A Classification of Neural Networks

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University Joseph Fourier – Grenoble 1

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Outline

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We extend this result for infinite word reading machines, and provide a refined classification of simple neural nets.

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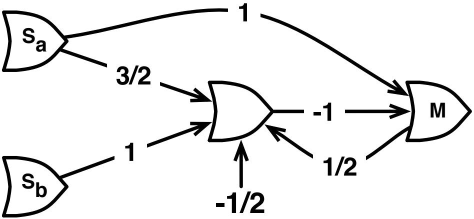
Neural nets and automata

McCulloch & Pitts' neural net



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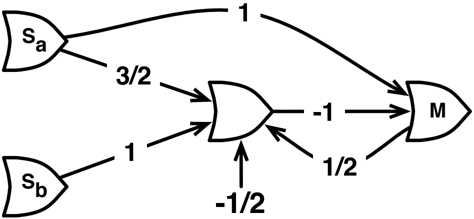

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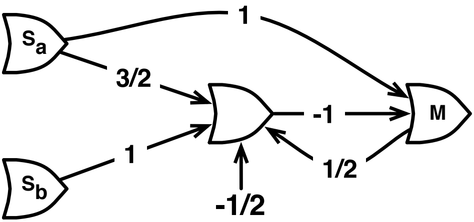

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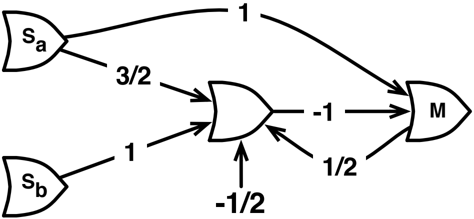

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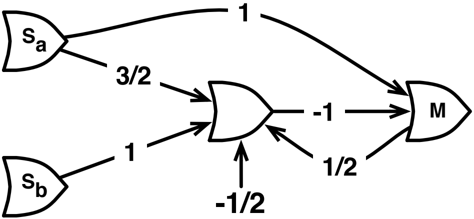
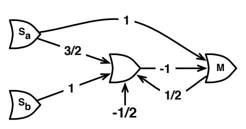


The set of stimuli accepted by this net is the *language recognized by this net.*

Definition

A *MP neural net* is a tuple $\mathcal{N} = (C, S, M, w, b)$, where



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- $S \subseteq C$ is a set of sensory cells,
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- $w : C \times C \to \mathbb{Q}$ gives the weights of the connections,
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The dynamic of cell i is given by

$$x_i(t+1) = \sigma\left(\sum_{j \in C} w(j,i) \cdot x_j(t) + b(i)\right), \text{ where } \sigma(x) = \begin{cases} 0 & \text{if } x < 1, \\ 1 & \text{if } x \geq 1. \end{cases}$$

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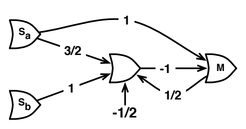

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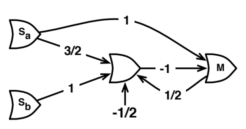

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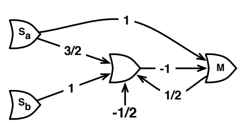

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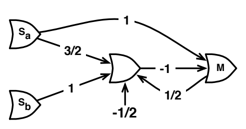

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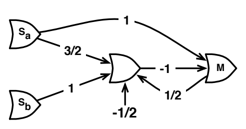

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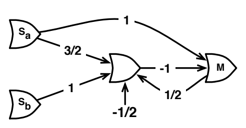

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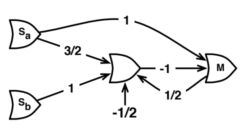

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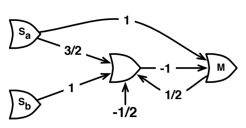

