An Attractor-Based Complexity Measurement for Boolean Recurrent Neural Networks

Application to a Simplified Model of the Basal Ganglia-Thalamocortical Network

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Introduction

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- The measurement reflects the complexity of the attractors' structure of the networks.
- We provide an application of this measurement to a simplified Boolean model of the basal ganglia-thalamocortical network

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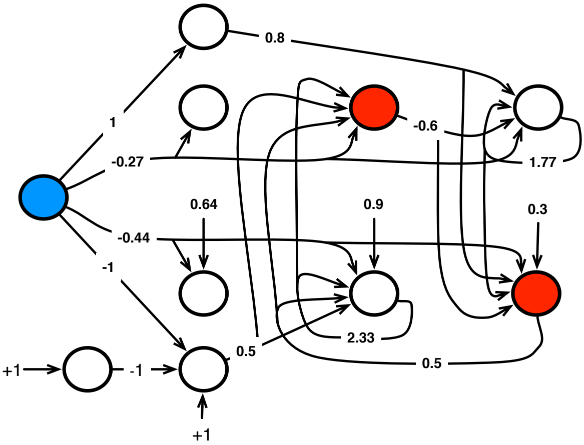
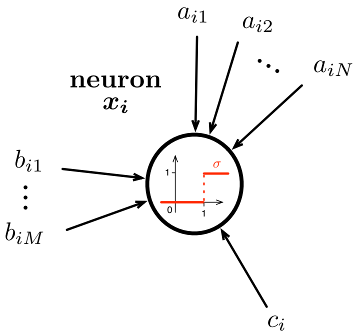
Boolean Recurrent Neural Networks



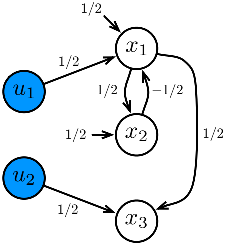
Dynamics



$$x_i(t+1) = \sigma\left(\sum_{j=1}^{N} a_{ij} \cdot x_j(t) + \sum_{j=1}^{M} b_{ij} \cdot u_j(t) + c_i\right)$$

From Boolean Neural Networks to Automata

Boolean Neural Network **Automaton**



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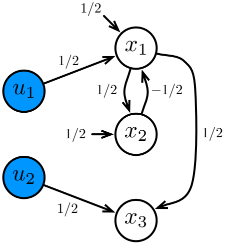

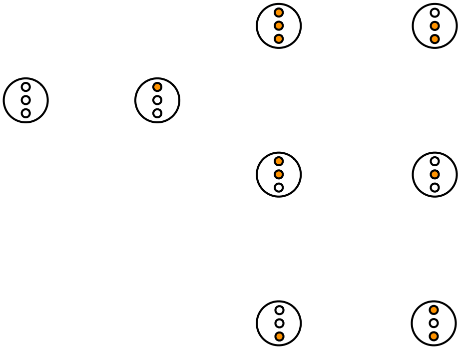

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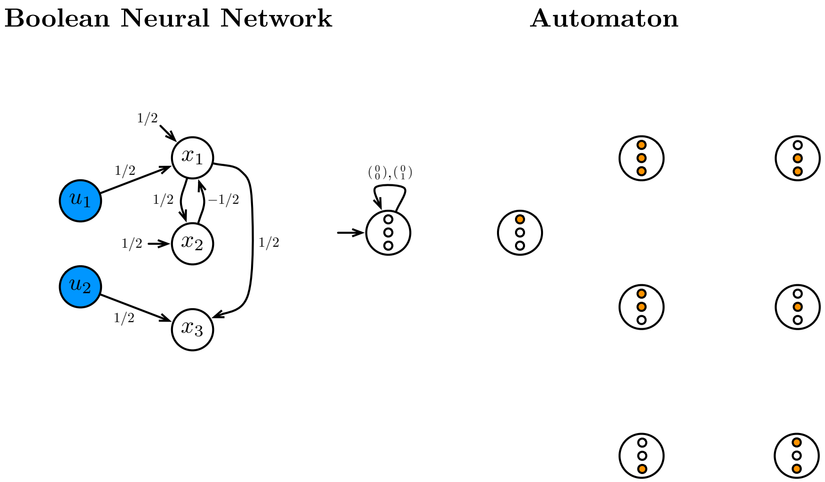

