Automata Computation with Hodgkin-Huxley Neural Networks Composed of Synfire Rings

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Introduction

- This work focuses on the simulation of digital computers by biological neural networks.
- It is known that first-order discrete-time recurrent neural networks with integer, rational or real weights are computationally equivalent to automata (Kleene 56, Minsky 67), Turing machines (Siegelmann & Sontag 95), and Turing machines with advices (super-Turing) (Siegelmann & Sontag 94, Cabessa & Siegelmann 14), respectively.
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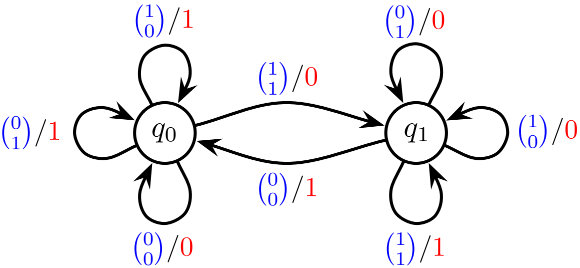
Binary Adder Automaton



	^{1}	1^1	0^1	1	1	0^1	1
+		1	1	1	0	0	1
	1	1	0	0	1	1	0

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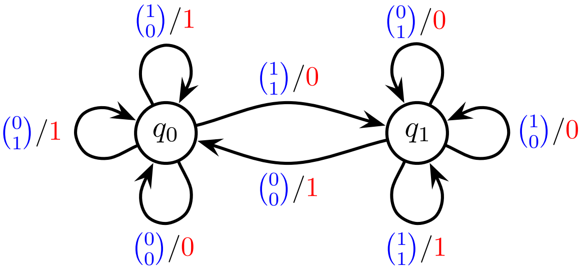

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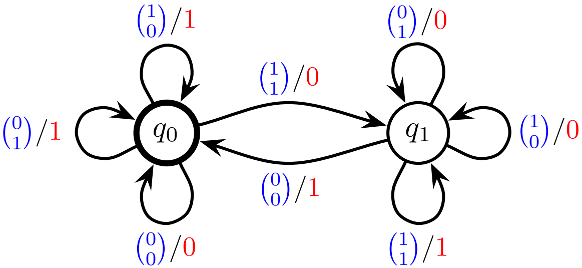

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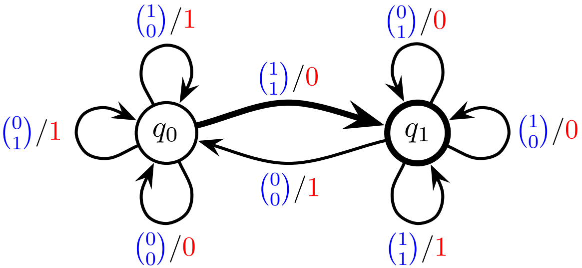

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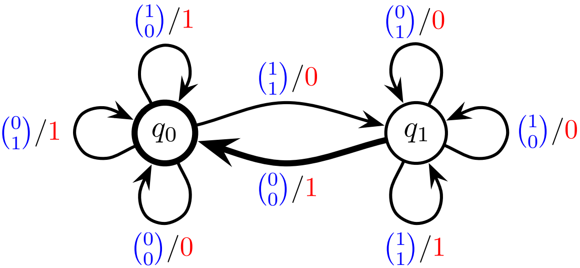

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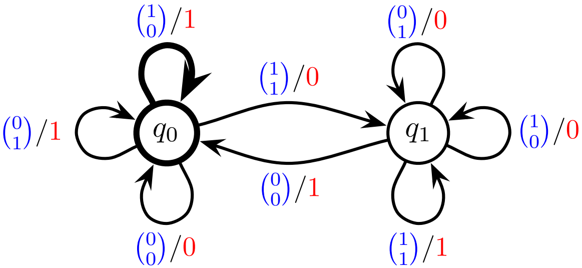

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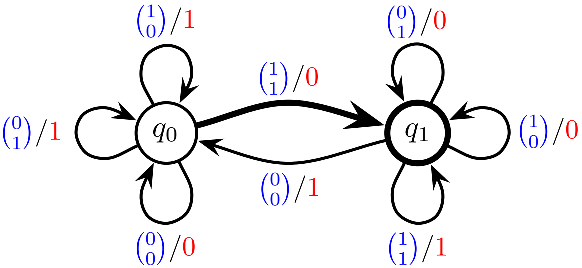

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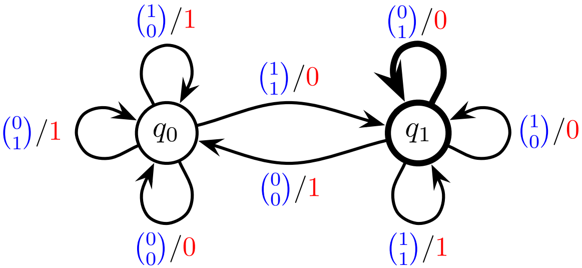

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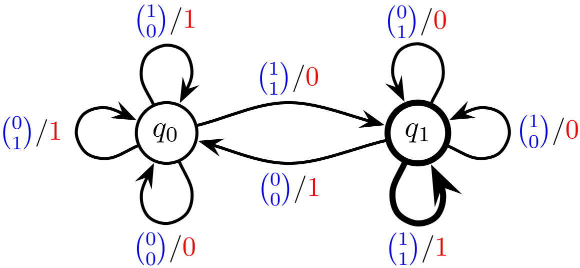

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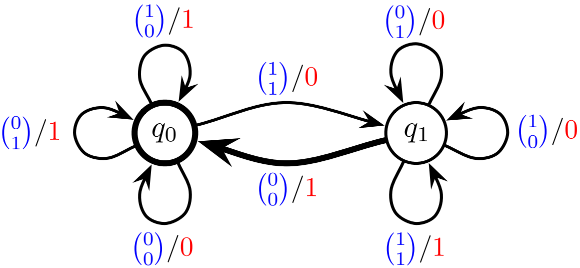

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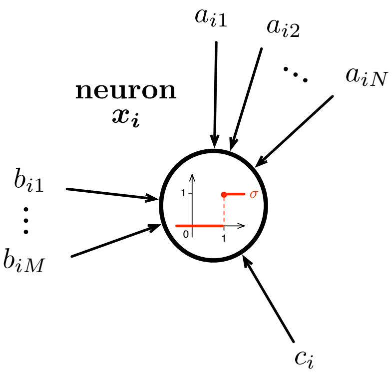
Automata & Boolean RNNs

Theorem (Minsky 1967)

Any finite state automaton can be simulated by some Boolean recurrent neural network.

