

# The Super-Turing Computational Power of Interactive Evolving Recurrent Neural Networks

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Joint work with Alessandro E.P. Villa

Department of Information Systems  
University of Lausanne  
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# Introduction

- ▶ We follow the so-called *mind-computer analogy* approach to cognitive science.
- ▶ We study the computational capabilities of basic models of recurrent neural networks.
- ▶ We show that recurrent neural networks provide a natural model of computation beyond the Turing limits.

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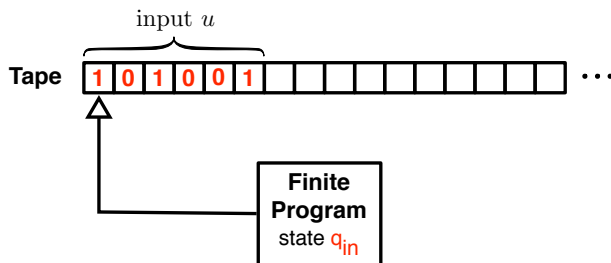
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# Turing machine

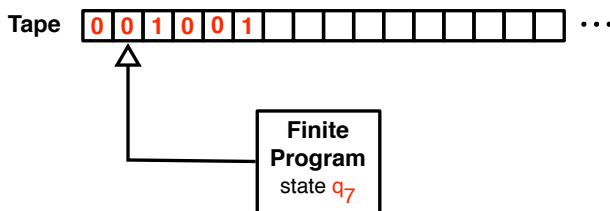
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- ▶ input  $u$  is *accepted* by  $\mathcal{M}$  if  $\mathcal{M}(u)$  reaches the state  $q_{acc}$
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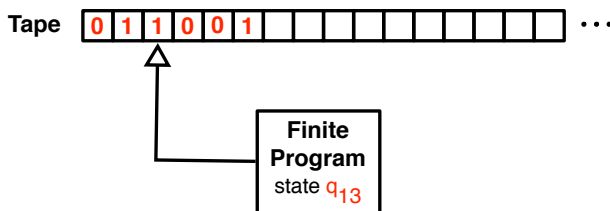
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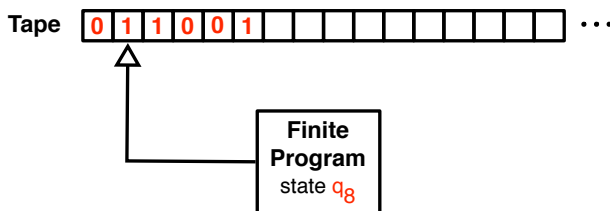
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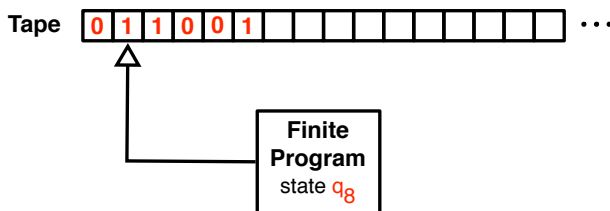


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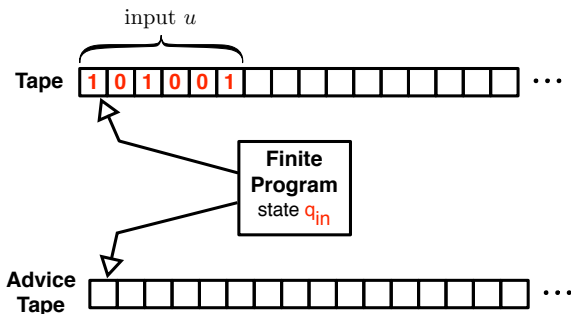
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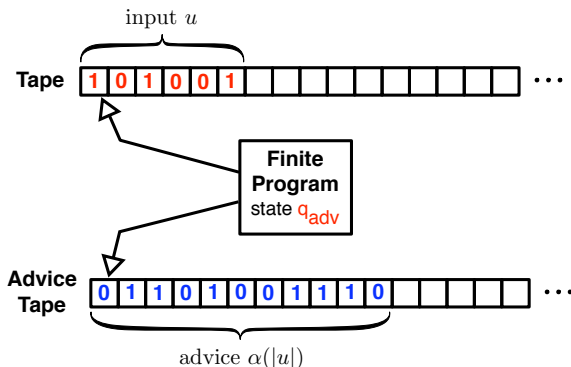
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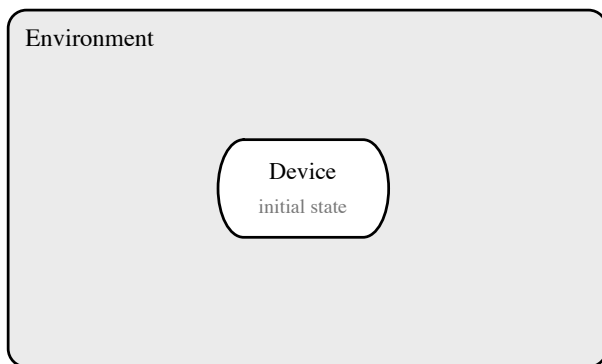