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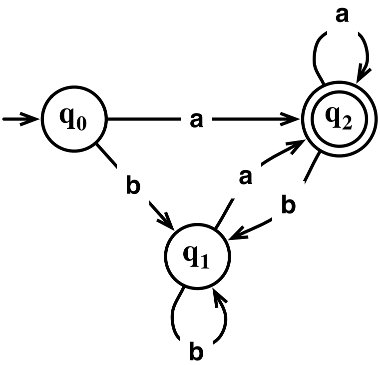
Büchi automaton



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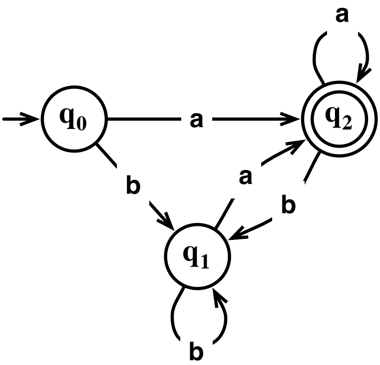

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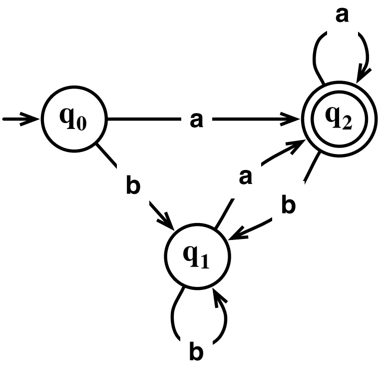

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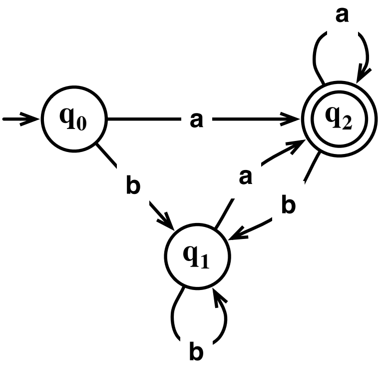

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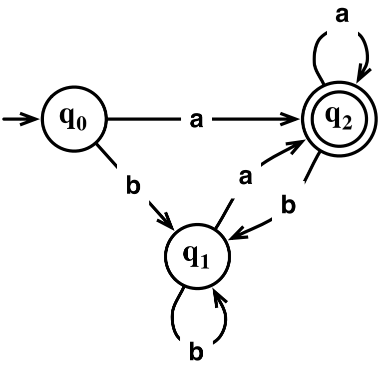

The set of words accepted by this automaton is the *language recognized by this automaton.*

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A *det. Büchi automaton* is a tuple $\mathcal{A} = (Q, A, \delta, i, F)$, where

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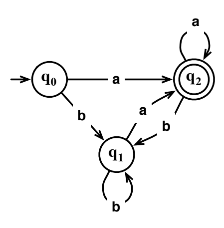

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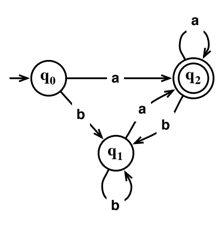

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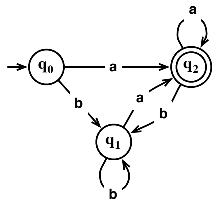

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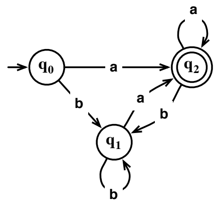

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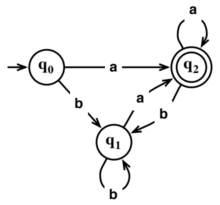

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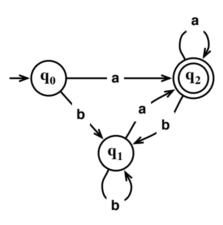

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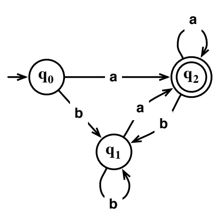

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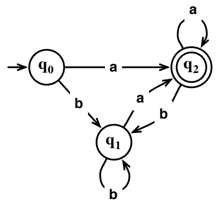

