Attractor-Based Complexity and Learning in Boolean Neural Networks

Application to a Simplified Model of the Basal Ganglia-Thalamocortical Network

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Introduction

- We introduce an attractor-based complexity measure and learning procedure for Boolean recurrent neural networks.
- We illustrate these concepts in a simplified Boolean model of the basal ganglia-thalamocortical network.

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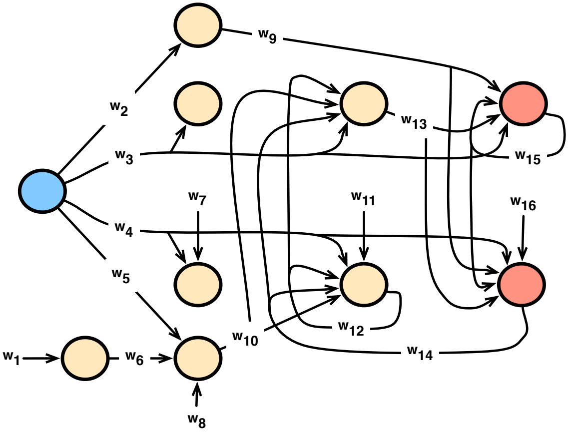
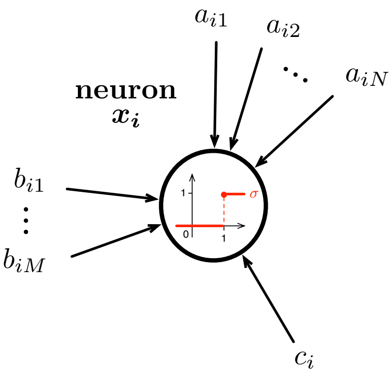
Recurrent Neural Network



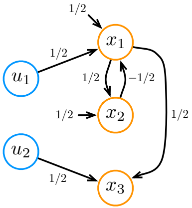
Boolean Recurrent Neural Network



$$x_i(t+1) = \theta \left(\sum_{j=1}^{N} a_{ij} \cdot x_j(t) + \sum_{j=1}^{M} b_{ij} \cdot u_j(t) + c_i \right)$$

Attractors and Cycles

Boolean Neural Network



Automaton

Attractors and Cycles

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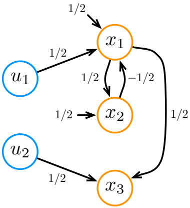

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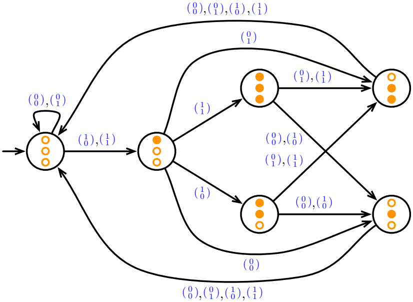
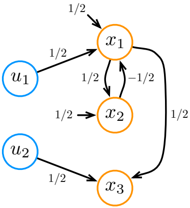
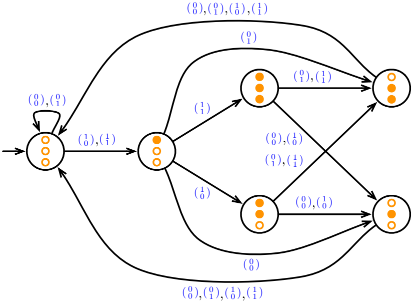


Attractors and Cycles

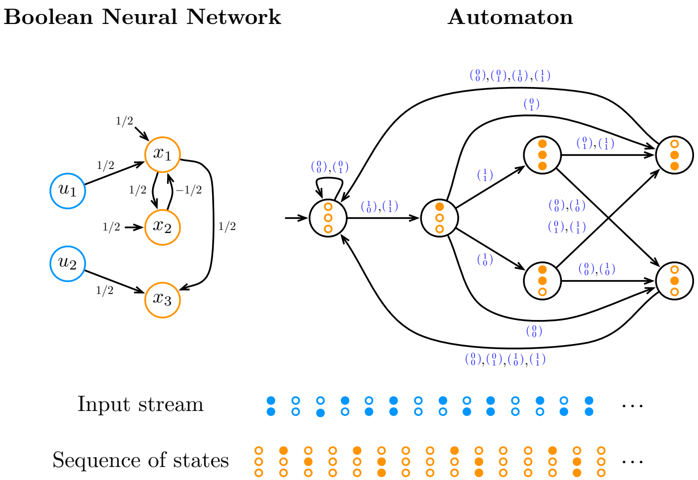
Boolean Neural Network **Automaton**





Input stream

Attractors and Cycles



Attractors and Cycles

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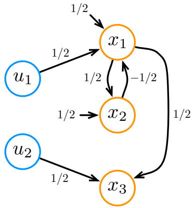

