Automata and Turing Complete Computation with Bio-Inspired Neural Networks

Jérémie Cabessa

LEMMA – Université Paris II

IBISC – Université d'Évry

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Introduction

- We recall important results about the computational capabilities of recurrent neural networks.
- We introduce a Turing complete bio-inspired paradigm for neural computation based on the concept of *synfire rings*.

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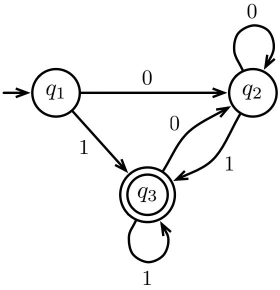
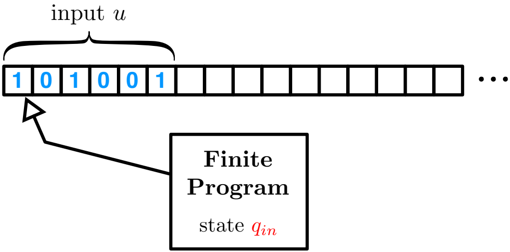
Finite State Automaton (FSA)



- input u is *accepted* by $\mathcal{A}$ if $\mathcal{A}(u)$ reaches a final state
- input u is *rejected* by $\mathcal{A}$ if $\mathcal{A}(u)$ otherwise

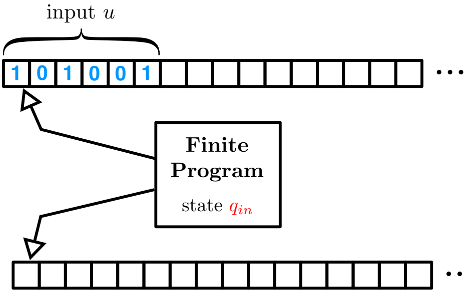
Turing Machine (TM)



- input u is *accepted* by $\mathcal{M}$ if $\mathcal{M}(u)$ reaches the state q_{acc}
- input u is *rejected* by $\mathcal{M}$ if $\mathcal{M}(u)$ reaches the state q_{rej}

Turing Machine with Advice (TM/A)

Additional advice tape and advice function $\alpha : \mathbb{N} \longrightarrow \{0,1\}^*$



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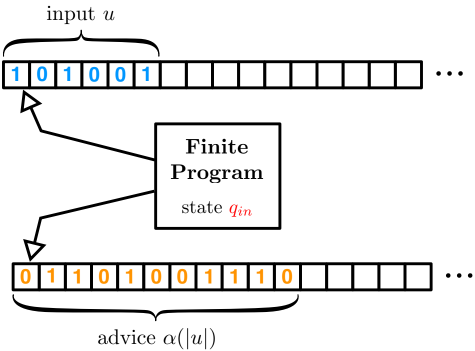

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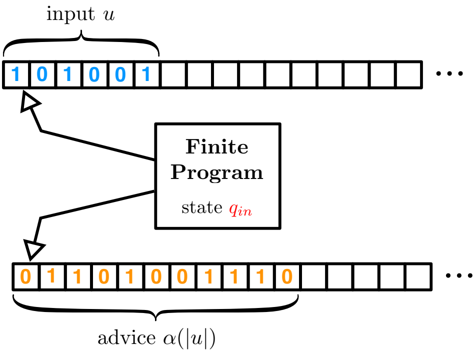

- The class of languages recognized in polynomial time by Turing machines with polynomial advices (TM/poly(A)) is **P/poly**.

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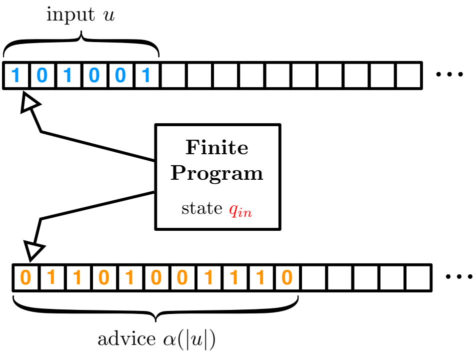
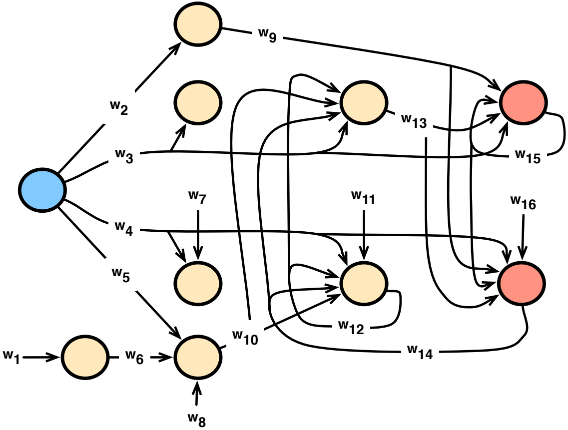
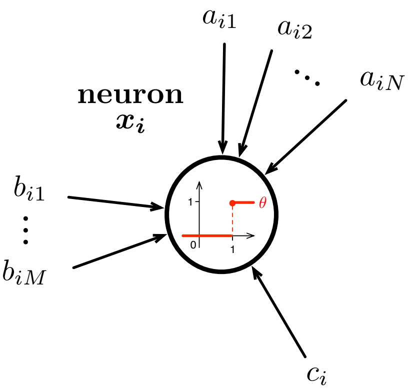


- TM/As are strictly more powerful than TMs: **P/poly** $\supsetneq$ **P**
  They are *super-Turing*...

Recurrent Neural Network

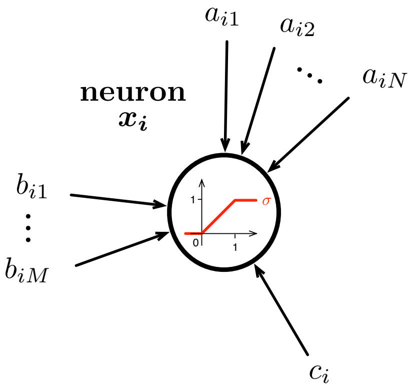


Boolean Recurrent Neural Networks



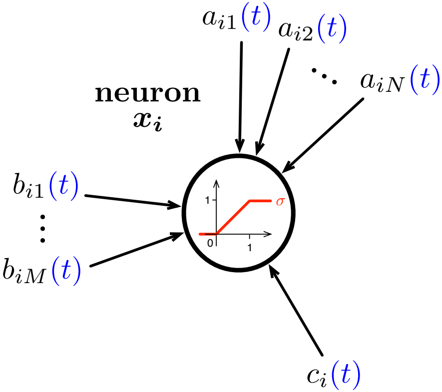
$$x_i(t+1) = \theta\left(\sum_{j=1}^{N} a_{ij} \cdot x_j(t) + \sum_{j=1}^{M} b_{ij} \cdot u_j(t) + c_i\right)$$

Sigmoid Recurrent Neural Networks



$$x_i(t+1) = \sigma\left(\sum_{j=1}^{N} a_{ij} \cdot x_j(t) + \sum_{j=1}^{M} b_{ij} \cdot u_j(t) + c_i\right)$$

Evolving Recurrent Neural Networks



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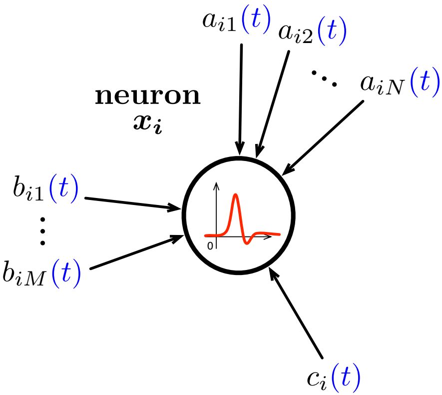
Bio-inspired Neural Networks



Izhikevich or Hodgkin-Huxley differential equations

Results: classical computation

	Boolean	Sigmoid		
		Static	Bi-valued Evolving	Evolving
$\mathbb{Q}$	FSA	TM	TM/poly(A)	TM/poly(A)
	REG	**P**	**P/poly**	**P/poly**
	Kl 56, Mi 67	Si & So 95	Ca & Si 11,14	Ca & Si 11,14
$\mathbb{R}$	FSA	TM/poly(A)	TM/poly(A)	TM/poly(A)
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Results: infinite computation / det.