Boolean Recurrent Neural Networks



$$x_i(t+1) = \theta\left(\sum_{j=1}^{N} a_{ij} \cdot x_j(t) + \sum_{j=1}^{M} b_{ij} \cdot u_j(t) + c_i\right)$$

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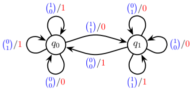

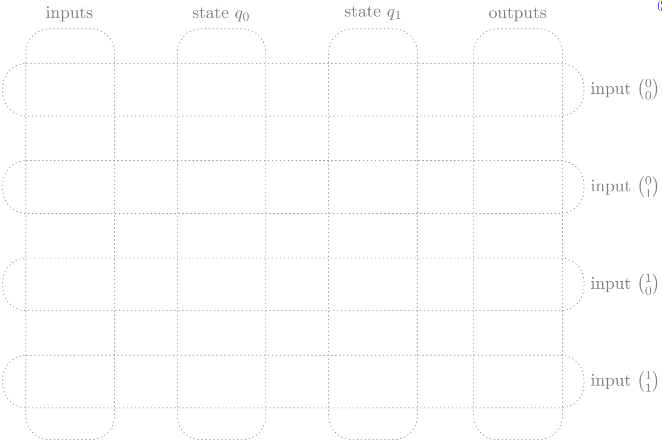

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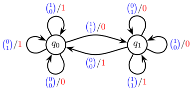

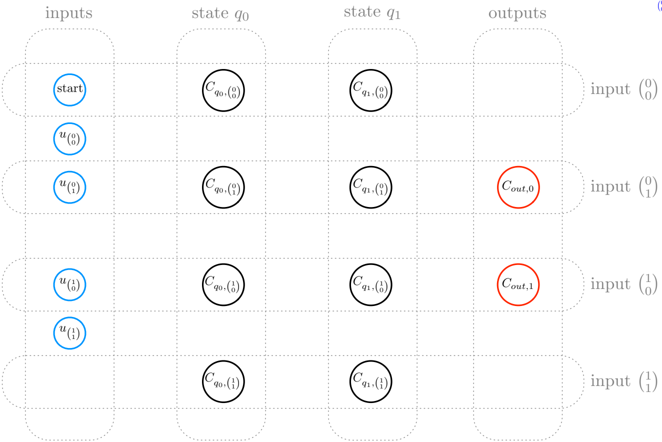

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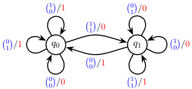

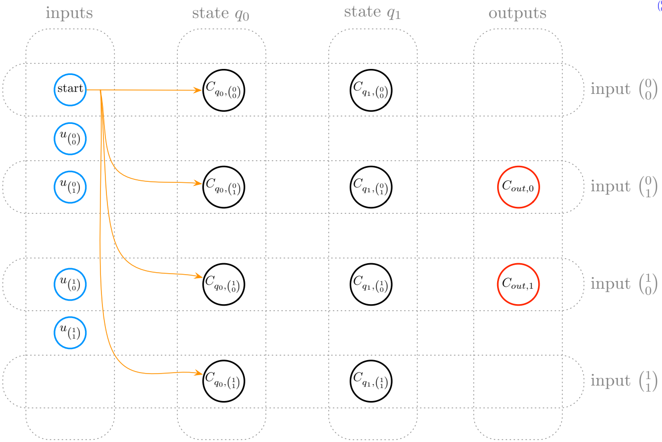

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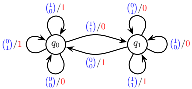

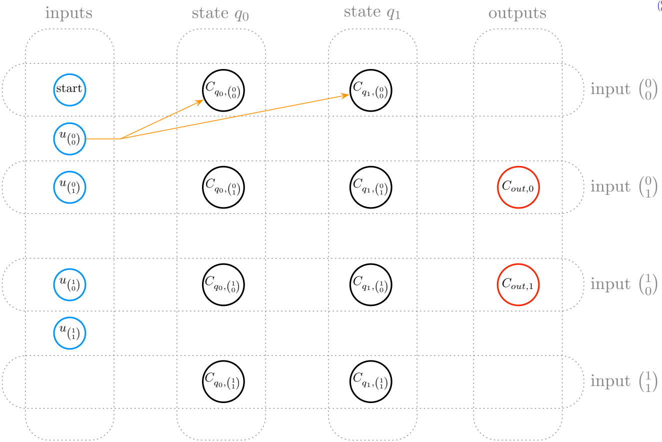

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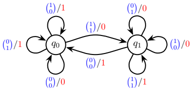

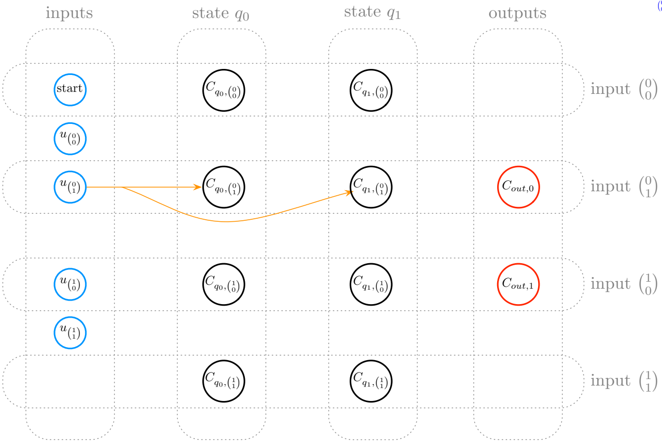

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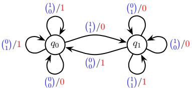

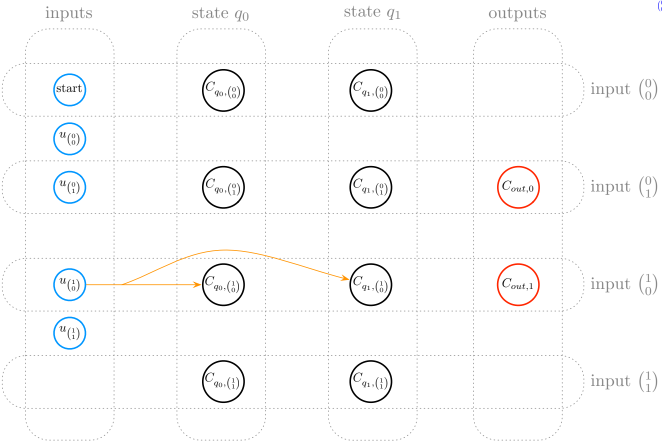

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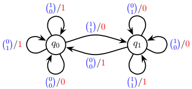