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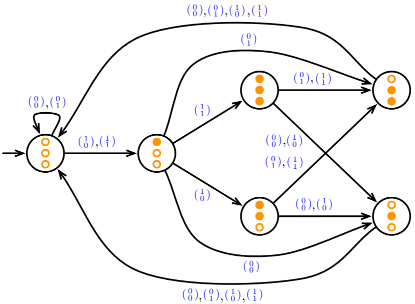
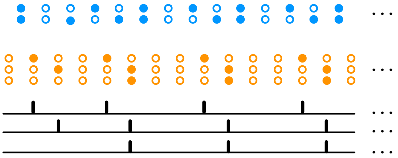


Input stream

Sequence of states

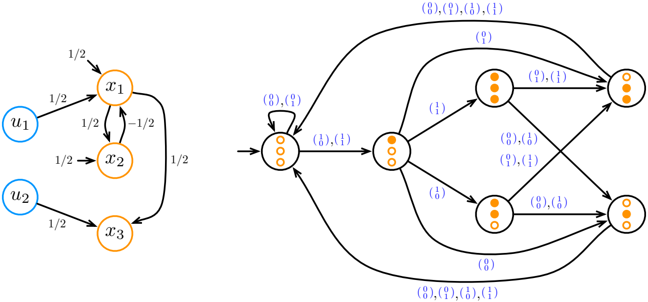
Raster plot

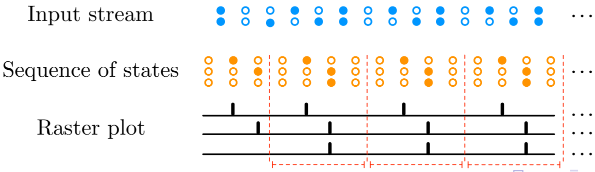


Attractors and Cycles

Boolean Neural Network **Automaton**

ATTRACTOR

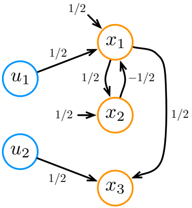


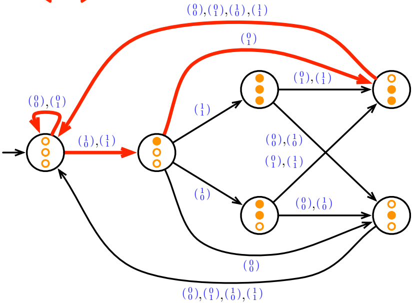


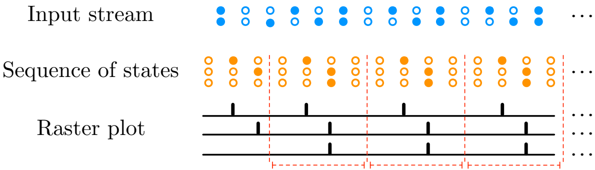
Attractors and Cycles

Boolean Neural Network **Automaton**

ATTRACTOR ⟺ **CYCLE**







Attractor-Based Complexity of RNNs

- We assume that some aspects of the computational capabilities of recurrent neural networks are related to their attractor dynamics.
- We introduce attractor-based complexity measure inspired from automata theory.
- We assume that the attractors are classified into two categories: meaningful or spurious.
- The attractor-based complexity refers to the maximal number of meaningful and spurious attractors that are included one into the other.

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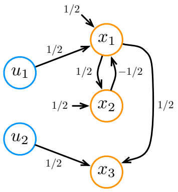
Attractor-Based Complexity of BRNNs

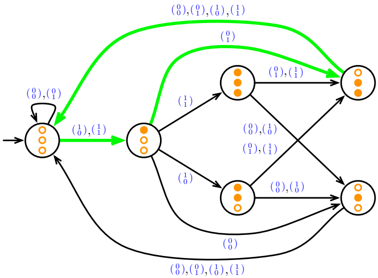
Example:

- Assume that $\mathcal{N}$ contains only one meaningful attractor; all others being spurious.
- Then, the attractor-based complexity of $\mathcal{N}$ is 3. Maximal "growing" and "alternating" sequence of 3 attractors.