	Boolean	Sigmoid		
		Static	Bi-valued Evolving	Evolving
$\mathbb{Q}$	Muller FSA $\in BC(\Pi^0_2)$ Ca & Vi 10,14	Muller TM $= BC(\Pi^0_2)$ Ca & Vi 15,16	super-Turing $= BC(\boldsymbol{\Pi^0_2})$ Ca & Vi 15,16	super-Turing $= BC(\boldsymbol{\Pi^0_2})$ Ca & Vi 15,16
$\mathbb{R}$	Muller FSA $\in BC(\Pi^0_2)$ Ca & Vi 10,14	super-Turing $= BC(\boldsymbol{\Pi^0_2})$ Ca & Vi 15,16	super-Turing $= BC(\boldsymbol{\Pi^0_2})$ Ca & Vi 15,16	super-Turing $= BC(\boldsymbol{\Pi^0_2})$ Ca & Vi 15,16

Results: infinite computation / nondet.

	Boolean	Sigmoid		
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Drawbacks of the Constructions

- Computational states of the machines are represented as Boolean states, i.e., spiking configurations of the networks.
  - ⋆ Computational states should rather be represented by *sustained activities of neural assemblies*, e.g., by *attractors*.
- Networks are not robust to cell death, synaptic plasticity, architectural plasticity in general.
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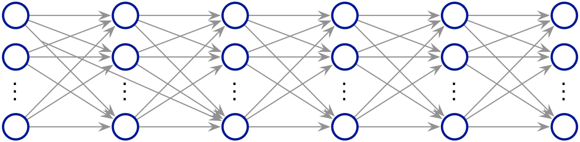
Synfire Chains

- *Synfire chains* have been theorized and experimentally validated as fundamental neuronal structures (Abeles 82).
- They allow for robust and highly precise transmission of information in neural networks.



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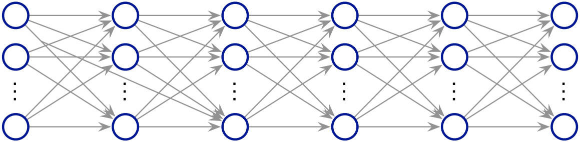

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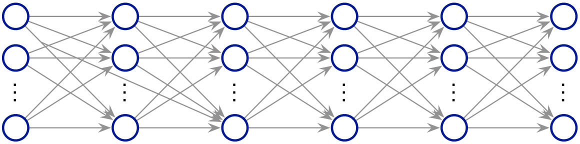

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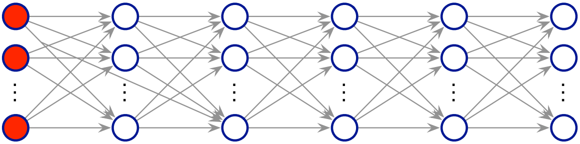

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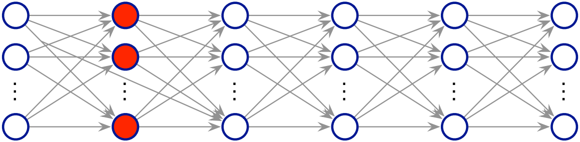

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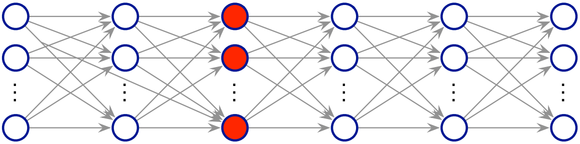

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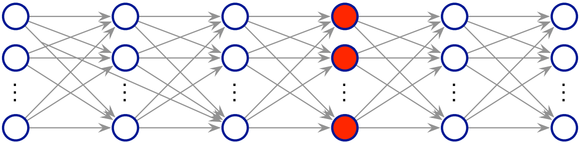

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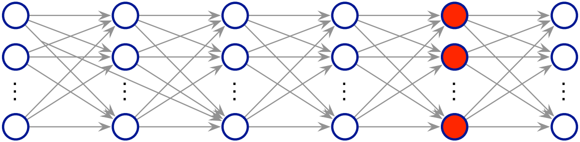

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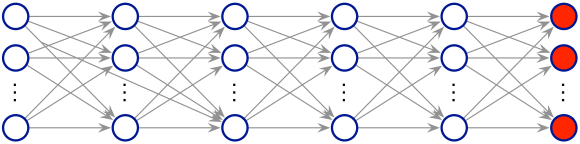

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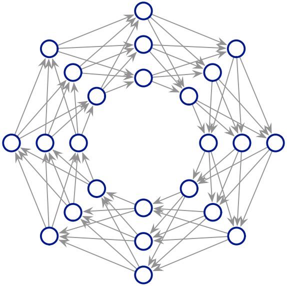

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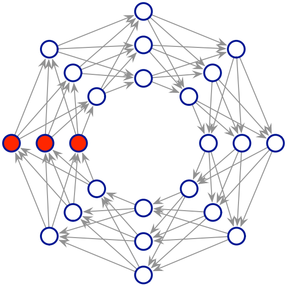

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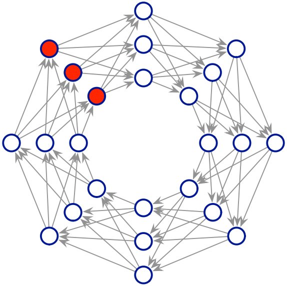

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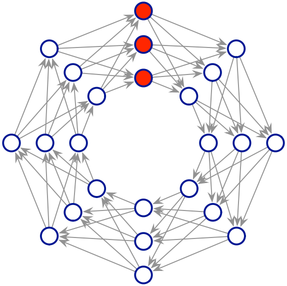

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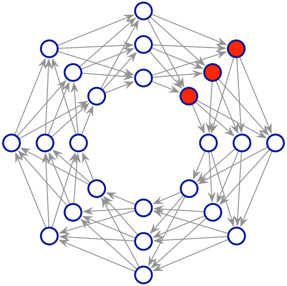

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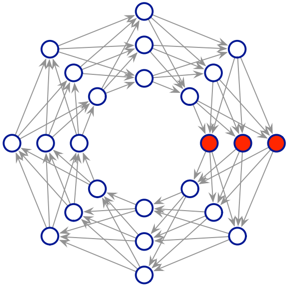

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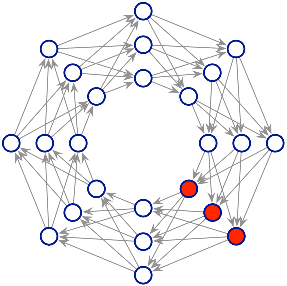

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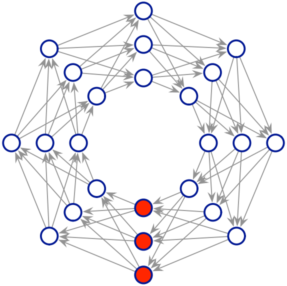

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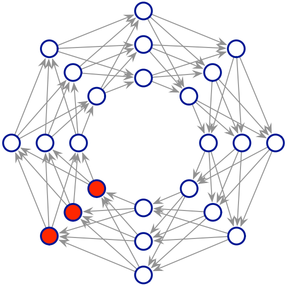

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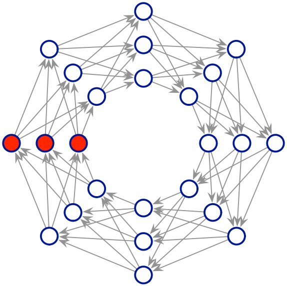
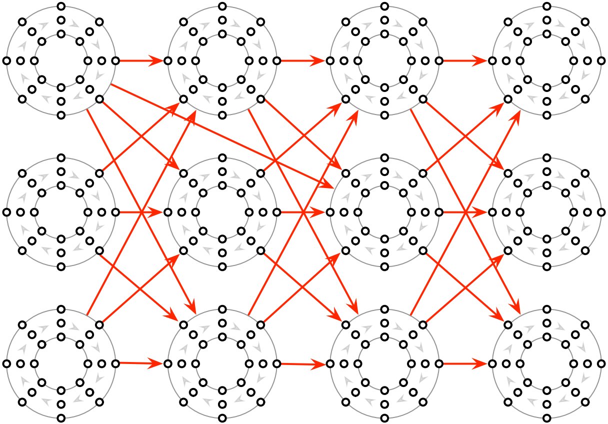
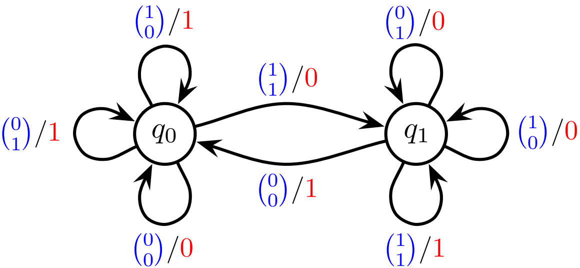