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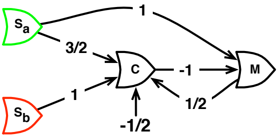
Theorem

Let L be an ω-language. Then the following are equivalent:

1. *L is recognizable by some MP neural net;*
2. *L is recognizable by some det. Büchi automata;*

Proof: $(1) \Rightarrow (2)$: From MP neural nets to det. Büchi automata…



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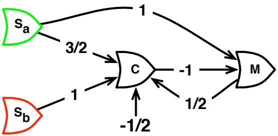

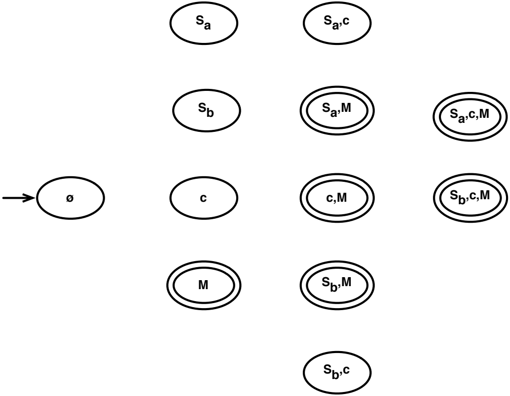

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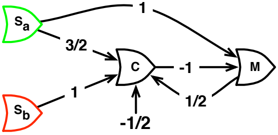

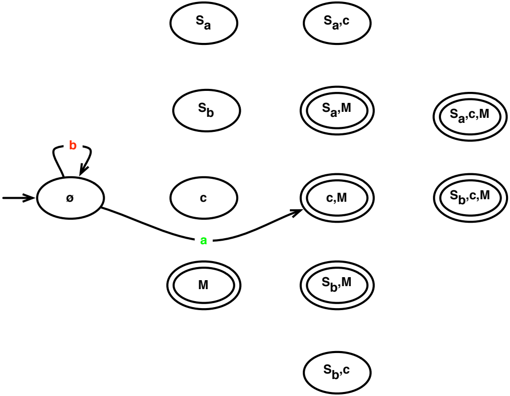

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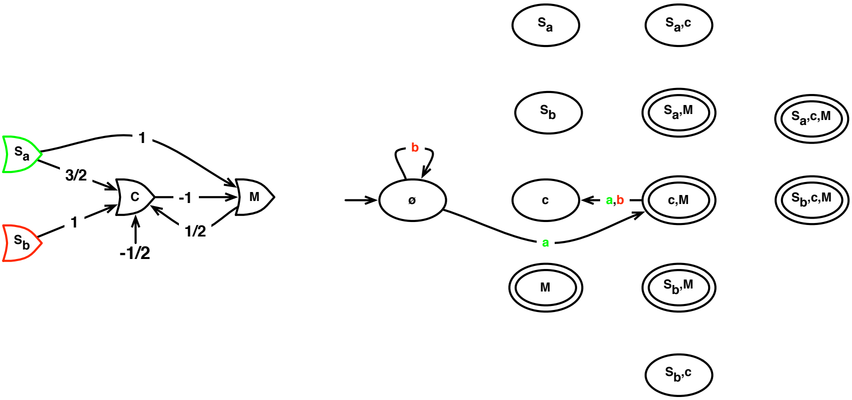

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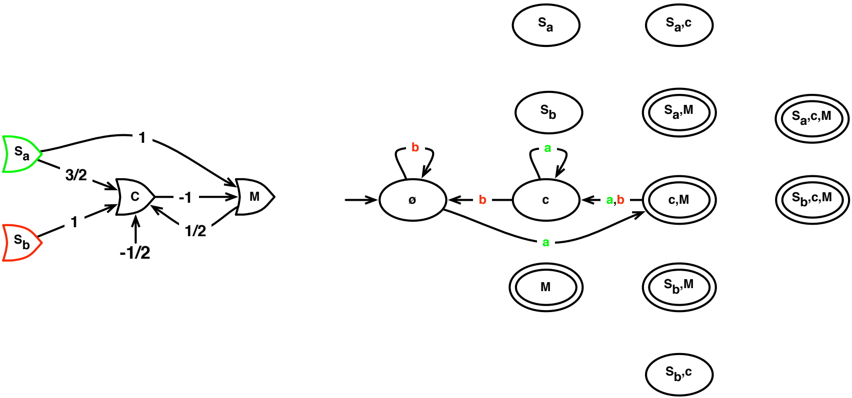