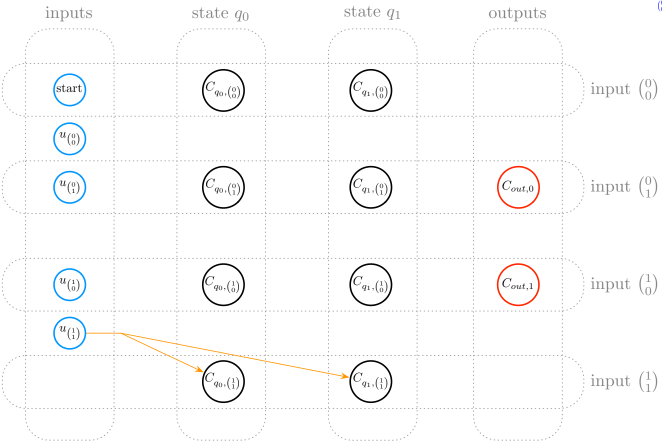

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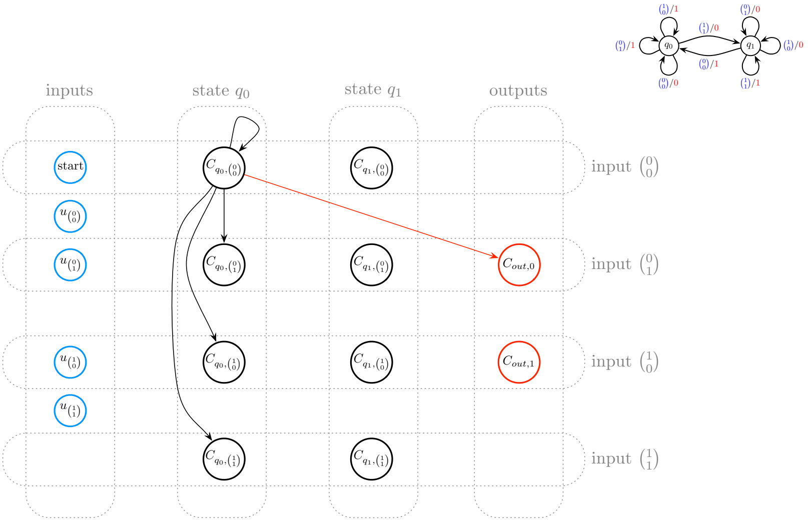

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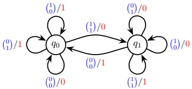

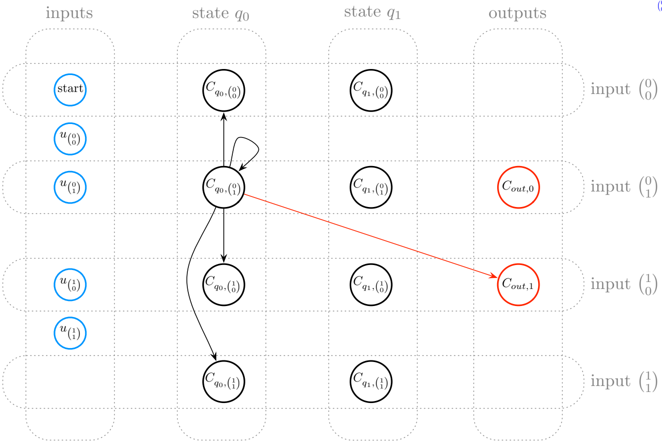

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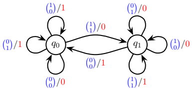

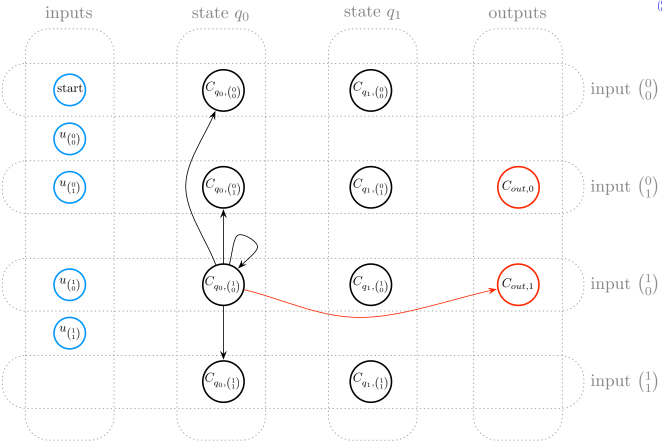

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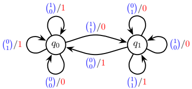

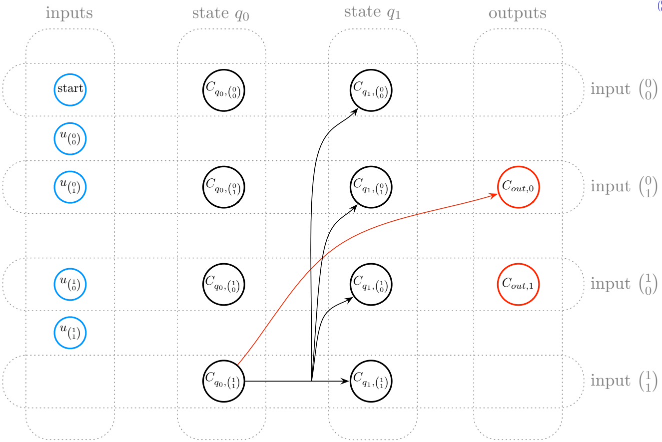

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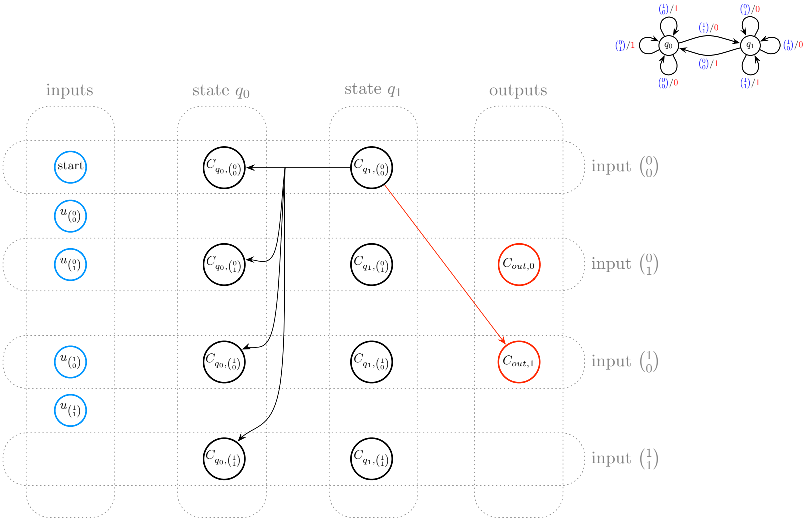