Boolean Neural Network



Automaton



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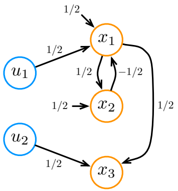

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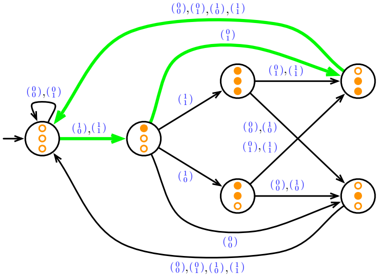

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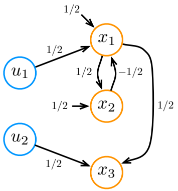

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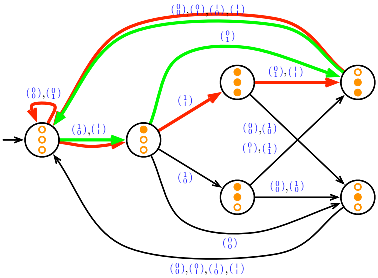

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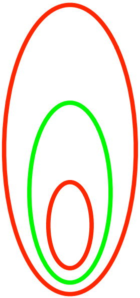
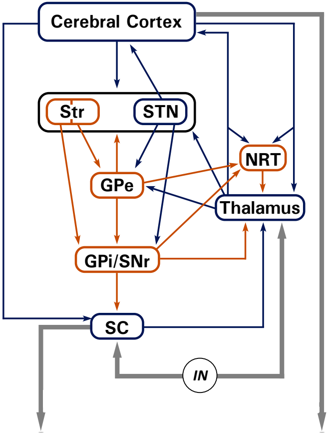


Boolean Model of the Basal Ganglia-Thalamocortical Network



IN	input node
SC	superior colliculus
GPi/SNr	output nuclei of the basal ganglia formed by the GABAergic projection neurons of the intermediate part of the pallidum and of the substantia nigra pars reticulata
Thalamus	thalamus
GPe	external part of the pallidum
NRT	thalamic reticular nucleus
Str-D1	striatopallidal component of the striatum
Str-D2	striatonigral component of the striatum
STN	subthalamic nucleus
Cerebral Cortex	cerebral cortex

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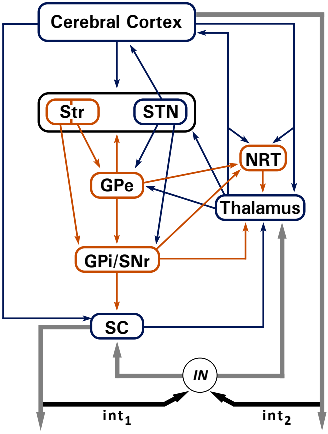

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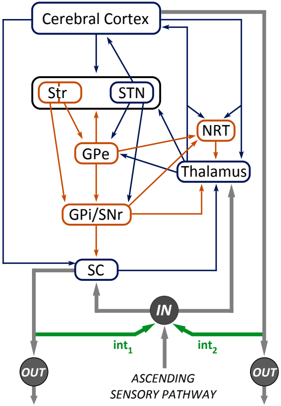
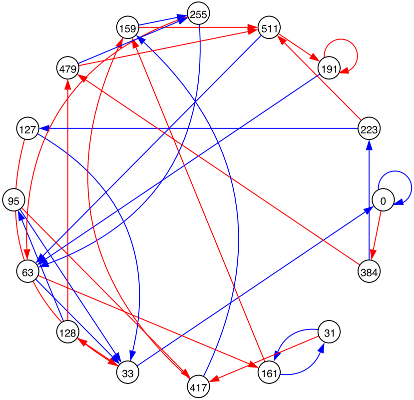
Source	**Target (Node #)**									
Node # (Name)	0	1	2	3	4	5	6	7	8	9
0 (IN)	·	1	1	·	·	·	·	·	·	·
1 (SC)	int_1	·	1	·	·	·	·	·	·	·
2 (Thalamus)	·	·	·	1	·	1	1	1	1	1
3 (RTN)	·	·	-1	·	·	·	·	·	·	·
4 (GPi/SNr)	·	-1	-1	-1	·	·	·	·	·	·
5 (STN)	·	·	·	·	2	·	2	·	·	2
6 (GPe)	·	·	·	-1/2	-1/2	-1/2	·	-1/2	-1/2	·
7 (Str-D2)	·	·	·	·	·	·	-1	·	·	·
8 (Str-D1)	·	·	·	·	-1/2	·	-1/2	·	·	·
9 (CCortex)	int_2	1/2	1/2	1/2	·	1/2	·	1/2	1/2	·

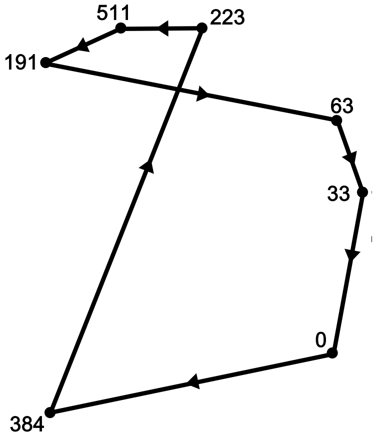
Table: Adjacency matrix

Corresponding Automaton





Attractors and Cycles



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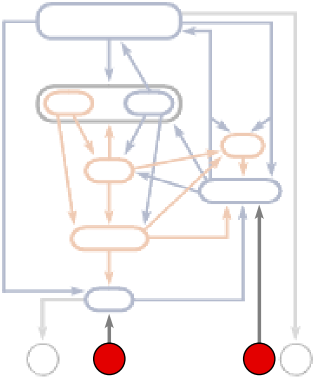