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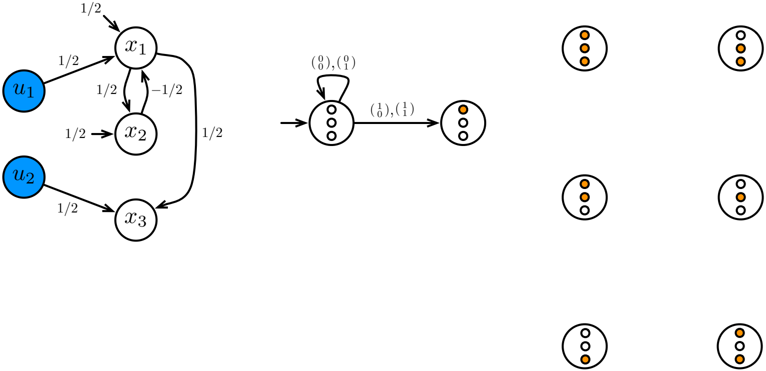

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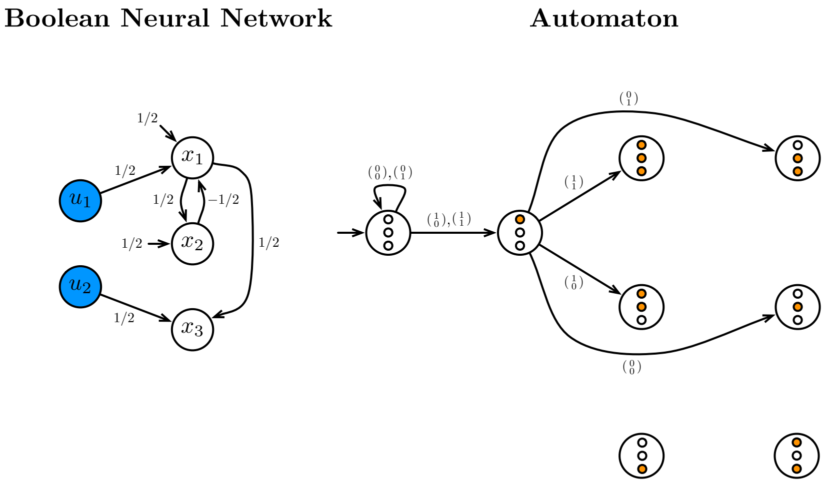

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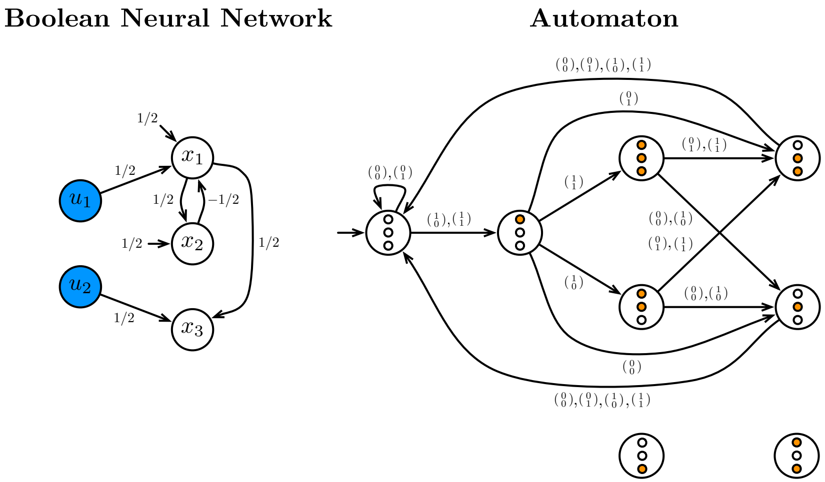
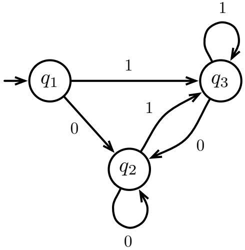


From Automata to Boolean Neural Networks

Automaton

Boolean Neural Network



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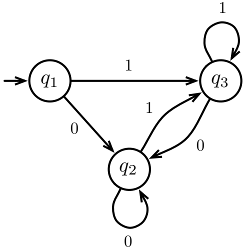
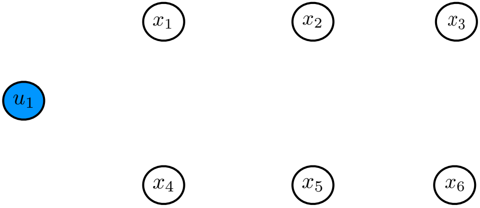