Neural Computation with Synfire Ring Architecture



Binary Adder Automaton



$$
\begin{array}{cccccccc}
 & {}^{1} & 1^{1} & 0^{1} & 1 & 1 & 0^{1} & 1 \\
+ & & 1 & 1 & 1 & 0 & 0 & 1 \\
\hline
 & 1 & 1 & 0 & 0 & 1 & 1 & 0
\end{array}
$$

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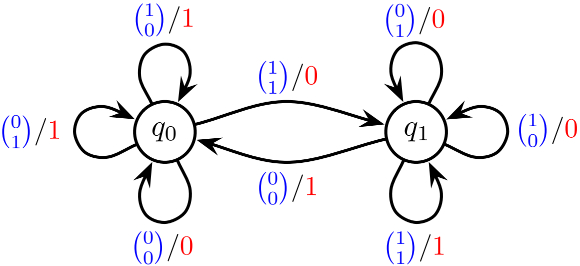
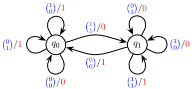
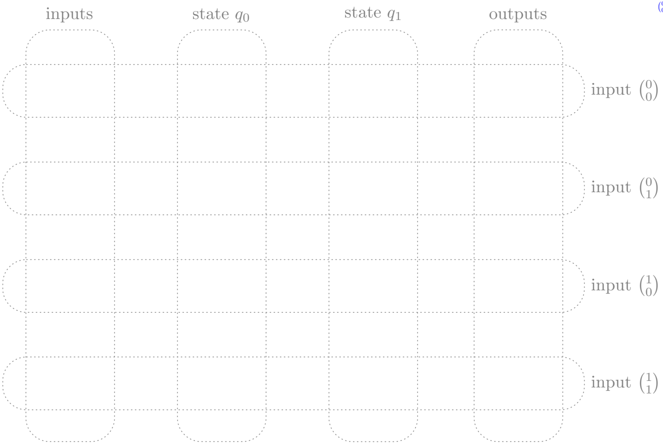


$$\begin{array}{cccccccc} & {}^{1} & 1^{1} & 0^{1} & 1 & 1 & 0^{1} & 1 \\ + & & 1 & 1 & 1 & 0 & 0 & 1 \\ \hline & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{array}$$

Binary Adder Boolean Neural Network





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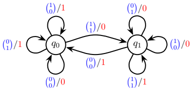

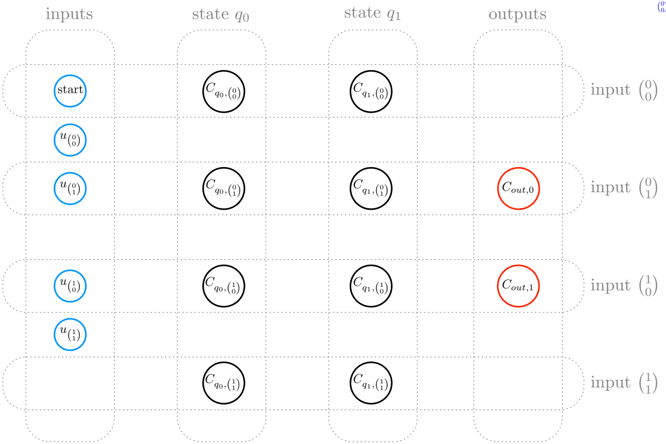

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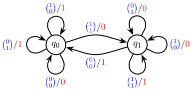

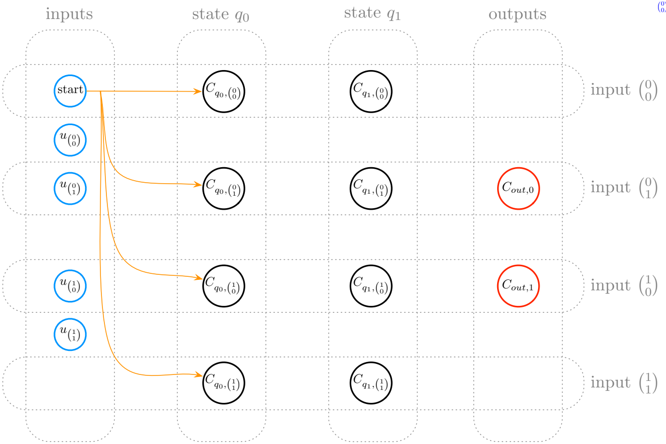

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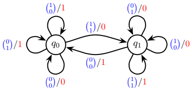

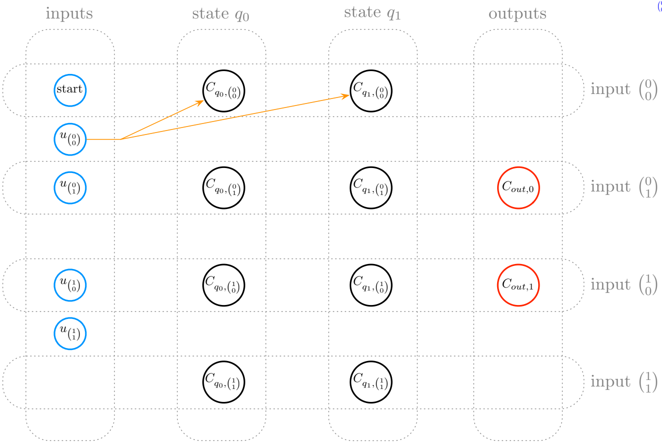

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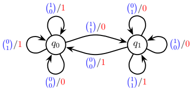
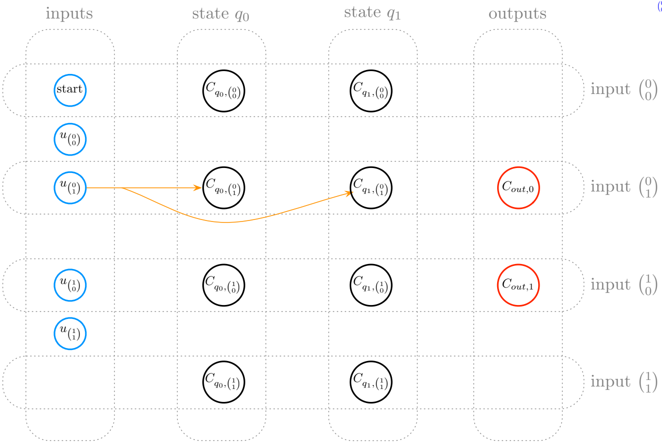
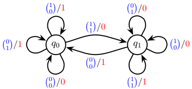
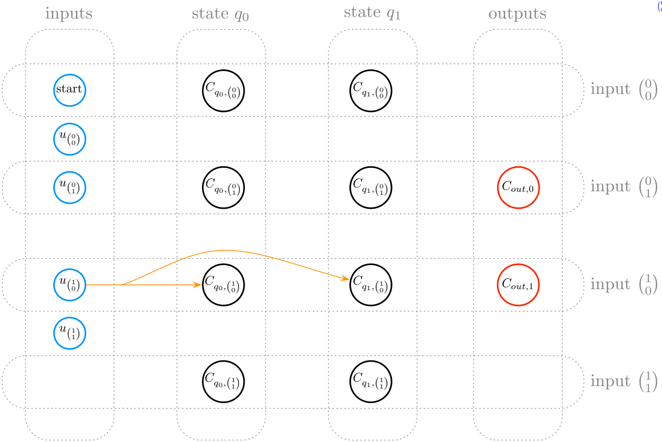