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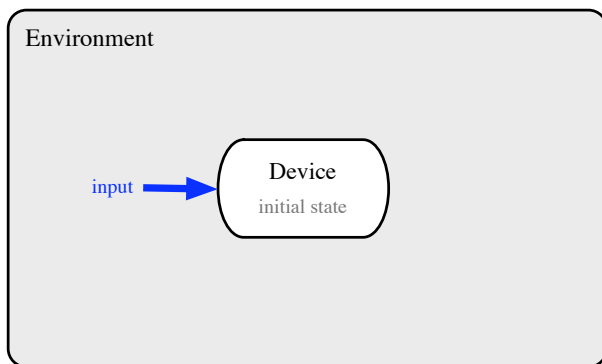


# Classical Computation



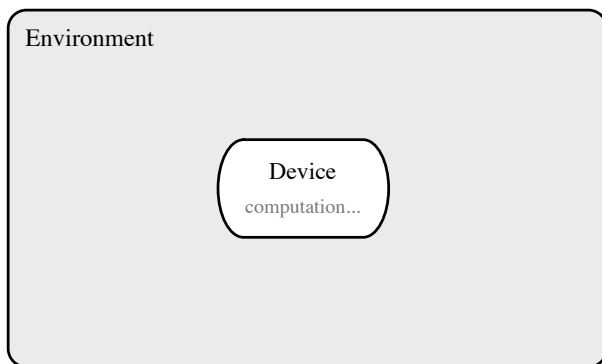
Closed-box and amnesic...

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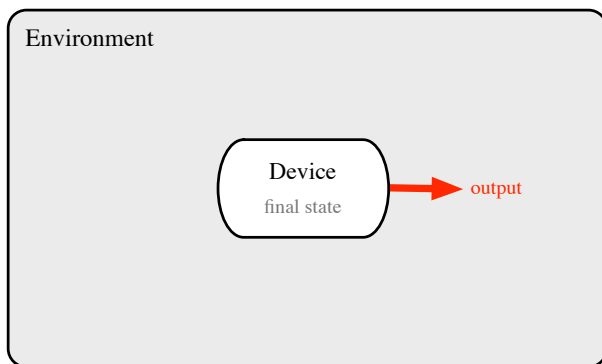
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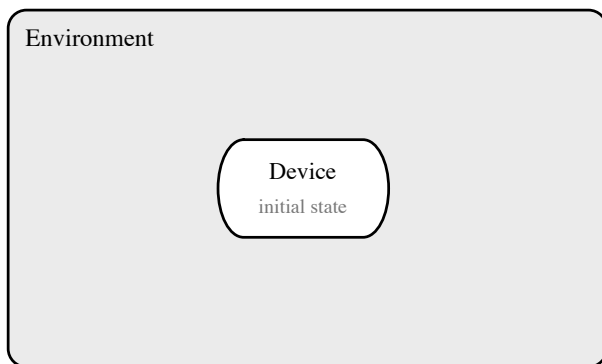
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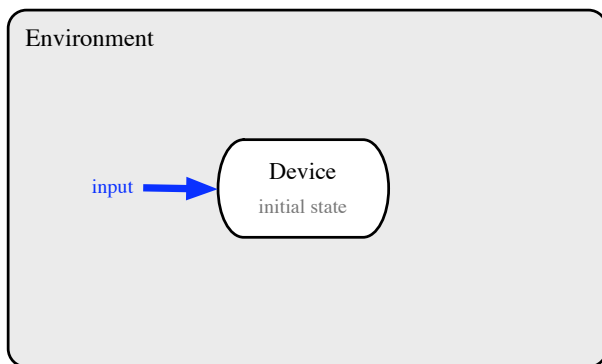
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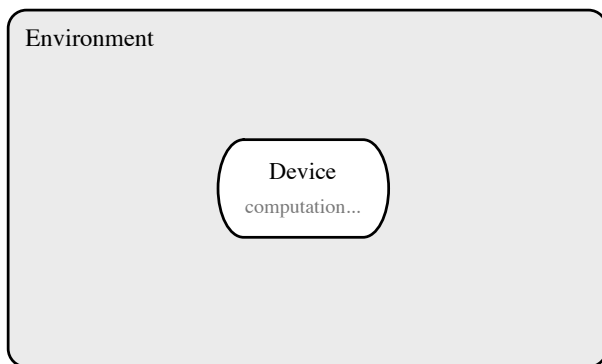