Boolean Neural Network



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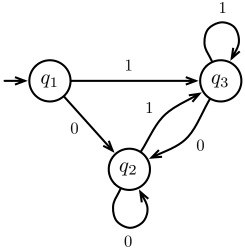

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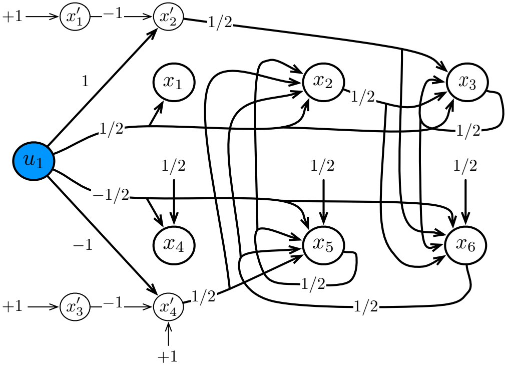

Equivalence between Boolean Neural Networks and Automata

Theorem (Minsky 67)

"It is evident that each neural network of the kind we have been considering is a finite-state machine."

"[...] It is interesting and even surprising that there is a converse to this. Every finite-state machine is equivalent to, and can be "simulated" by, some neural net."

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Cycles and Attractors

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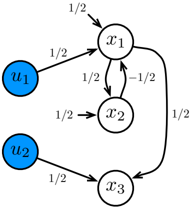

Cycles and Attractors

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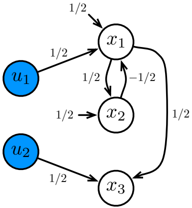
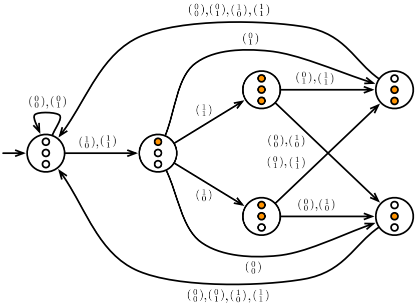


Automaton



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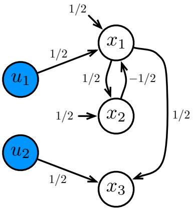

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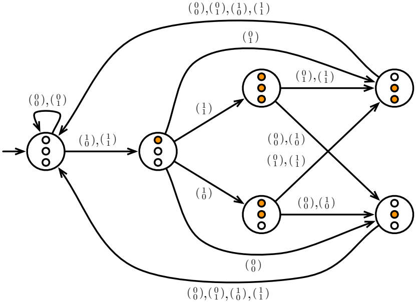
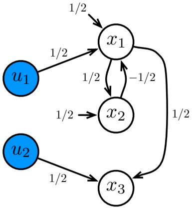
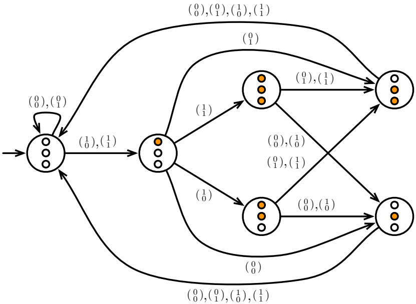


Input stream $\binom{1}{1}\binom{0}{1}\binom{1}{1}\binom{0}{0}\binom{1}{0}\binom{0}{1}\binom{0}{0}\binom{0}{1}\ \cdots$

Cycles and Attractors

Boolean Neural Network | **Automaton**



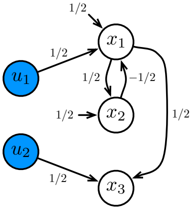


Input stream $\binom{1}{1}\binom{0}{1}\binom{1}{1}\binom{0}{0}\binom{1}{0}\binom{0}{1}\binom{0}{0}\binom{0}{1}\cdots$