Binary Adder Boolean Neural Network





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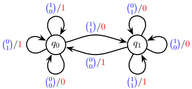
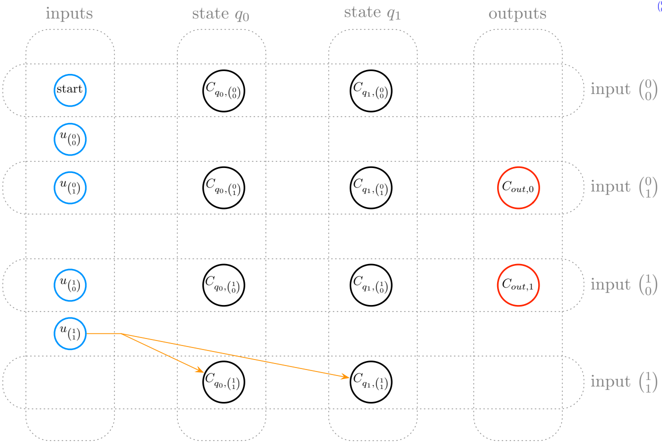
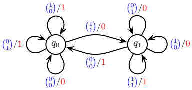
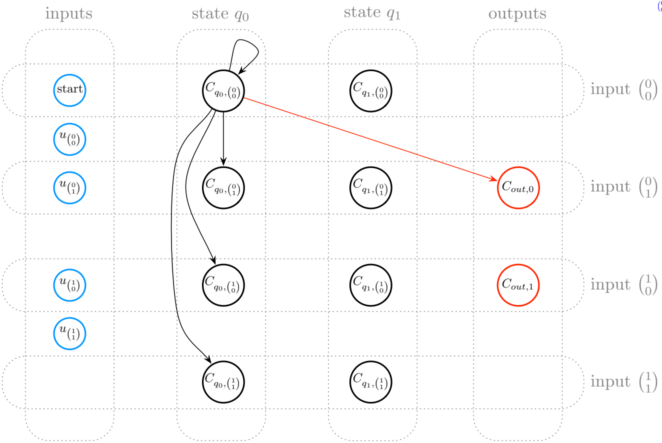




Binary Adder Boolean Neural Network





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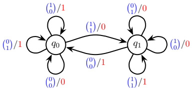

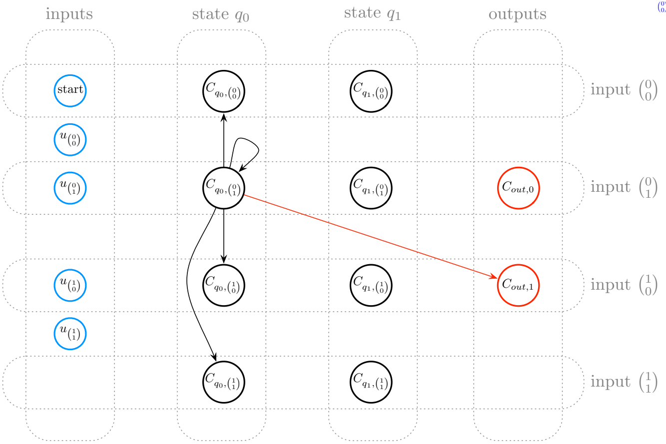

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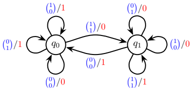

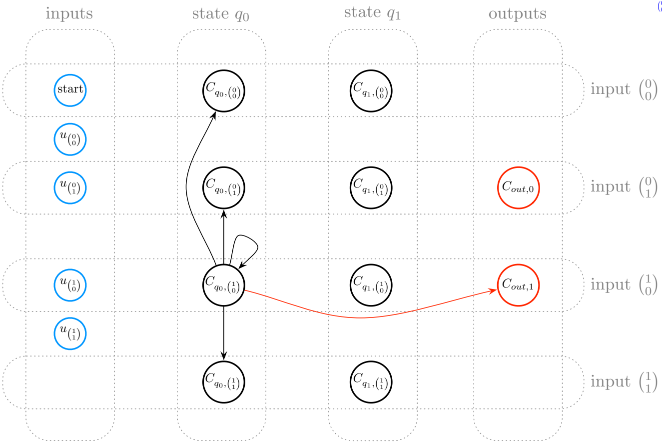

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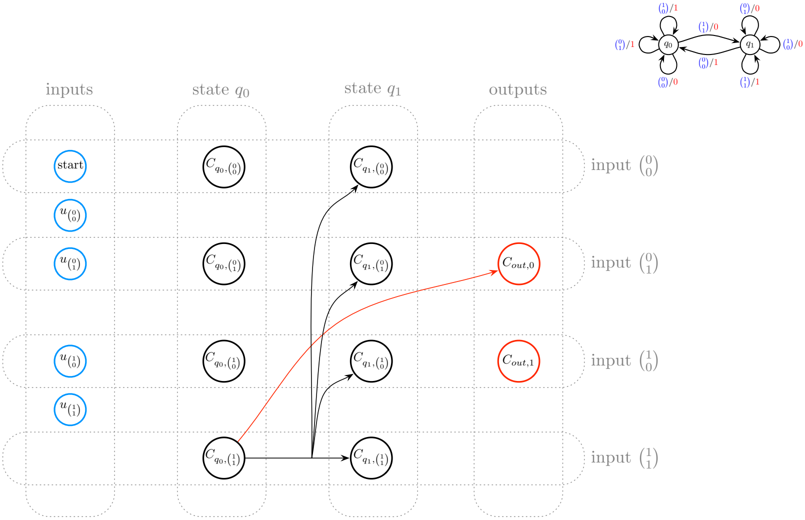

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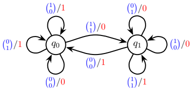

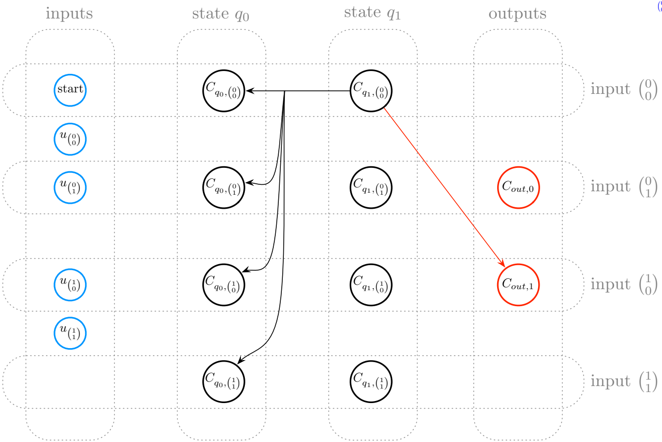

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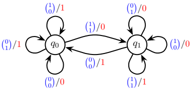

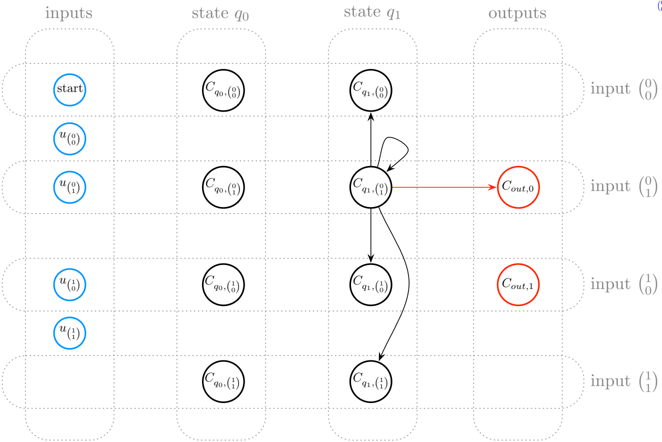

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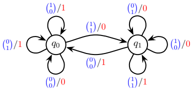
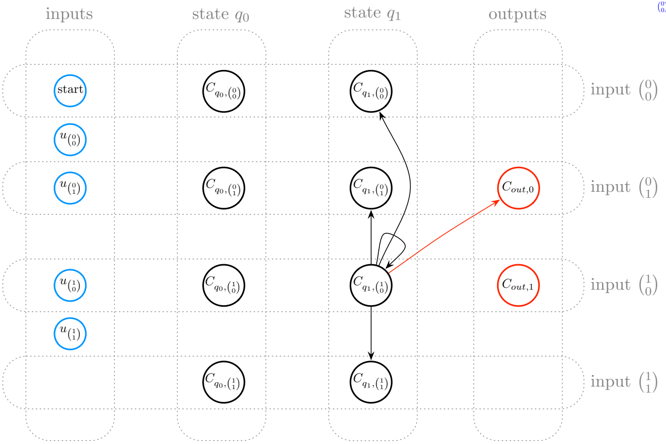
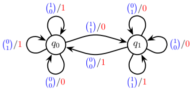
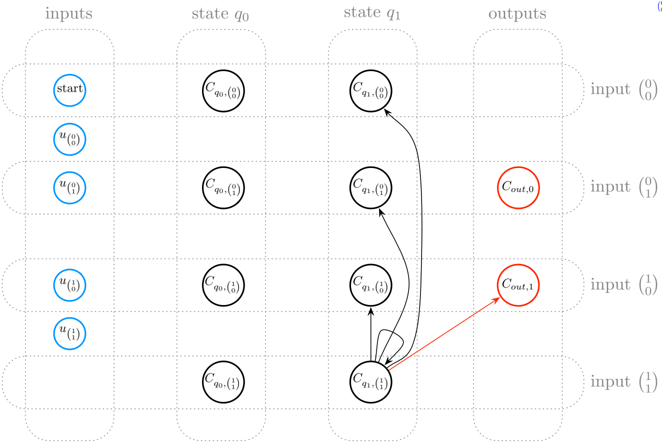