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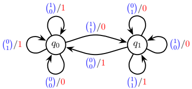

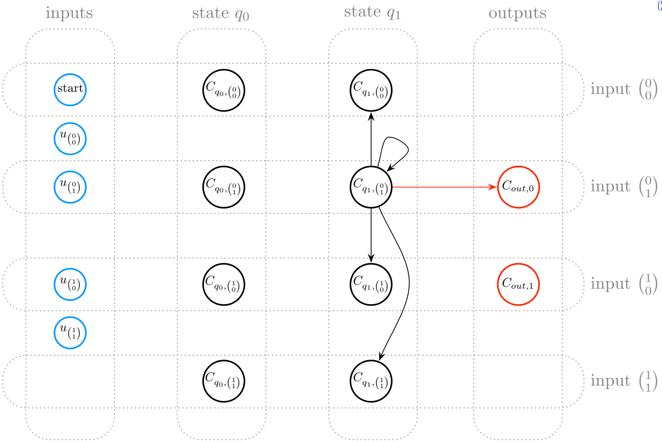

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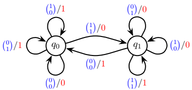

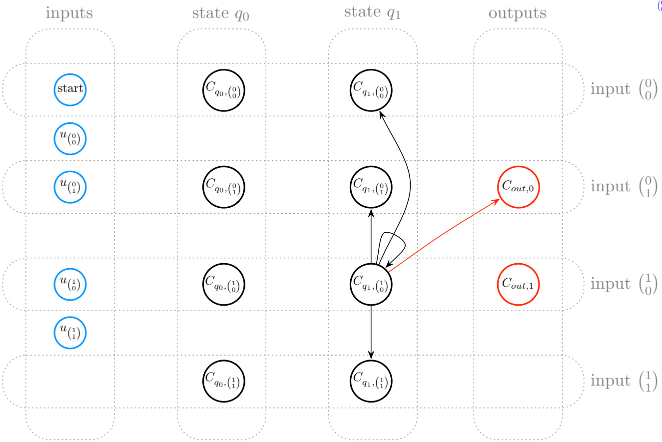

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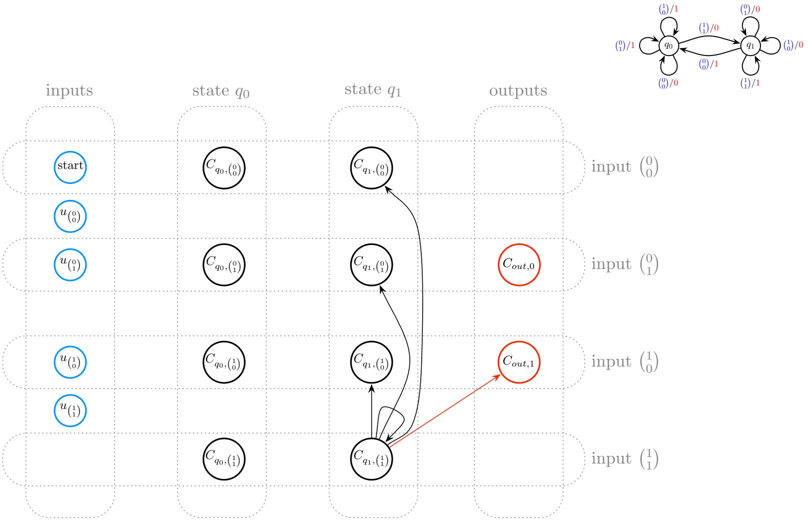

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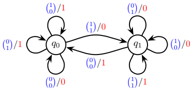

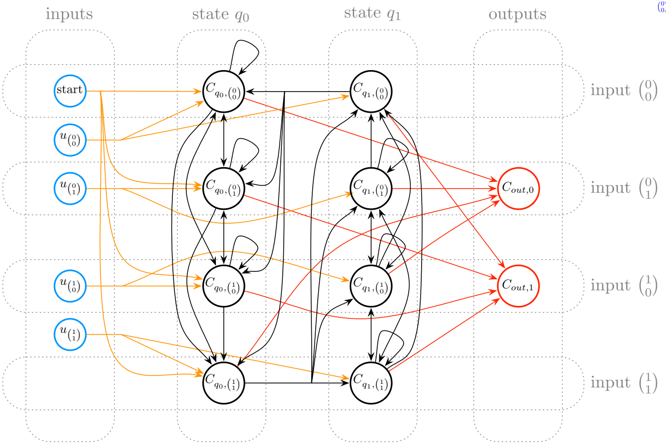

Simulation

time	0	1	2	3	4	5	6	7	8
states	q_0	q_1	q_0	q_0	q_1	q_1	q_1	q_0	–
inputs	$\binom{1}{1}$	$\binom{0}{0}$	$\binom{1}{0}$	$\binom{1}{1}$	$\binom{0}{1}$	$\binom{1}{1}$	$\binom{0}{0}$	–	–
outputs	0	1	1	0	0	1	1	–	–
$start$	1	0	0	0	0	0	0	0	0
$u_{\binom{0}{0}}$	0	1	0	0	0	0	1	0	0
$u_{\binom{0}{1}}$	0	0	0	0	1	0	0	0	0
$u_{\binom{1}{0}}$	0	0	1	0	0	0	0	0	0
$u_{\binom{1}{1}}$	1	0	0	1	0	1	0	0	0
$C_{s,i}$	–	$q_0, \binom{1}{1}$	$q_1, \binom{0}{0}$	$q_0, \binom{1}{0}$	$q_0, \binom{1}{1}$	$q_1, \binom{0}{1}$	$q_1, \binom{1}{1}$	$q_1, \binom{0}{0}$	–
$C_{out,0}$	0	0	1	0	0	1	1	0	0
$C_{out,1}$	0	0	0	1	1	0	0	1	1

Drawbacks of the Construction

- Computational states of the automaton are represented as Boolean states, i.e., spiking configurations of the network.