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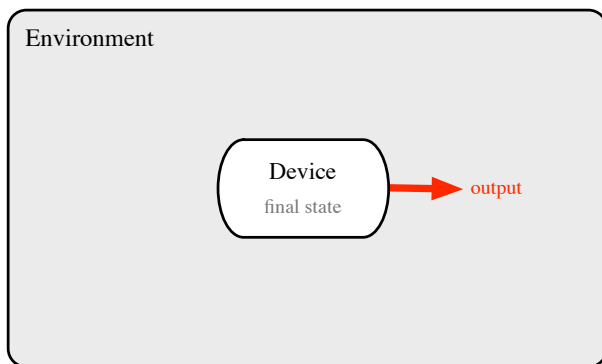
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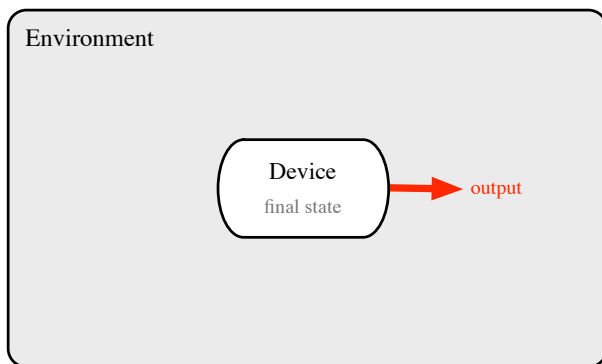
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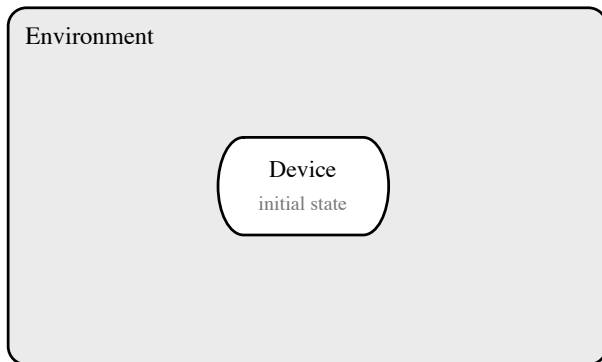
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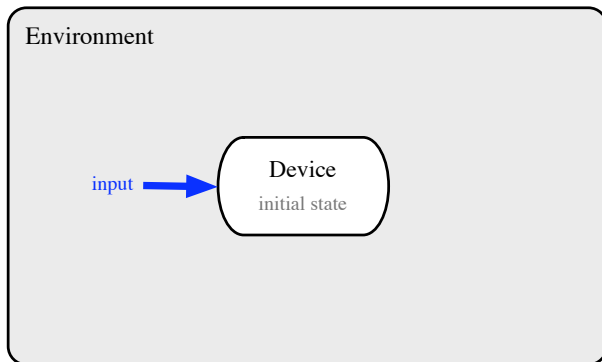
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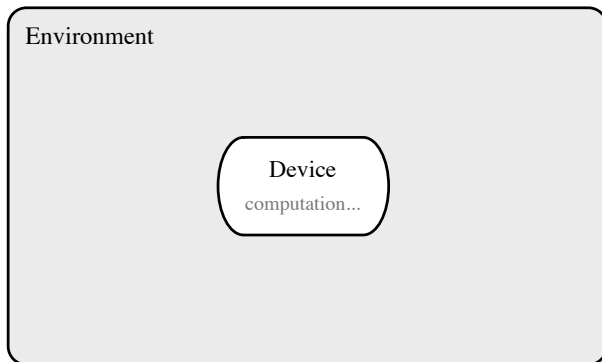
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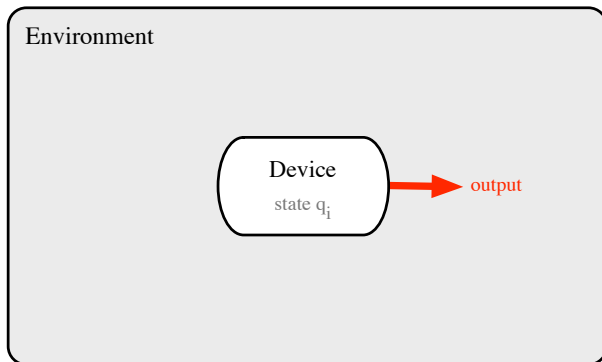
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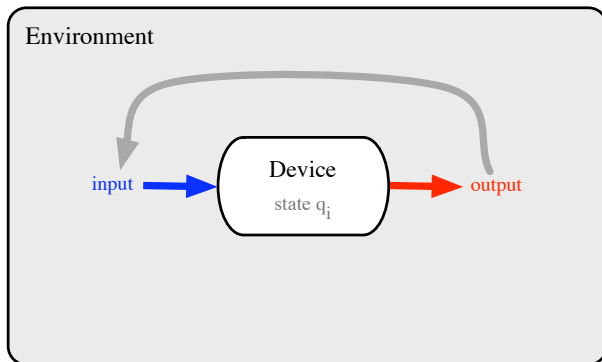
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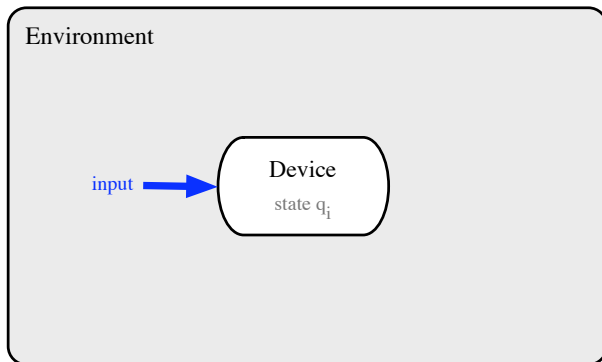