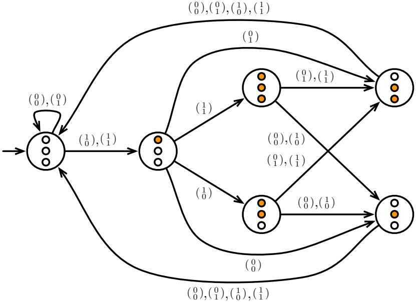
Sequence of states

Cycles and Attractors

Boolean Neural Network

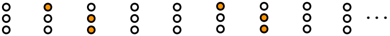


Automaton

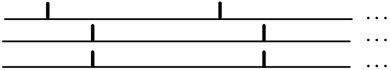


Input stream $\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\cdots$

Sequence of states



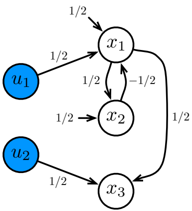
Spatio-temporal pattern

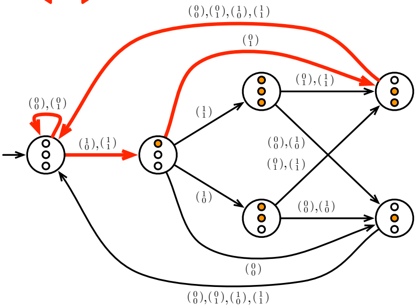


Cycles and Attractors

Boolean Neural Network — **Automaton**

ATTRACTOR ⟺ **CYCLE**





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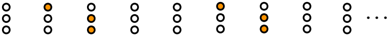
Sequence of states



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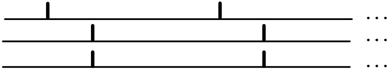

The Wagner Hierarchy

- In ω-automata theory, there is a transfinite classification of ω-automata according to the way their cycles are intricated one into the other...
- **The Wagner hierarchy**
- By translating the Wagner hierarchy from the ω-automata to the Boolean neural network context, one obtains a transfinite classification of Boolean neural networks according to the way their attractors are intricated one into the other...
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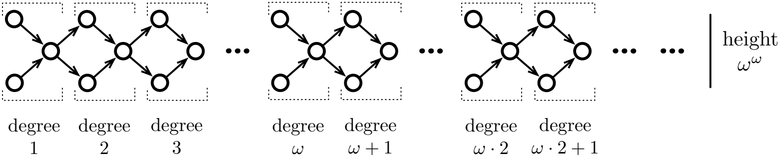
The Wagner Hierarchy

- A transfinite classification of *Muller automata* according to the topological complexity of their underlying language
- Equivalently, a transfinite classification of *Muller automata* according to the graph-theoretical complexity of their cycles
- Quasi well-ordering of transfinite height ω^ω



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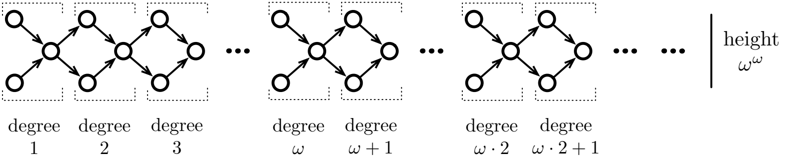

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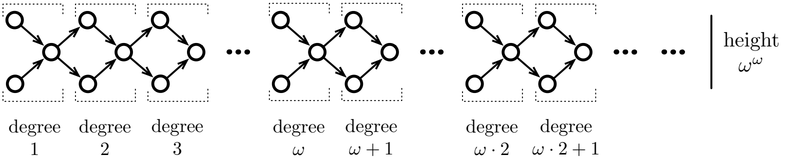

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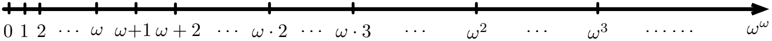
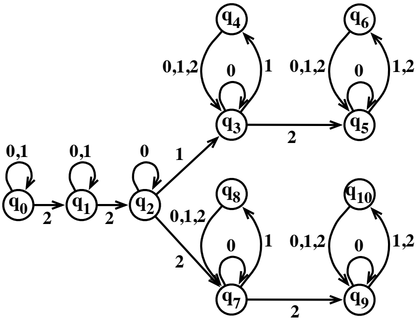


The Wagner Hierarchy – Muller Automata

- A *Muller automaton* consists of an automaton provided with an additional specification of every of its *cycles* into an accepting or a rejecting mode



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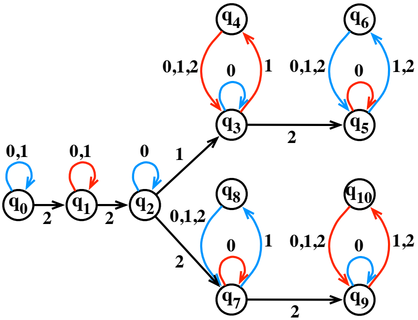
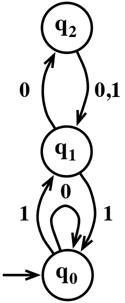