⋆ Computational states should rather be represented by *sustained activities of neural assemblies*, e.g., by *cyclic attractors*.

- Network is not robust to cell death, synaptic plasticity, architectural plasticity in general.

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- We introduce a paradigm of neural computation based on *synfire rings*.
- Computational states are represented by sustained activities within synfire rings.
- Hence, the successive computational states are encoded into cyclic attractors.
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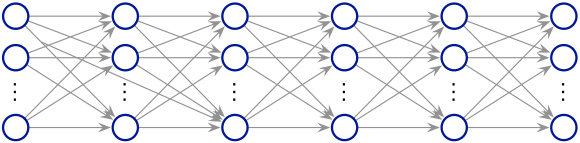
Synfire Chains

- *Synfire chains* allow for robust and highly precise transmission of information in neural networks (Abeles 82).
- *Synfire chains* are likely to be crucially involved in the processing and coding of information in neural networks.



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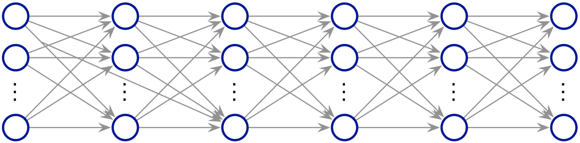

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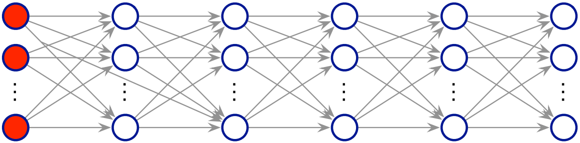

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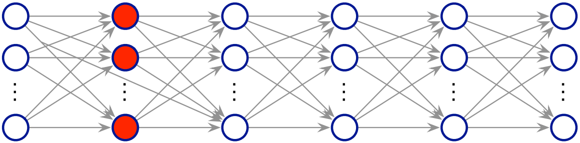

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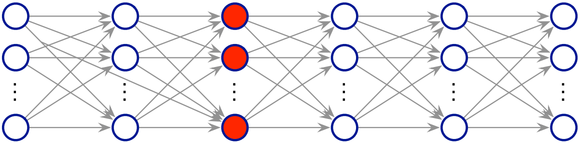

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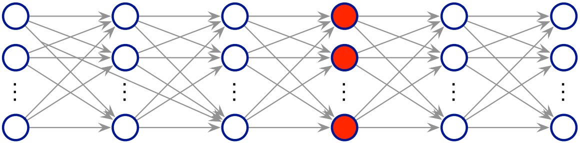

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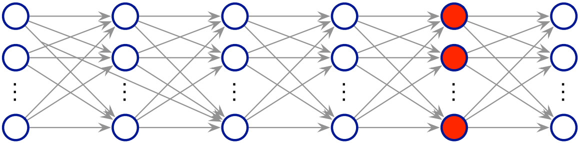

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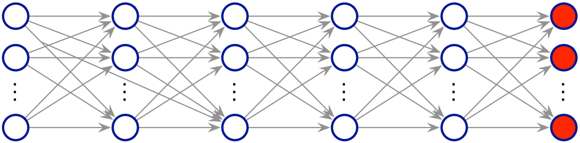
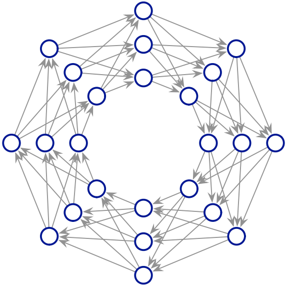


Synfire Rings

- *Synfire rings* are looping synfire chains that have been observed in self-organizing neural networks (Zheng & Triesch 14).
- *Synfire rings* allow for robust and temporally precise *self-sustained activities*.



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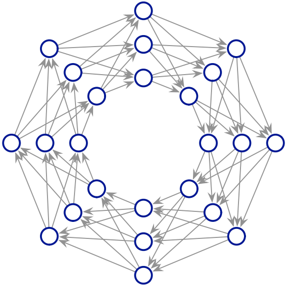

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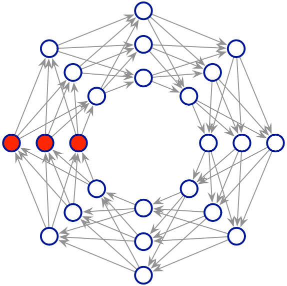

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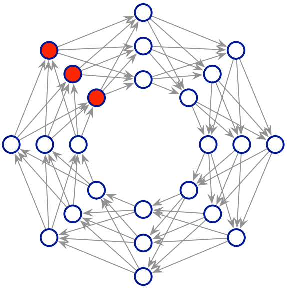

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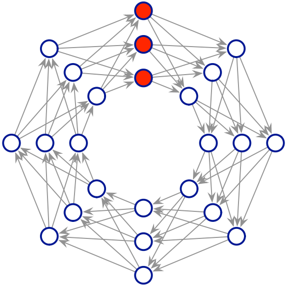

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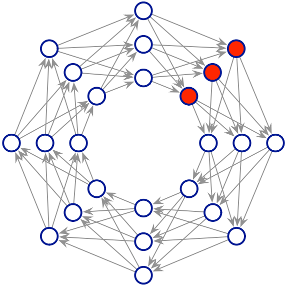

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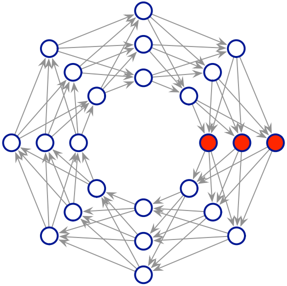

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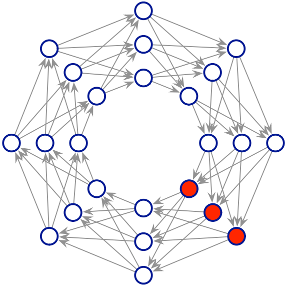