Binary Adder Boolean Neural Network





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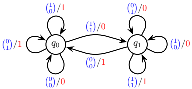
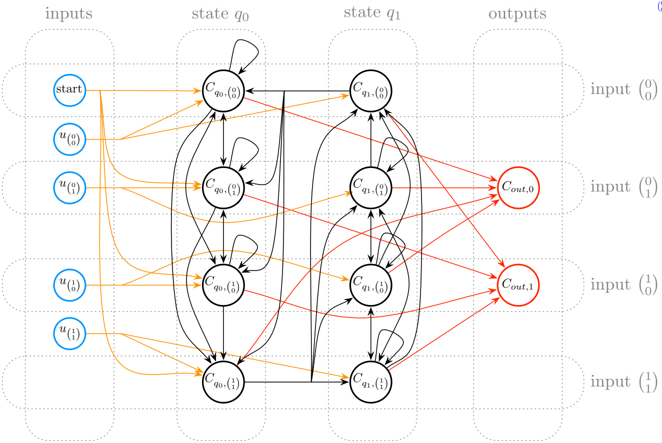
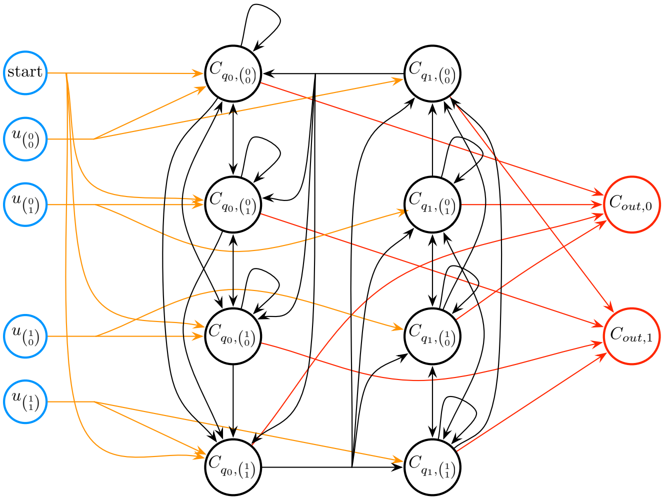
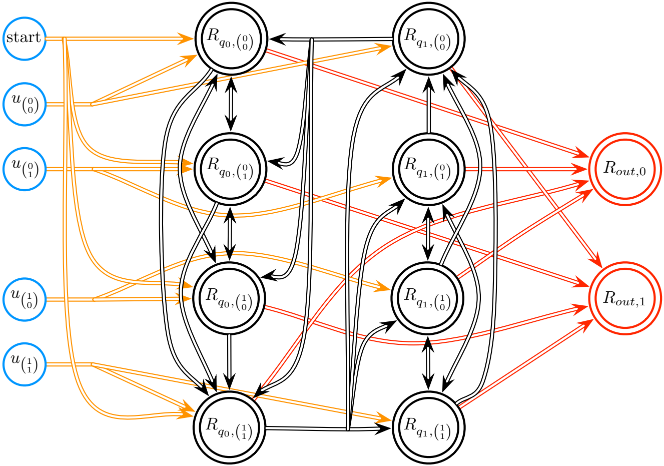




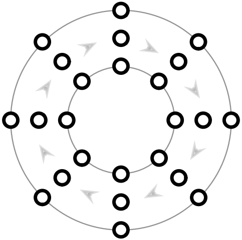
Generalization to Synfire Ring RNNs

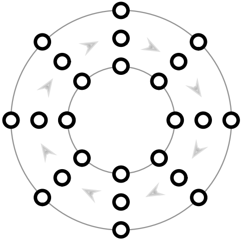


Generalization to Synfire Ring RNNs



Fibres of connections & Inhibitory System





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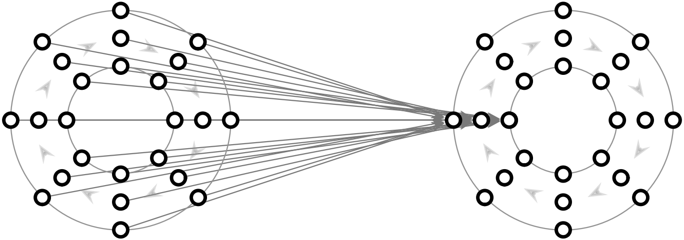

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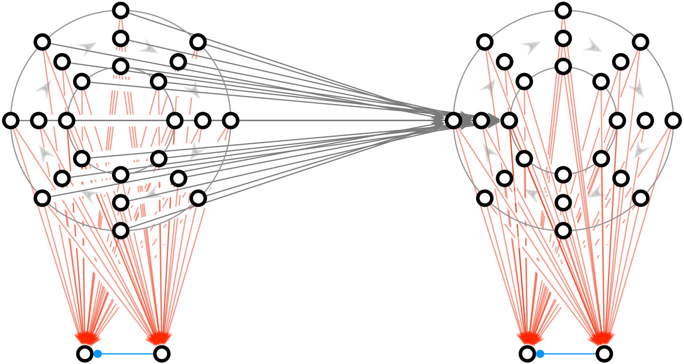

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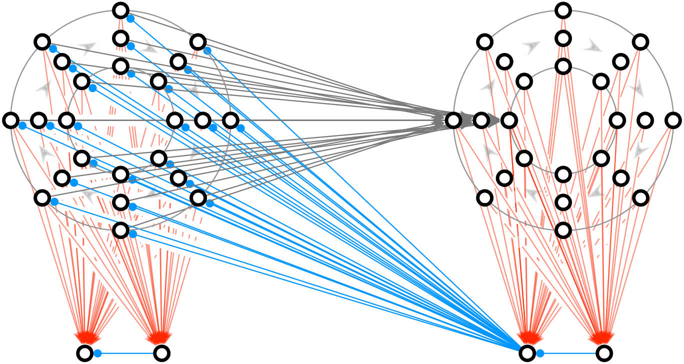

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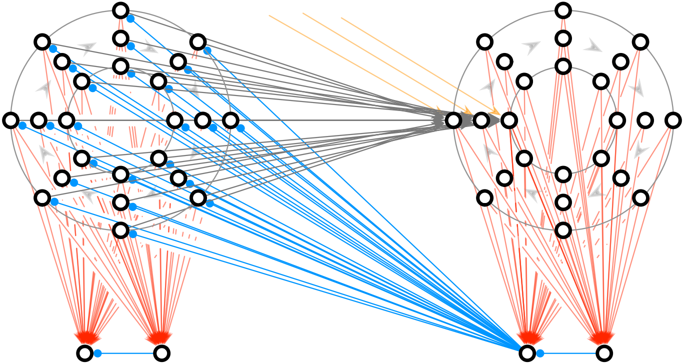