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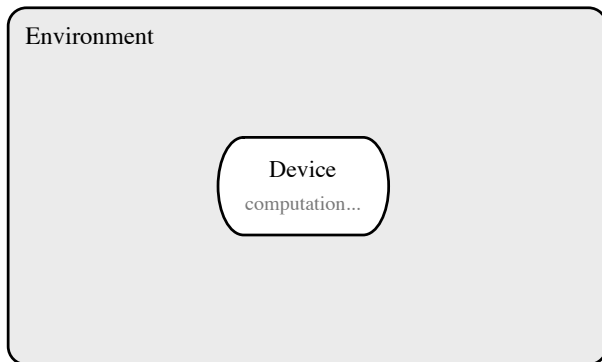
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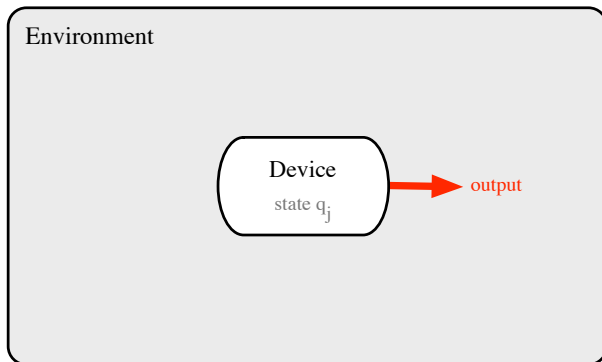
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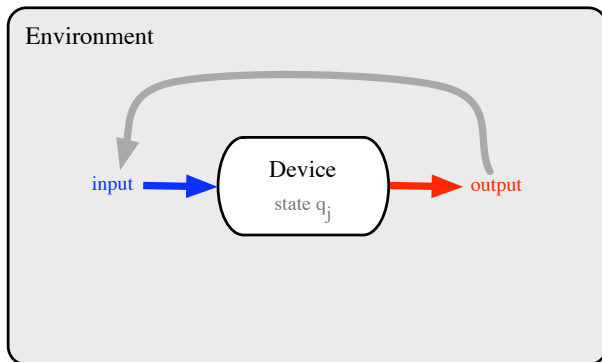
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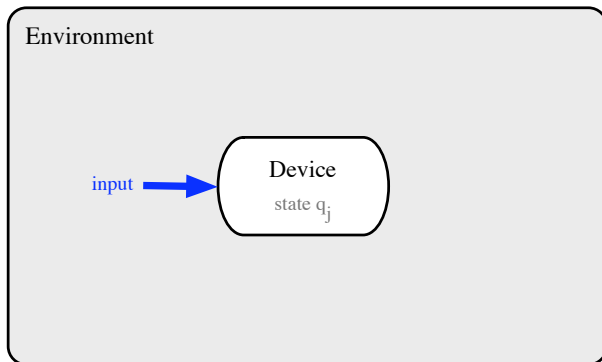