The Wagner Hierarchy – Degrees ω^n

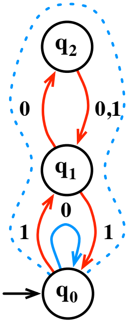
Muller automaton	Wagner degree



The Wagner Hierarchy – Degrees ω^n

Muller automaton

Wagner degree



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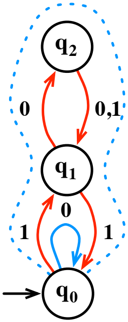
Muller automaton



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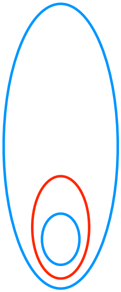
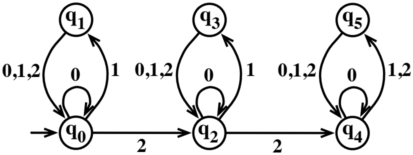


$[+]\ \omega^2$

The Wagner Hierarchy – Degrees $\omega^n \cdot k$

Muller automaton

Wagner degree



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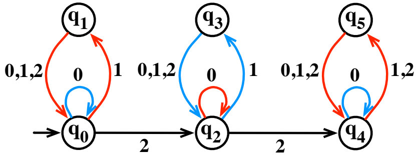
Muller automaton

Wagner degree



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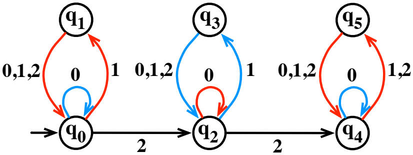
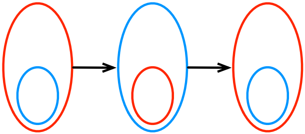


Wagner degree

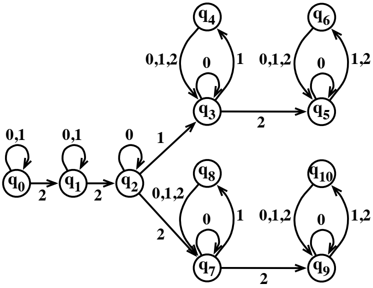


$[+]\ \omega^1 \cdot 3$

The Wagner Hierarchy – Degrees $\omega^n \cdot k + \omega^{n'} \cdot k'$

Muller automaton

Wagner degree



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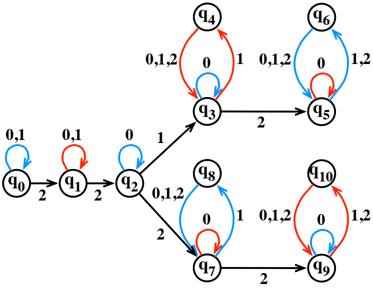
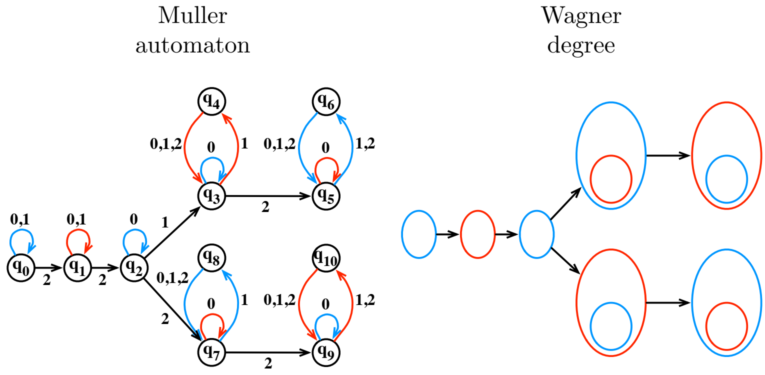


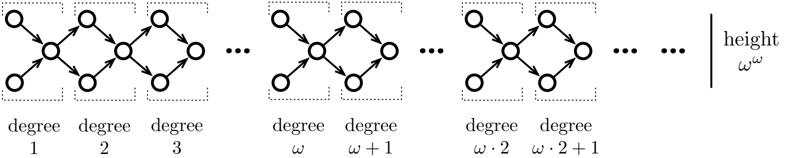
The Wagner Hierarchy – Degrees $\omega^n \cdot k + \omega^{n'} \cdot k'$