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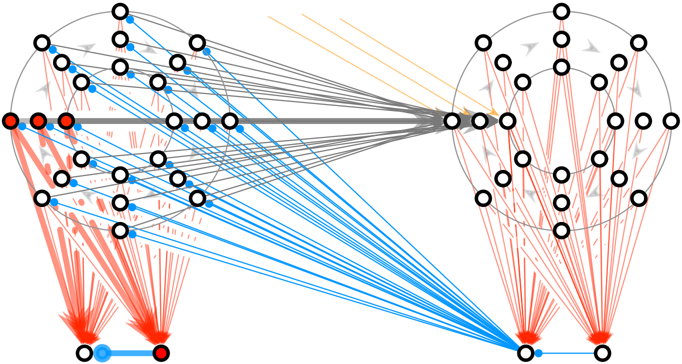

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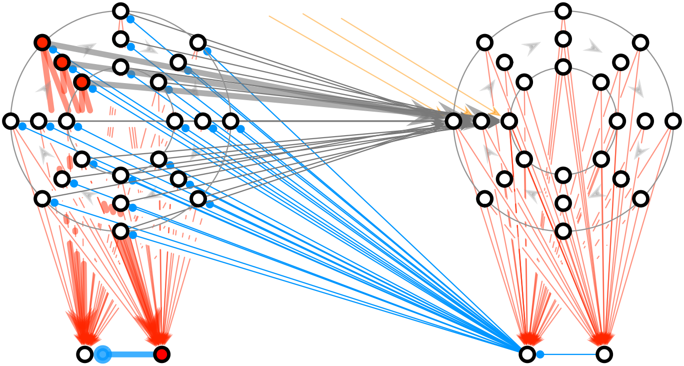

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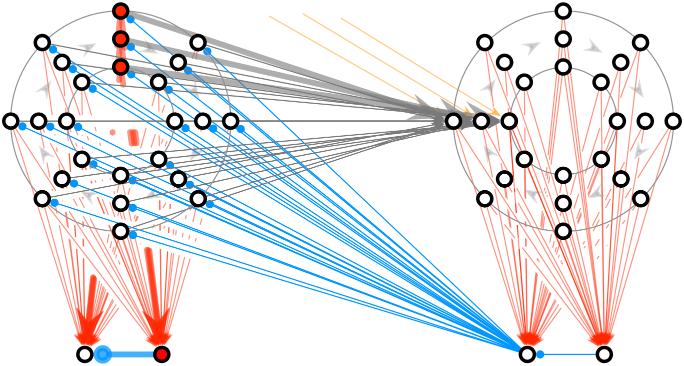

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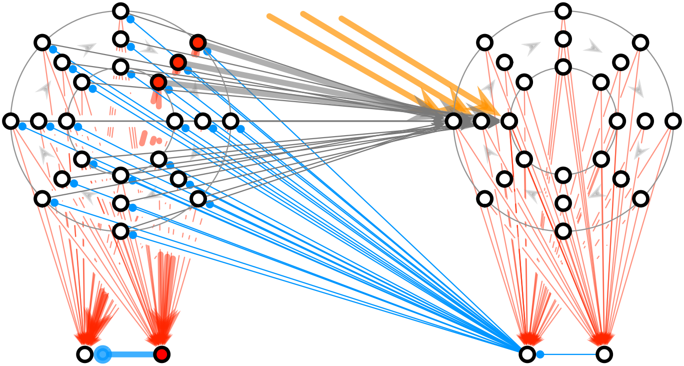

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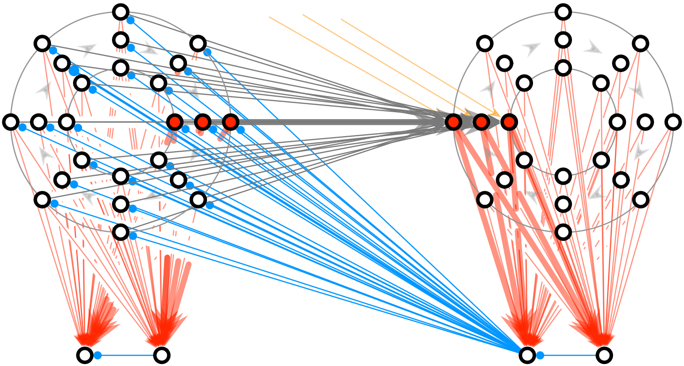

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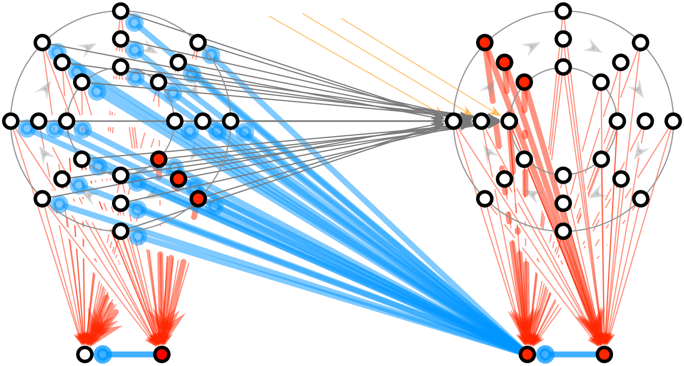

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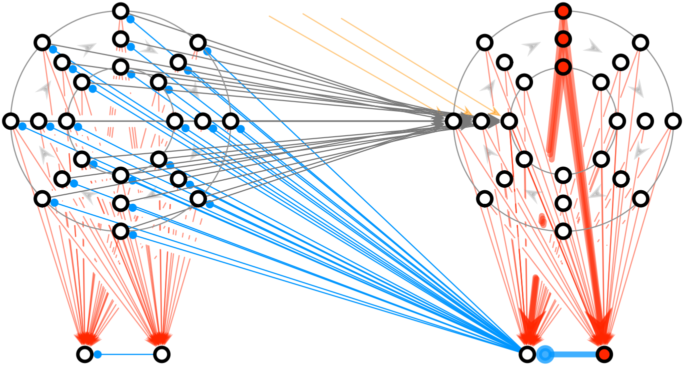

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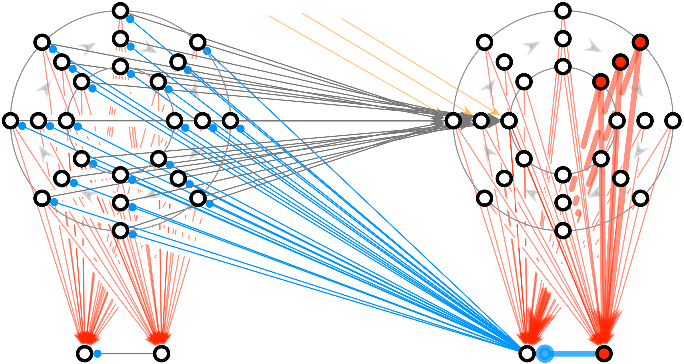
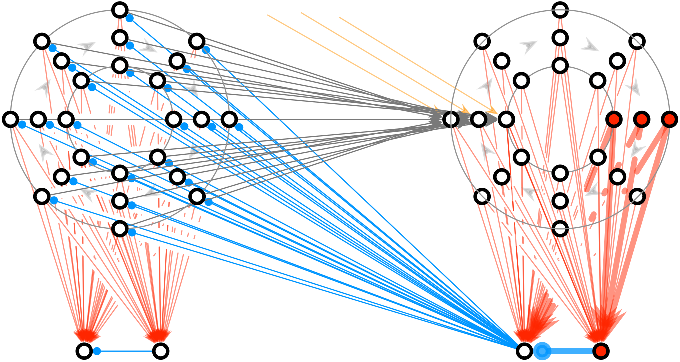


Fibres of connections & Inhibitory System



Simulation

Play movie...

Automata & Boolean RNNs with Synfire Rings

Since the construction is generic, one has the following result:

Theorem

Any finite state automaton can be simulated by some Boolean neural network composed of synfire rings.