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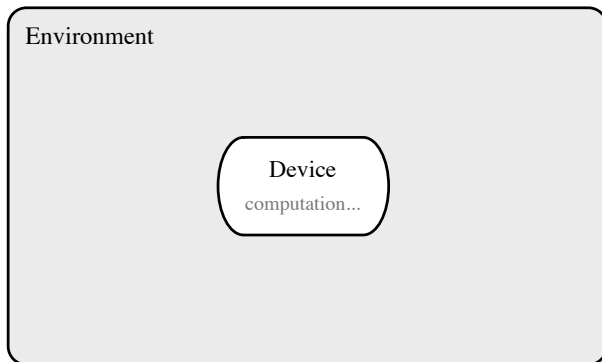
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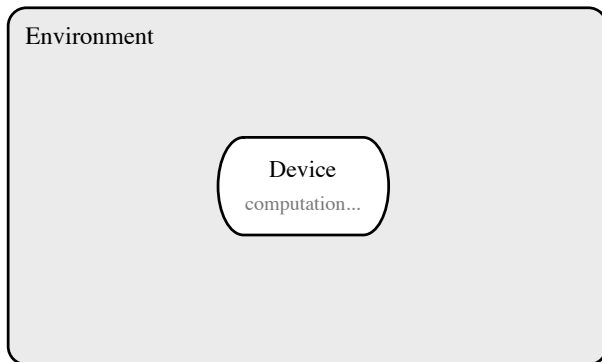
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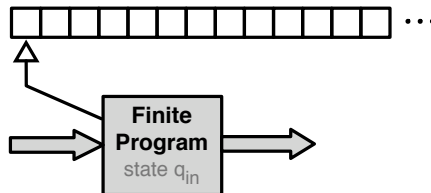
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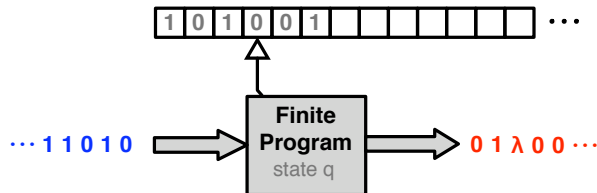


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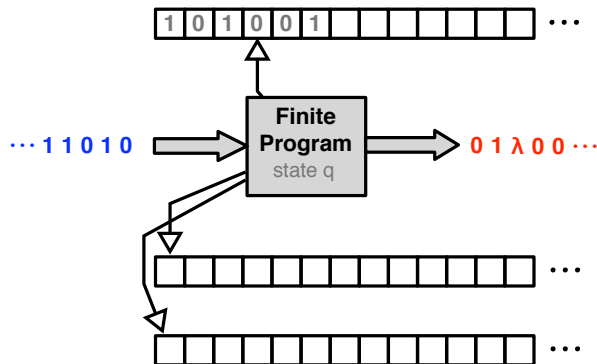