$[+]\ \omega^1 \cdot 2 + \omega^0 \cdot 3$

The Wagner Hierarchy – Summary

- A quasi well-ordering of transfinite height ω^ω
- Every ordinal $\alpha < \omega^\omega$ has a unique Cantor normal form $\alpha = \omega^{n_0} \cdot p_0 + \omega^{n_1} \cdot p_1 + \cdots + \omega^{n_k} \cdot p_k$, where $n_0 > n_1 > \cdots > n_k$
- The degree α of a Muller automaton $\mathcal{M}$ in the Wagner hierarchy is the maximal "tree of cycles" $\mathcal{T}_\alpha$ in $\mathcal{M}$



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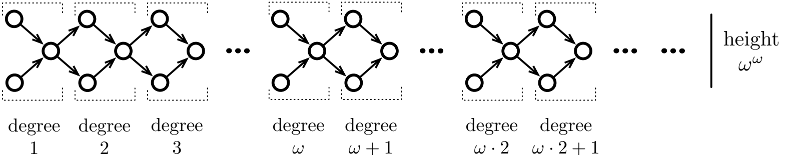

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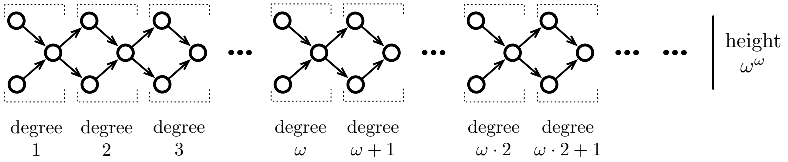

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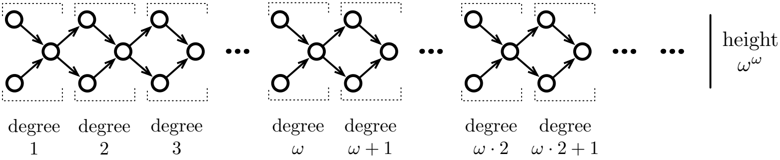

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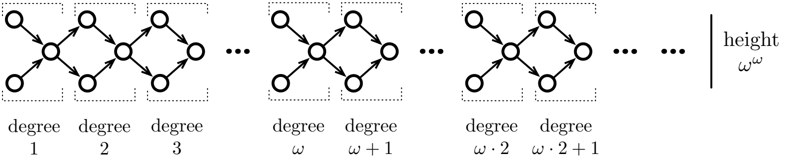

The Boolean RNNs Hierarchy

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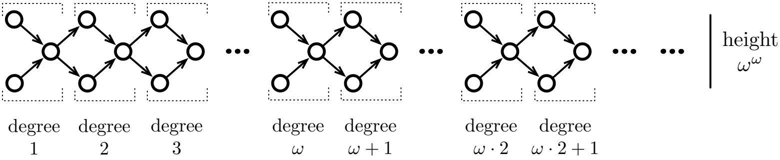
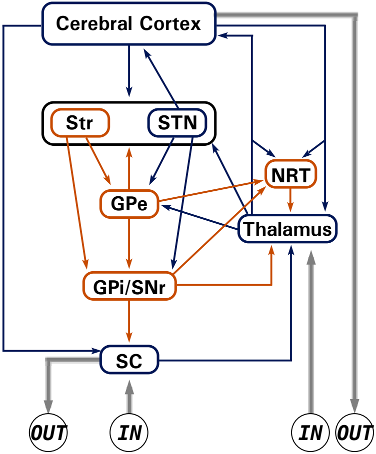
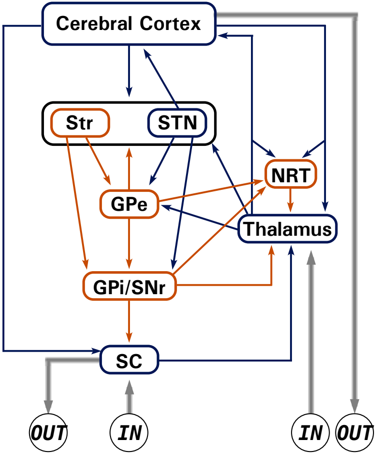


Boolean Model of the Basal-Ganglia Thalamo-cortical Network



Corresponding Automaton

- 9 activation nodes and 1 input node in the network $\Rightarrow$ 512 states and a binary alphabet for the automaton