Generalizations

- We generalize these results to the contexts of more biological neural networks:

1. Izhikevich spiking neural networks (not here)
2. Hodgkin-Huxley neural networks

Hodgkin-Huxley Neurons

$$\alpha_n(V_m) = \frac{0.01(10 - V_m)}{\exp(\frac{10-V_m}{10}) - 1} \qquad \beta_n(V_m) = 0.125\exp(\frac{-V_m}{80})$$

$$\alpha_m(V_m) = \frac{0.1(25 - V_m)}{\exp(\frac{25-V_m}{10}) - 1} \qquad \beta_m(V_m) = 4\exp(\frac{-V_m}{18})$$

$$\alpha_h(V_m) = 0.07\exp(\frac{-V_m}{20}) \qquad \beta_h(V_m) = \frac{1}{\exp(\frac{30-V_m}{10}) + 1}$$

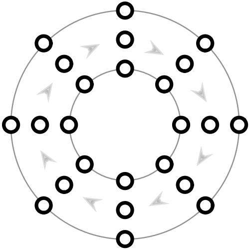
$$\frac{dn}{dt} = \alpha_n(V_m)(1-n) - \beta_n(V_m)n$$

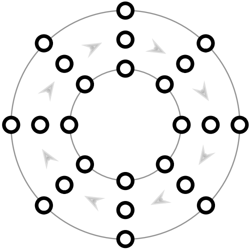
$$\frac{dm}{dt} = \alpha_m(V_m)(1-m) - \beta_m(V_m)m$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1-h) - \beta_h(V_m)h$$

$$C_m\frac{dV_m}{dt} = I - \bar{g}_{\mathsf{K}}n^4(V_m - V_K) - \bar{g}_{\mathsf{Na}}m^3h(V_m - V_{Na}) - \bar{g}_l(V_m - V_l)$$

Fibres of connections & Inhibitory System





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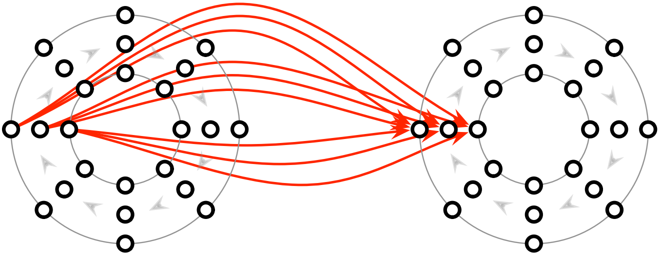

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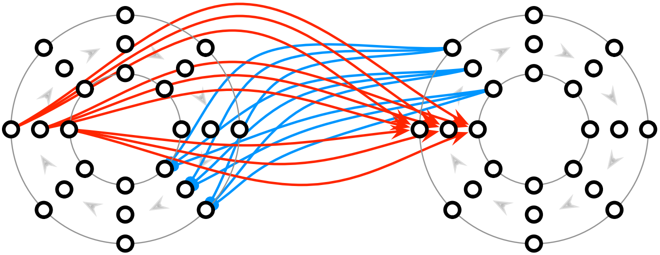

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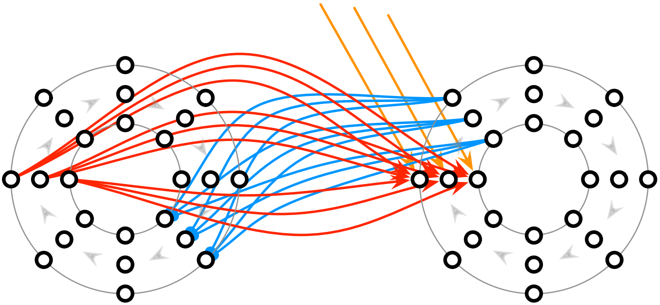

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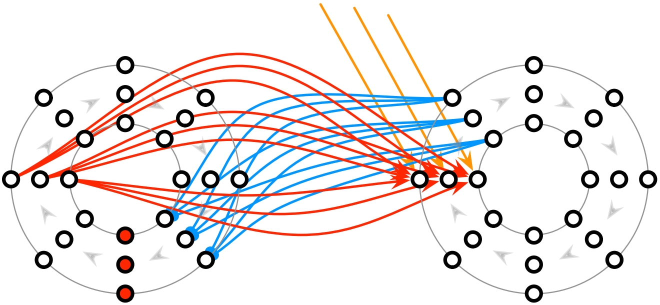

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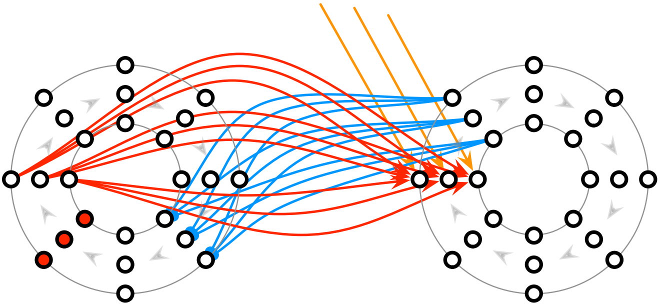

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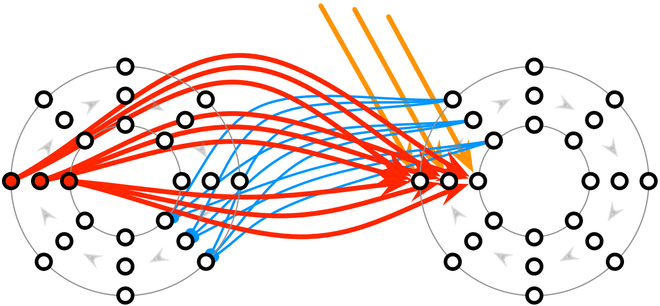

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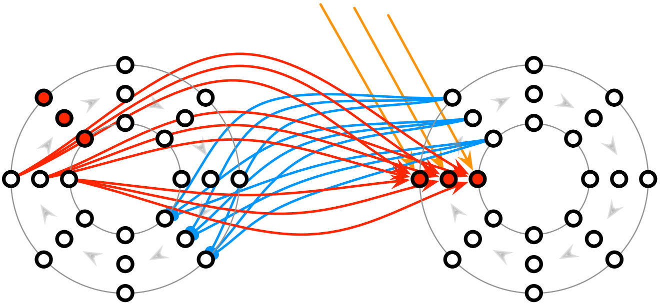

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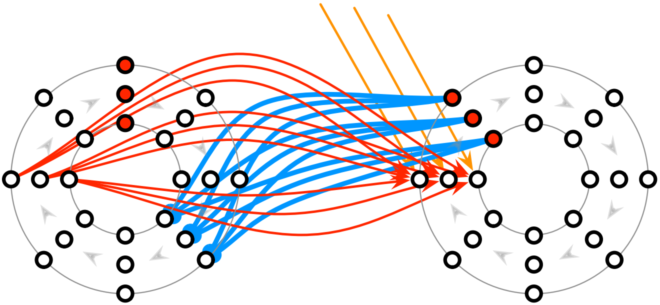

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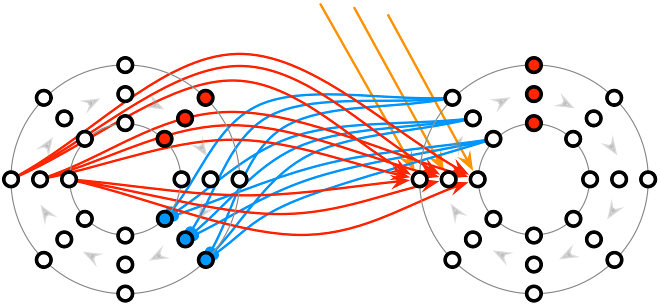

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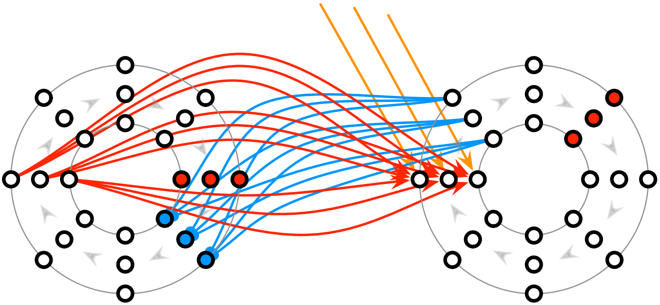
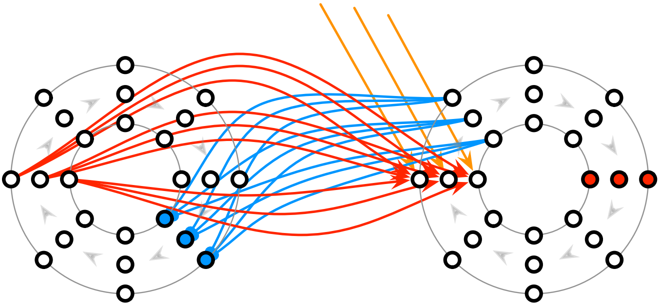


