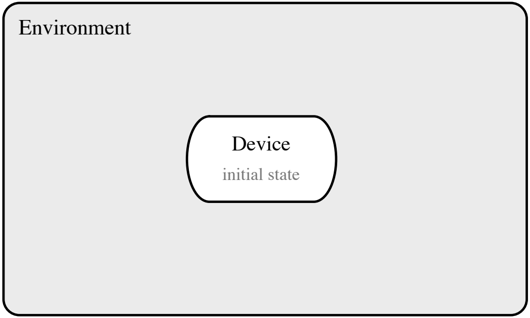

Sequentially interactive and memory active... More appropriate for bio-inspired complex information processing systems.

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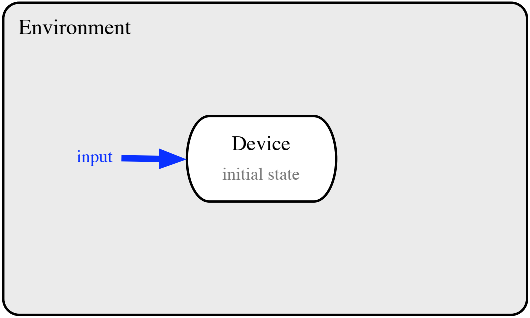

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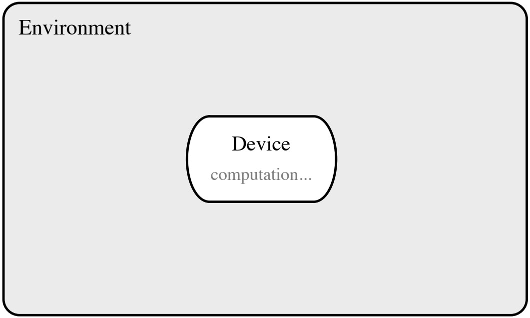

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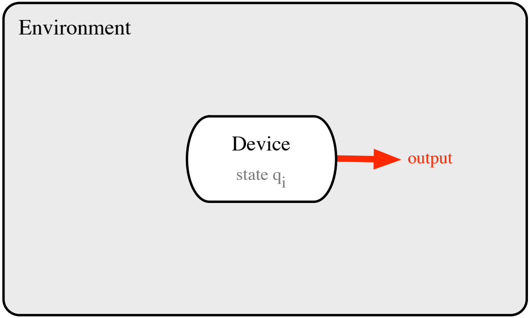

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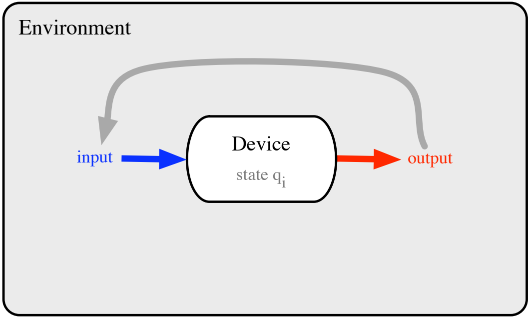

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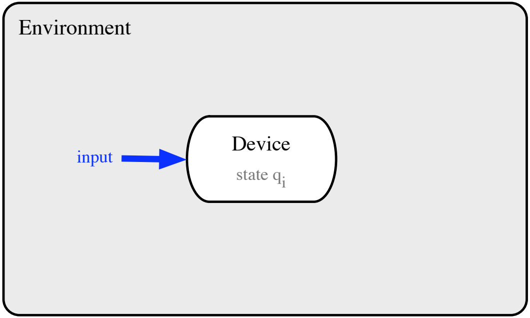

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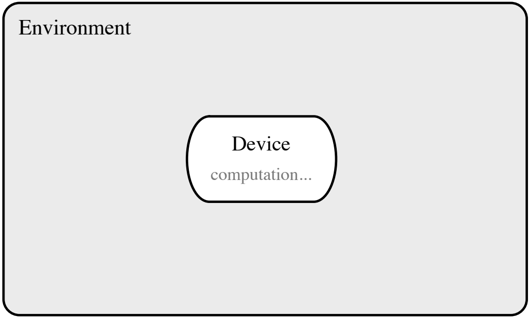

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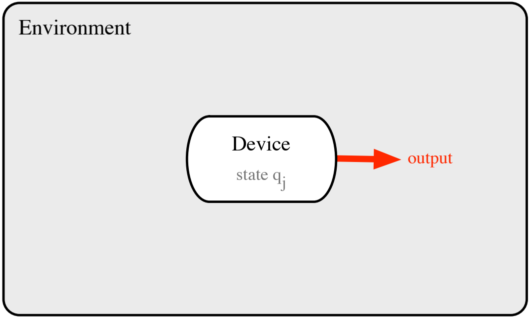

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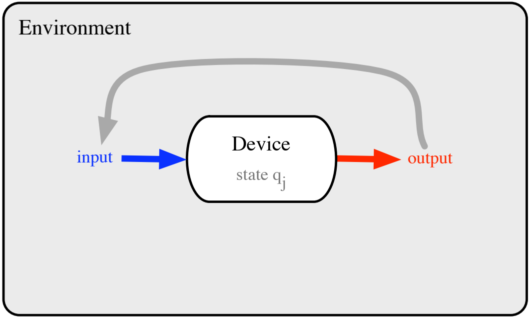

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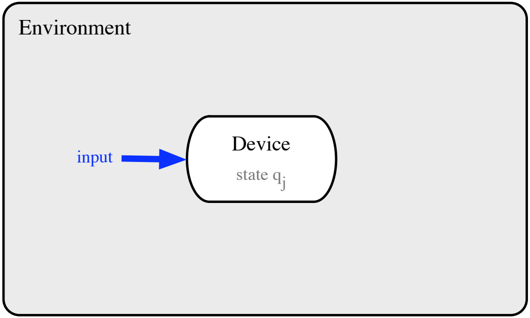

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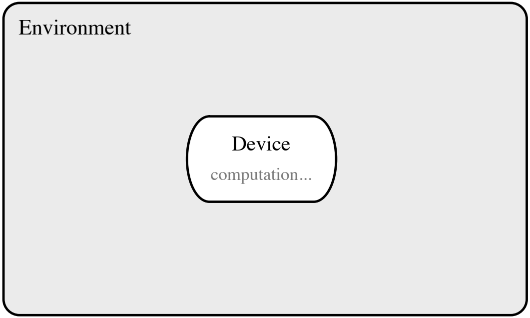

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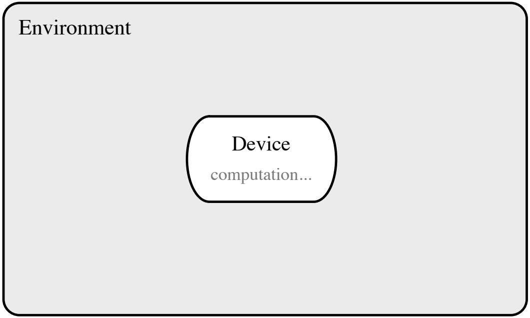

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Computational power

The results concerning the computational power of RNNs in classical computation generalize to the interactive computational framework.

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Conclusions

- Recurrent neural networks provide a natural abstract model of computation beyond the Turing limits.
- *Architectural Evolution* represents an equivalent alternative to the *power of the continuum* towards the achievement of super-Turing capabilities.
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- The question of the existence of hypercomputational capabilities in the brain and in nature is a matter of scientific and philosophical debate (e.g. Penrose and Searle vs Copeland).
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