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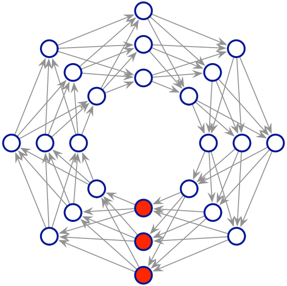

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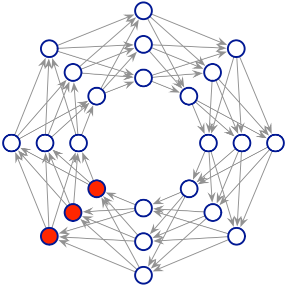

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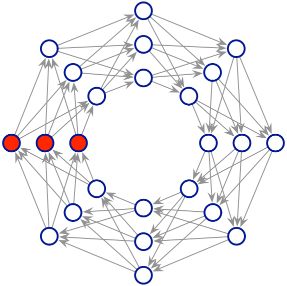

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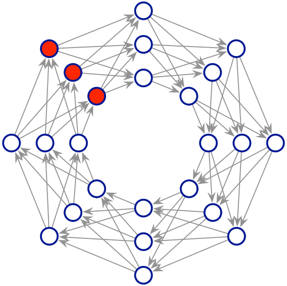


Hodgkin-Huxley Neurons (software demo)

$$\alpha_n(V_m) = \frac{0.01(10 - V_m)}{\exp(\frac{10-V_m}{10}) - 1} \qquad \beta_n(V_m) = 0.125\exp(\frac{-V_m}{80})$$

$$\alpha_m(V_m) = \frac{0.1(25 - V_m)}{\exp(\frac{25-V_m}{10}) - 1} \qquad \beta_m(V_m) = 4\exp(\frac{-V_m}{18})$$

$$\alpha_h(V_m) = 0.07\exp(\frac{-V_m}{20}) \qquad \beta_h(V_m) = \frac{1}{\exp(\frac{30-V_m}{10}) + 1}$$

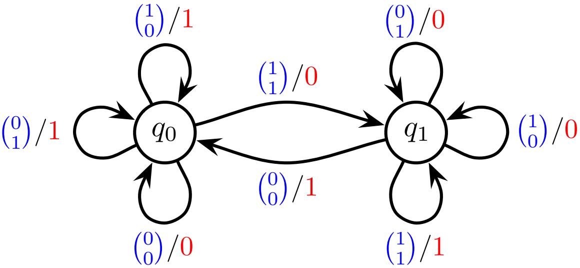
$$\frac{dn}{dt} = \alpha_n(V_m)(1-n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1-m) - \beta_m(V_m)m$$

$$\frac{dh}{dt} = \alpha_h(V_m)(1-h) - \beta_h(V_m)h$$

$$C_m\frac{dV_m}{dt} = I - \bar{g}_{\mathsf{K}}n^4(V_m - V_K) - \bar{g}_{\mathsf{Na}}m^3h(V_m - V_{Na}) - \bar{g}_l(V_m - V_l)$$

General Construction



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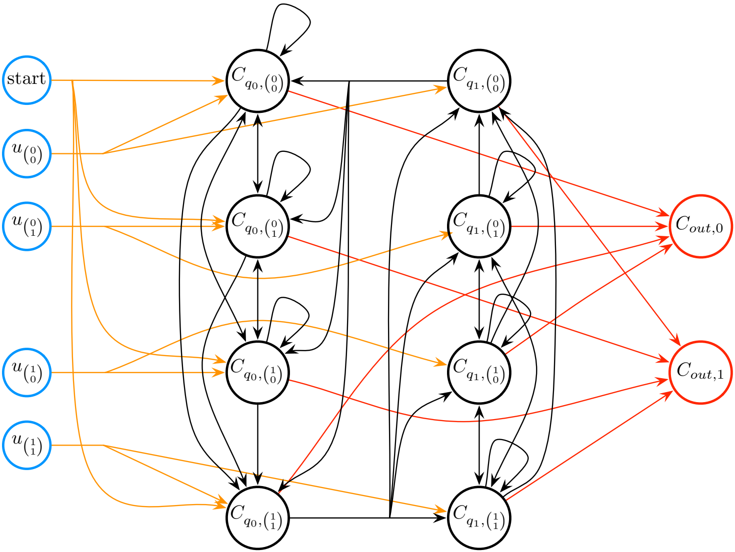

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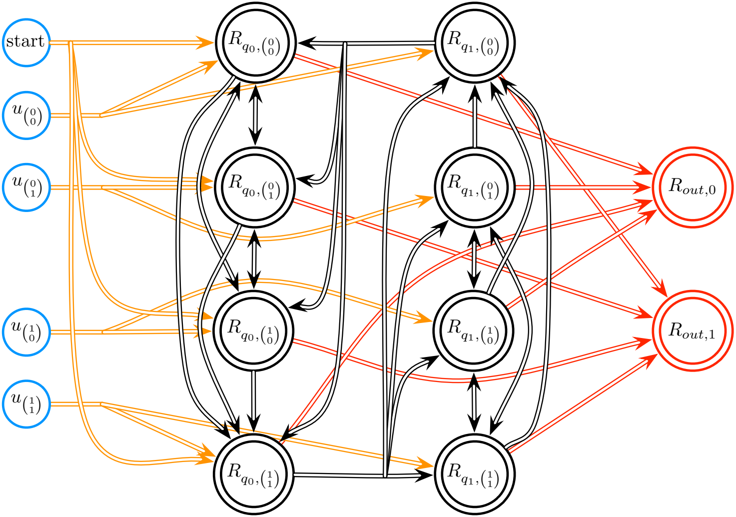
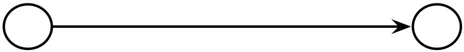


Fibres of connections & Inhibitory System



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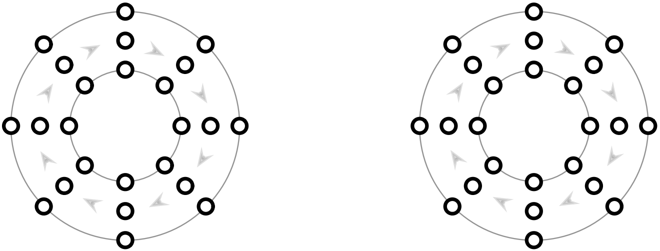