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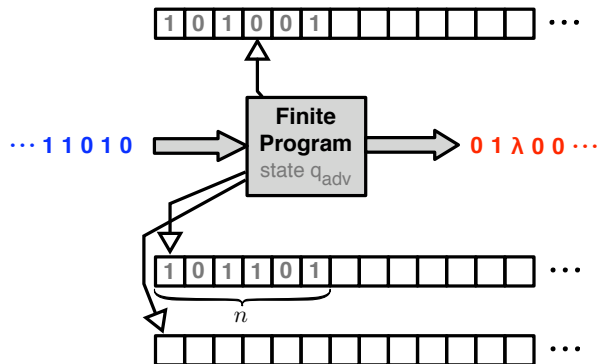
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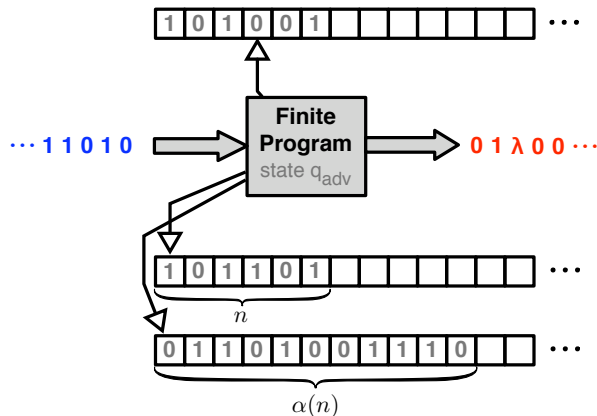
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*Turing machines with advice are strictly more powerful than Turing machines.*

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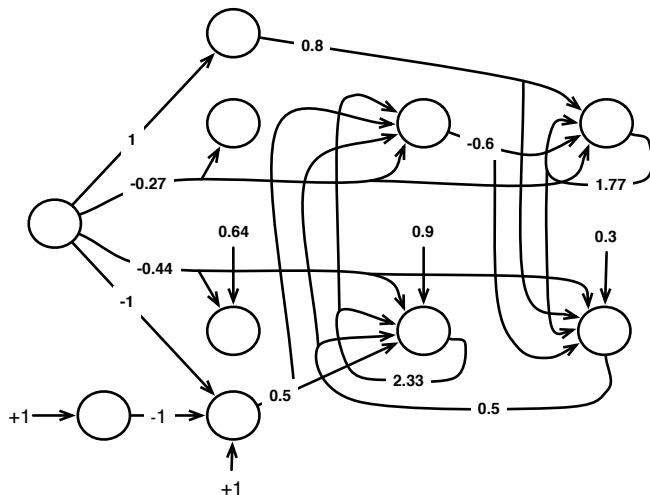
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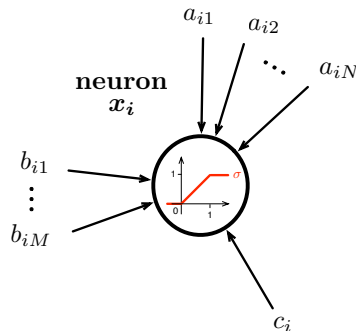
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# Recurrent Neural Networks





# Dynamics: static synaptic weights

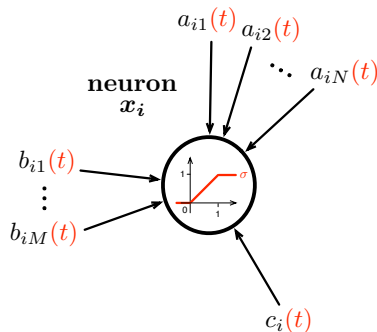


$$x_i(t+1) = \sigma \left( \sum_{j=1}^N a_{ij} \cdot x_j(t) + \sum_{j=1}^M b_{ij} \cdot u_j(t) + c_i \right)$$

# Results

	Static Architecture
$\mathbb{Q}$	<p><b>Turing</b></p> <p>Siegelmann &amp; Sontag 95 (classical comp.) Cabessa &amp; Siegelmann 12 (interactive comp.)</p>
$\mathbb{R}$	<p><b>Super-Turing</b></p> <p>Siegelmann &amp; Sontag 94 (classical comp.) Cabessa &amp; Siegelmann 12 (interactive comp.) Cabessa &amp; Villa 12 (interactive comp.)</p>

# Dynamics: evolving synaptic weights



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# Summary

	Static	Evolving
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$\mathbb{R}$	Super-Turing	Super-Turing

# Conclusions

- ▶ Evolving-RNNs provide a natural abstract computational model beyond the Turing limits.
- ▶ *Architectural Evolution* is an alternative way to the *power of the continuum* to achieve super-Turing capabilities.
- ▶ The results support the idea that *architectural evolution* might play a crucial role in the computational capabilities of biological neural networks.
- ▶ Future work: study the computational power of more biologically oriented neural models involved in more bio-inspired computational frameworks.
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