Fibres of connections & Inhibitory System



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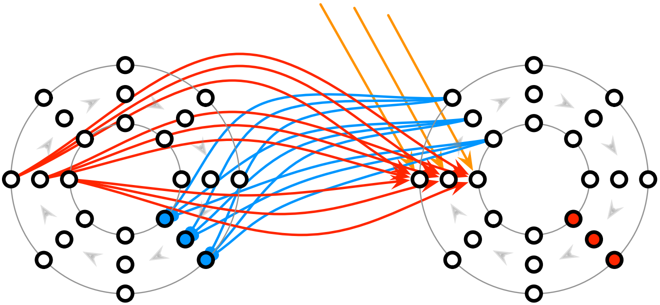

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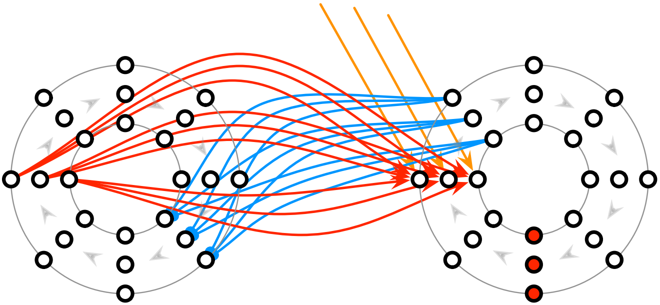

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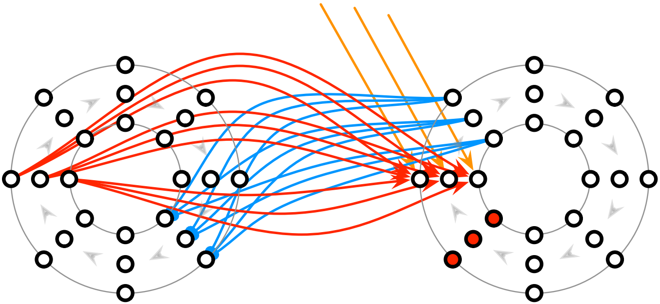


Simulation

Play movie...

Automata & Hodgkin-Huxley RNNs with Synfire Rings

Since the construction is generic, one has the following result:

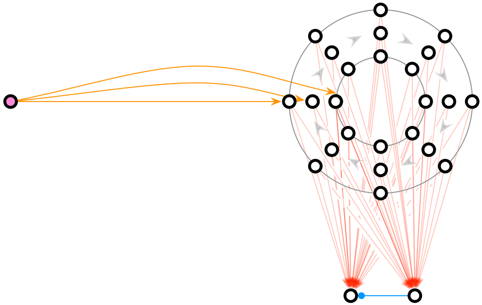
Result

Any finite state automaton can be simulated by some Hodgkin-Huxley based neural network composed of synfire rings.

Fibres of connections & Inhibitory System

We consider the following kinds of (fibres of) connections.

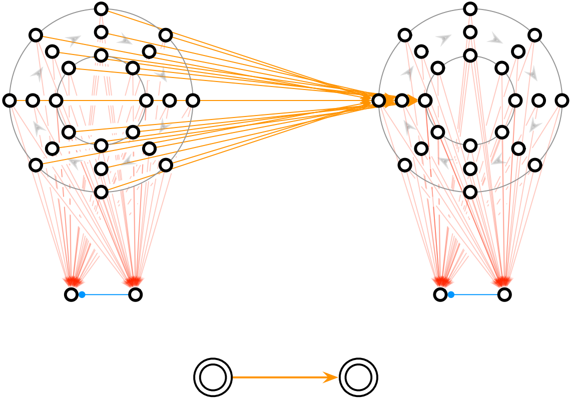
- cell to ring excitatory



Fibres of connections & Inhibitory System

We consider the following kinds of (fibres of) connections.

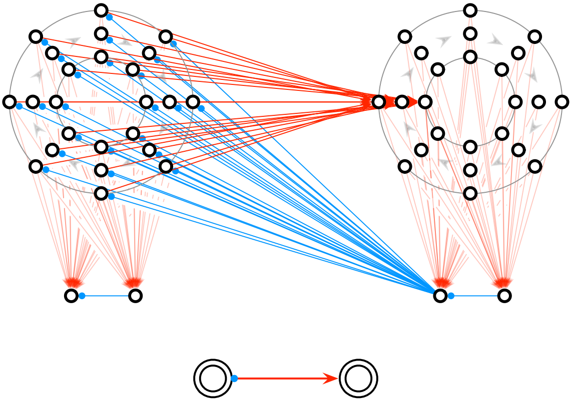
- constant excitatory



Fibres of connections & Inhibitory System

We consider the following kinds of (fibres of) connections.

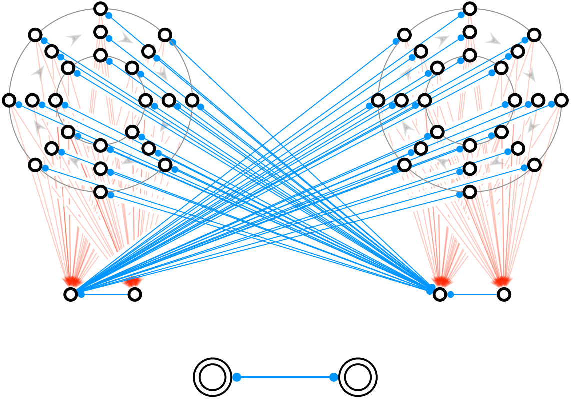
- constant excitatory / one-shot inhibitory



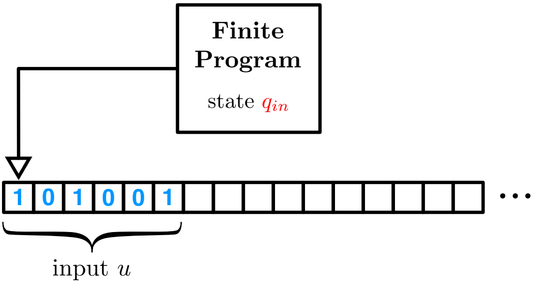
Fibres of connections & Inhibitory System

We consider the following kinds of (fibres of) connections.

- one-shot inhibitory / one-shot inhibitory



General Architecture



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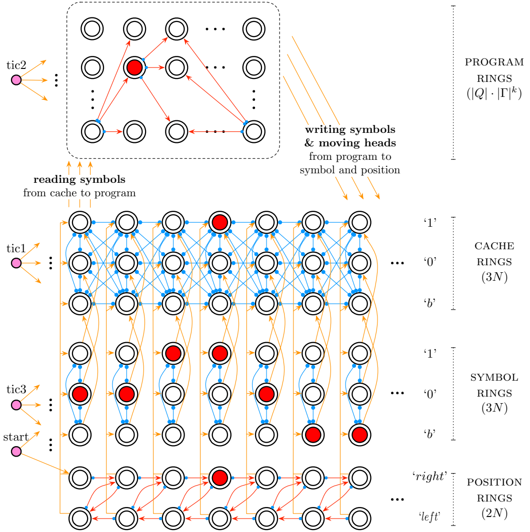

Turing machines & Boolean RNNs with Synfire Rings

Since the construction is generic, one has the following result:

Theorem

Any bounded-space Turing machine can be simulated by some Boolean neural network composed of synfire rings.

Simulation of the bounded-space Turing machine recognizing the non-regular language $L = \{0^n 1^n 0^n : n \geq 0\}$

Play movie...

Conclusions

- We introduced a new paradigm for abstract neural computation based on the concept of synfire rings.
- Research project:

⋆ Develop bio-inspired and ML-based learning algorithms on the synfire ring architecture.

⋆ Neuromorphic implementation of these bio-inspired neural networks.

⋆ Biological implementation via cultures of neurons: TOWARDS NEURONAL COMPUTERS...

Conclusions

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  - ⋆ Neuromorphic implementation of these bio-inspired neural networks.
  - ⋆ Biological implementation via cultures of neurons: TOWARDS NEURONAL COMPUTERS...

Conclusions

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- Research project:

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⋆ Neuromorphic implementation of these bio-inspired neural networks.

⋆ Biological implementation via cultures of neurons: towards neuronal computers...

Conclusions

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Thank you!