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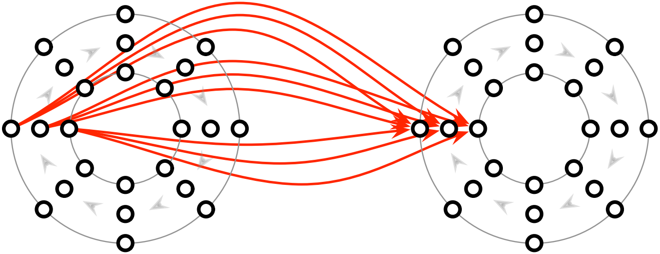

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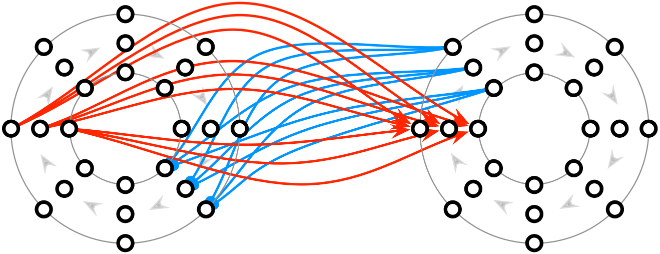

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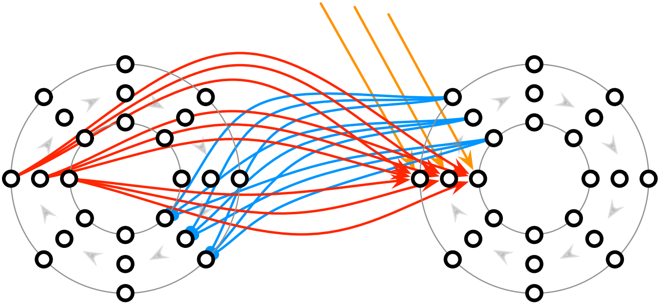

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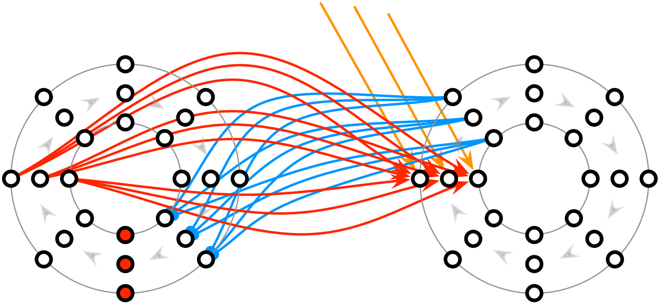

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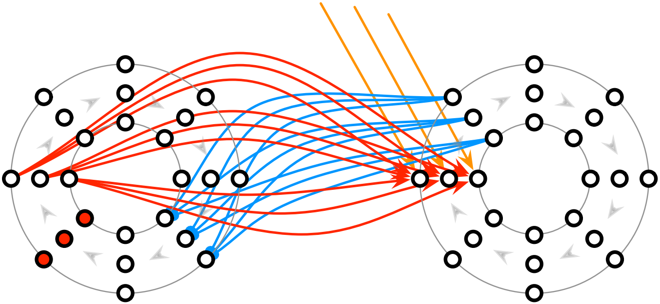

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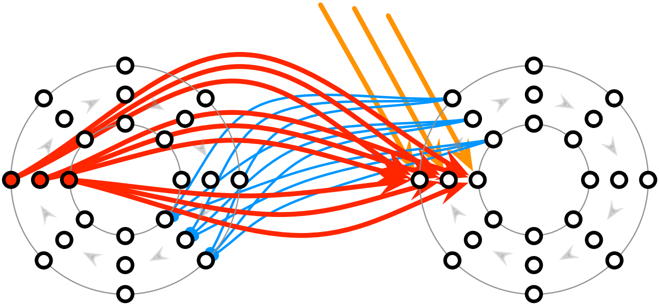

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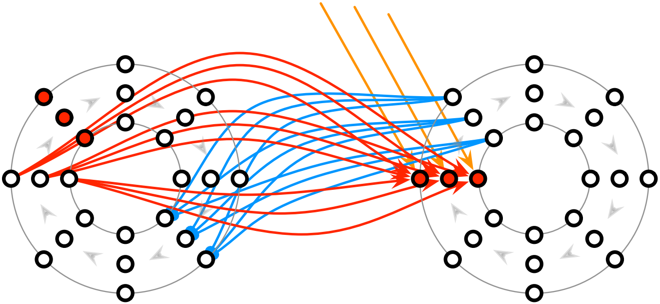

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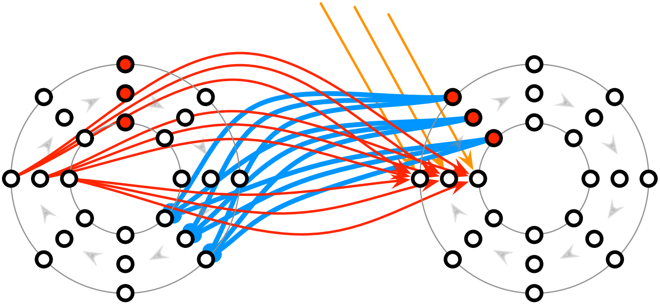

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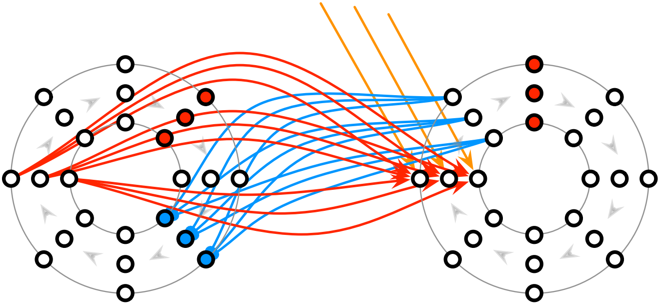

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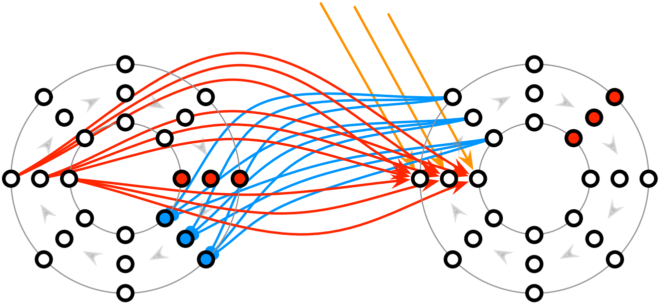