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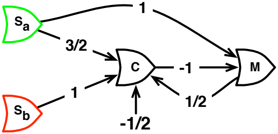
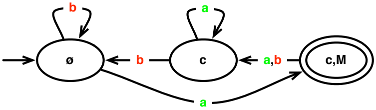
Proof: $(1) \Rightarrow (2)$: From MP neural nets to det. Büchi automata…





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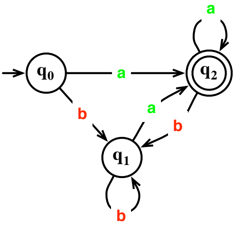

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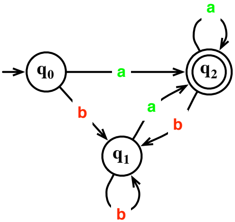

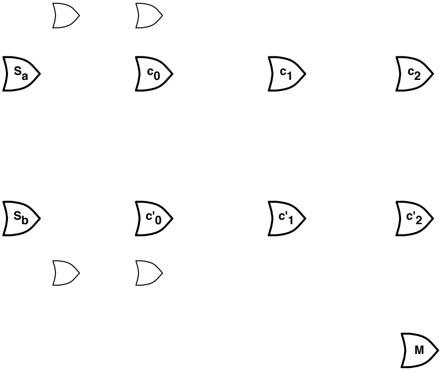

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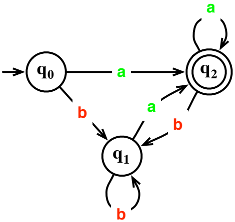

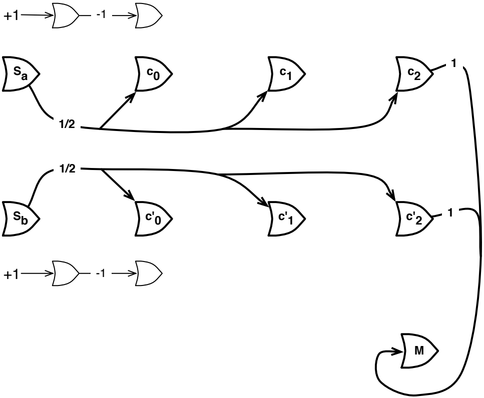

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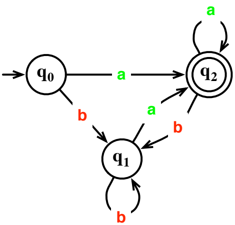