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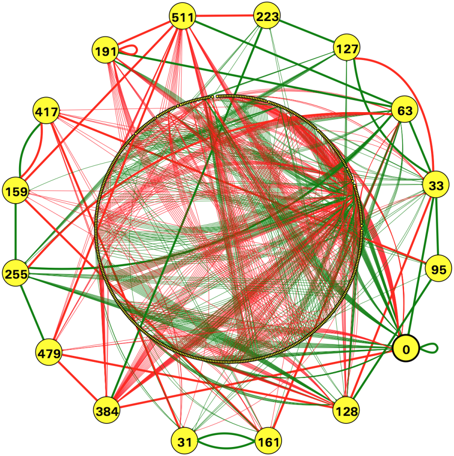

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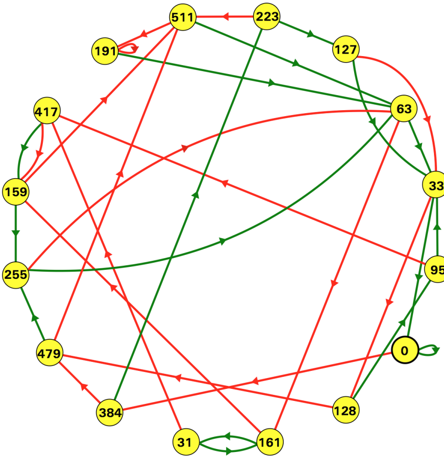

Corresponding Muller Automaton

- We provide an accepting or rejecting value to each *cycle* of the automaton, i.e., we provide an meaningful or spurious value to each *attractor* of the network
- A constitutive cycle is considered to be spurious if it is characterised either by active SC and quiet Thalamus at the same time step or by a quiet GPi/SNR during the majority of the duration of the constitutive cycle. A constitutive cycle is meaningful otherwise
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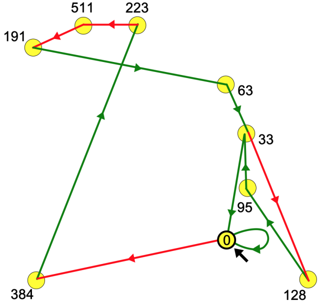

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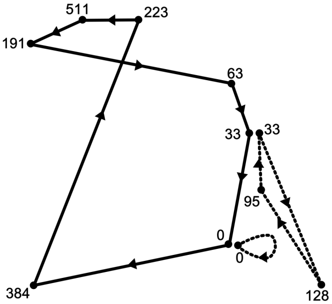

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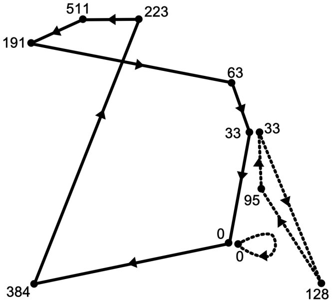

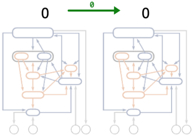

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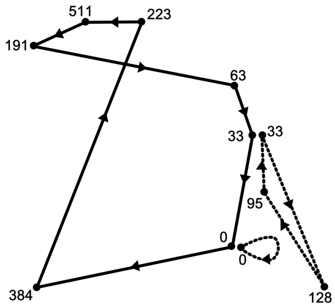

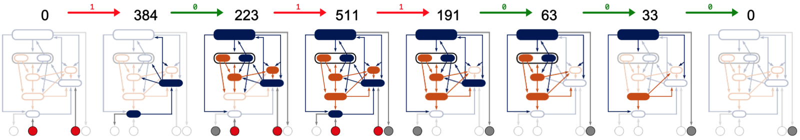

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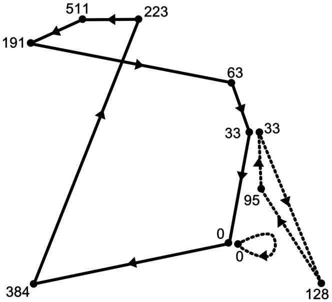

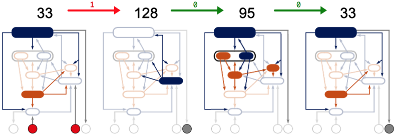

Attractor-Based Complexity of the Model

- In the corresponding Muller automaton, we found a maximal "tree of cycles" of length ω^6
- $\Rightarrow$ **Degree of the Boolean model is ω^6**