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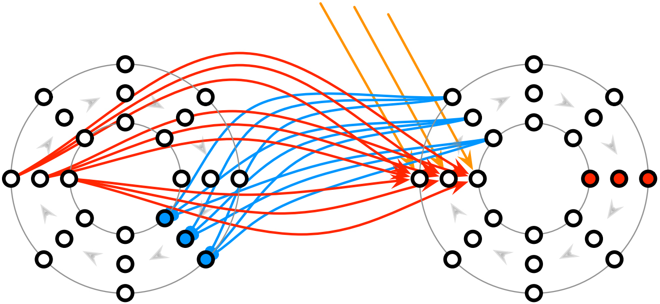

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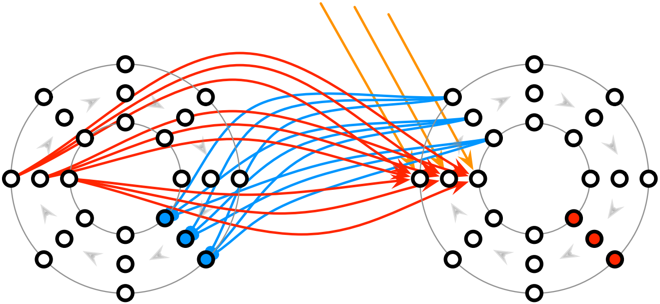

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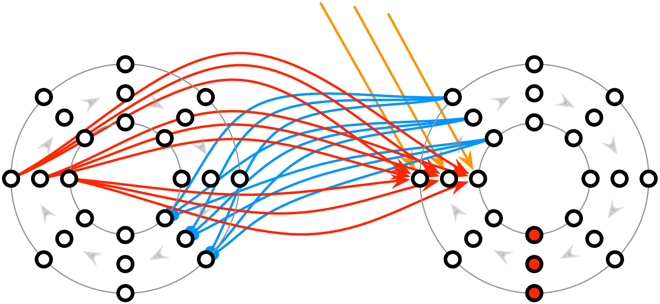

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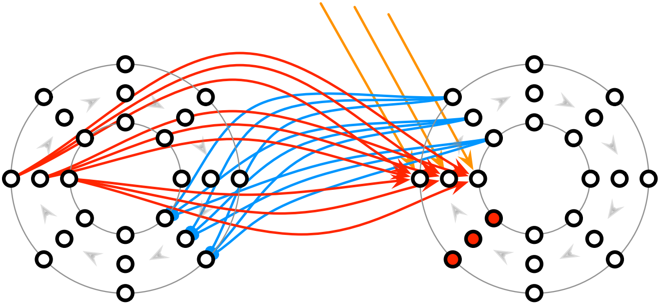
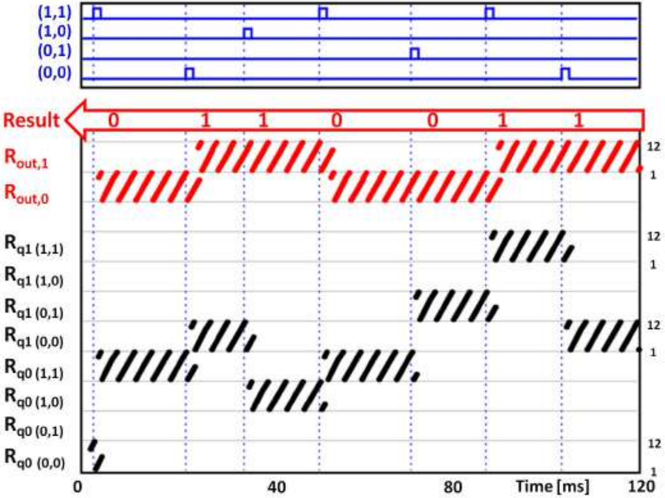


Simulation


(1,1)
(1,0)
(0,1)
(0,0)
Result
0 1 1 0 0 1 1
$R_{out,1}$
$R_{out,0}$
$R_{q1\,(1,1)}$
$R_{q1\,(1,0)}$
$R_{q1\,(0,1)}$
$R_{q1\,(0,0)}$
$R_{q0\,(1,1)}$
$R_{q0\,(1,0)}$
$R_{q0\,(0,1)}$
$R_{q0\,(0,0)}$
12
1
0
40
80
Time [ms]
120

Simulation

Play movie...

Results

Algorithm 1

Require: DFSA $\mathcal{A} = (Q, \Sigma, \delta_{\mathcal{A}}, q_0, F)$ (resp. DFST $\mathcal{T} = (Q, \Sigma, \delta_{\mathcal{T}}, q_0)$)

consider K input cells $(u_a)_{a \in \Sigma}$, where $K = |\Sigma|$
consider $I \times J$ synfire rings $(R_{q,a})_{q \in Q, a \in \Sigma}$, where $I = |\Sigma|$ and $J = |Q|$
consider K synfire rings $(R_{out,a})_{a \in \Sigma}$, where $K = |\Sigma|$
for all state $q \in Q$ **do**
  for all input symbol $a \in \Sigma$ **do**
    add a fibre of input connections from u_a to $R_{q,a}$
  end for
end for
for all transition $(q, a, q') \in graph(\delta_{\mathcal{A}})$ (resp. $(q, a, q', o) \in graph(\delta_{\mathcal{T}})$) **do**
  for all input symbol $a' \in \Sigma$ **do**
    add a fibre of inter-ring connections from $R_{q,a}$ to $R_{q',a'}$
    add a fibre of output connections from $R_{q,a}$ to $R_{out,o}$
  end for
end for
set weights w^{exc}_{input}, w^{exc}_{inter} and w^{exc}_{output} appropriately
set weights w^{inh}_{inter} and w^{inh}_{output} appropriately

Automata & Hodgkin-Huxley RNNs with Synfire Rings

Since the construction is generic, one has the following result:

Theorem

Any finite state automaton can be simulated by some Hodgkin-Huxley based neural network composed of synfire rings.

Conclusions

- We introduced a *bio-inspired paradigm of neural computation* based on the concept of *synfire rings*.
- We intend to extend the results towards the simulation of *Turing machines*.
- We intend to study the issue of *learning* within the synfire ring architecture.
- Utopia: achieve the realization of *biological neural computers*.

Thank you!

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