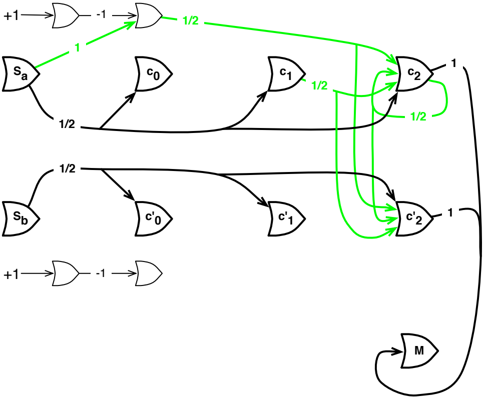

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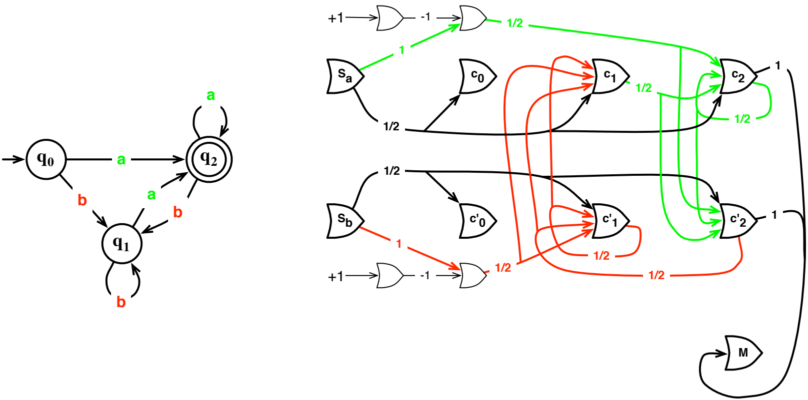

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A classification of neural nets

Let A and B be two ω-languages:

$$A \leq_W B \quad \text{iff} \quad \text{there exists } f \text{ continuous s.t. } A = f^{-1}(B)$$
$$\text{iff} \quad \text{there exists } f \text{ continuous s.t. } x \in A \Leftrightarrow f(x) \in B$$

Then as usual

$$A <_W B \quad \text{iff} \quad A \leq_W B \text{ and } B \not\leq_W A$$
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Indeed we have …

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Definition

- The collection of ω-languages recognized by Det-Büchi automata ordered by $\leq_W$ is called the *Det. Büchi hierarchy*.
- The collection of ω-languages recognized by MP-neural nets ordered by $\leq_W$ is called the *MP-neural hierarchy*.

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- *The Det-Büchi hierarchy is well-founded, has width 2 and height $\omega + 1$.*
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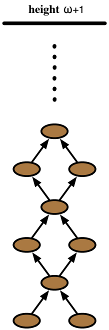

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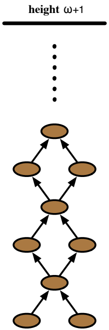

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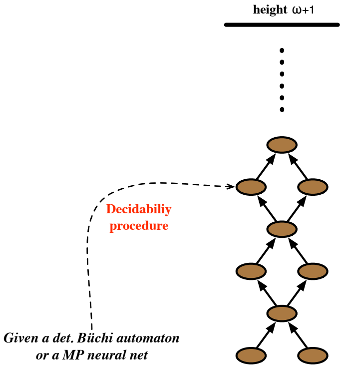

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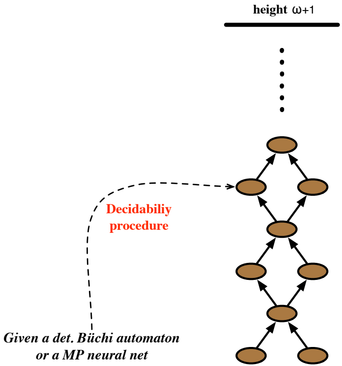
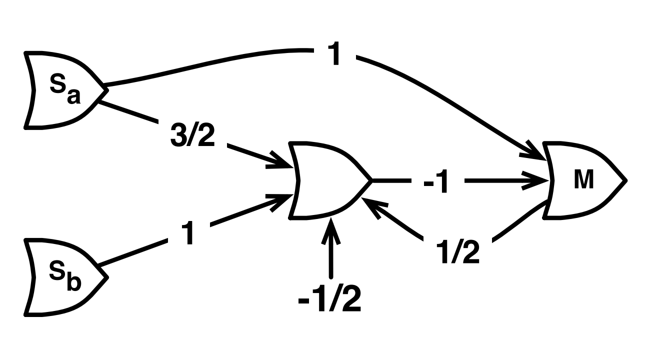


Example

Consider the following MP neural net $\mathcal{N}$.



Then the degree of $L(\mathcal{N})$ in the MP-neural hierarchy is ω.

Significant extension of this work:

we are also able to establish another equivalence between some kind of McCulloch and Pitts' neural nets and Muller automata; this induces another decidable hierarchy of networks of height ω^ω.

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