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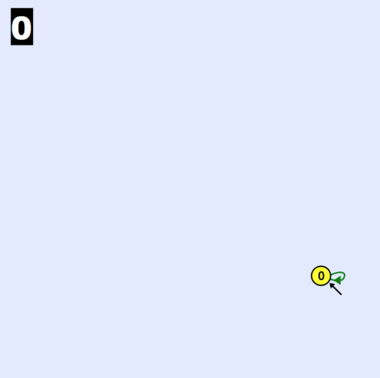

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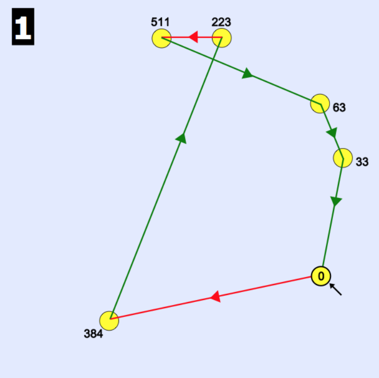

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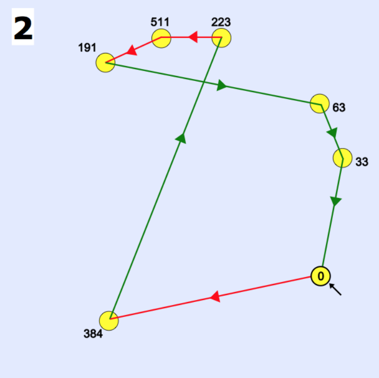

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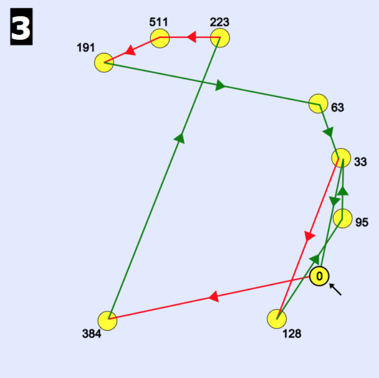

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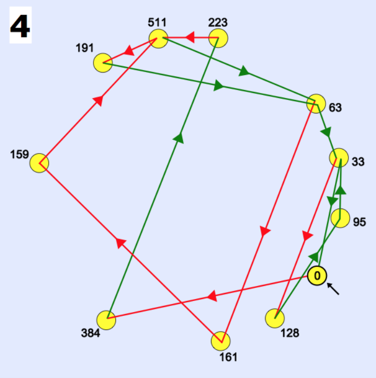

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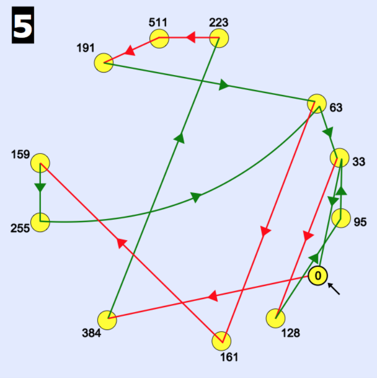

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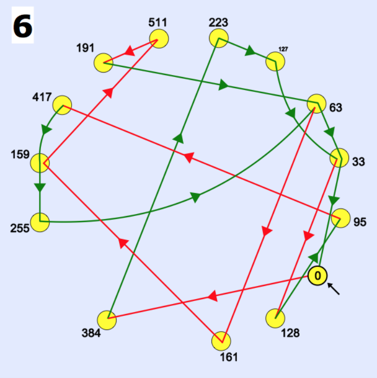

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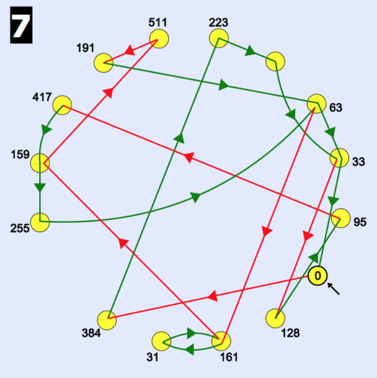

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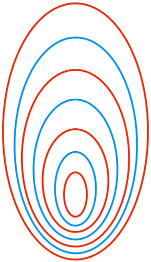
Wagner
degree



$[-]\ \omega^6$

Conclusions

- We introduced a new attractor-based complexity measurement for Boolean recurrent neural networks
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