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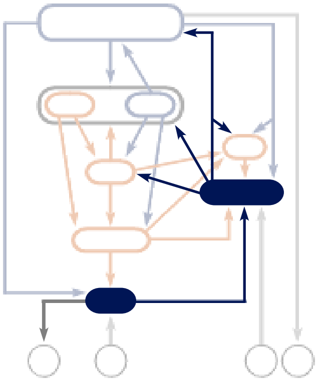

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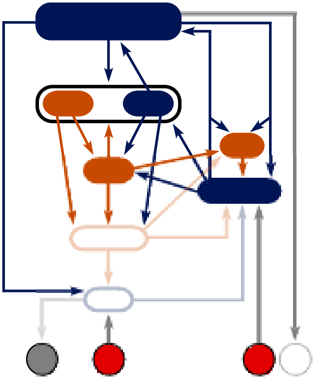

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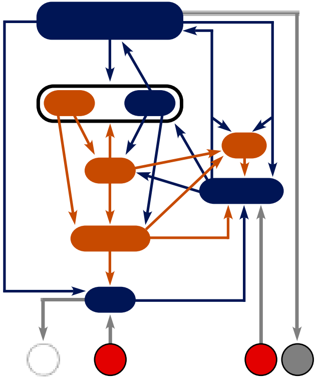

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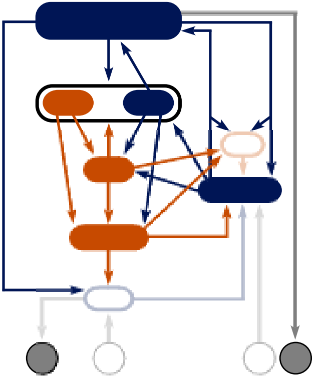

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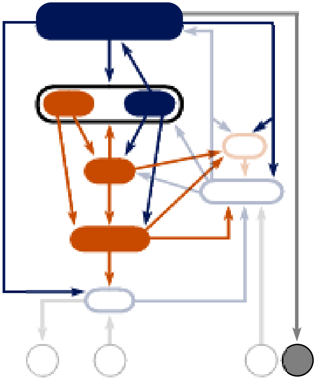

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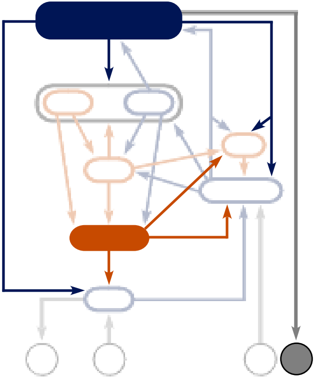

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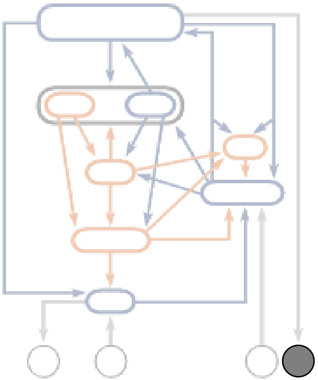

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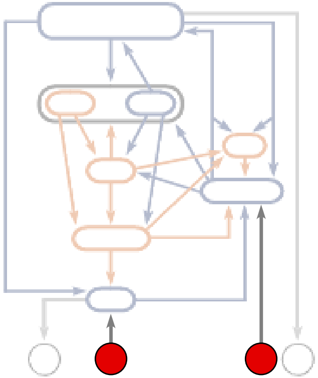

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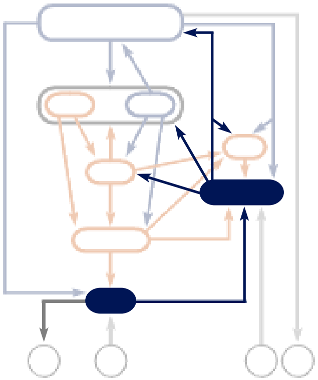

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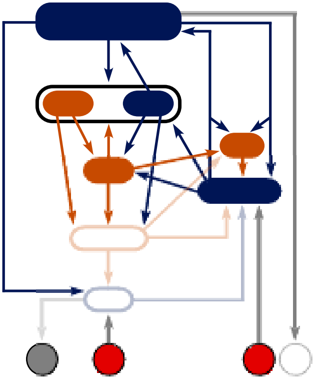

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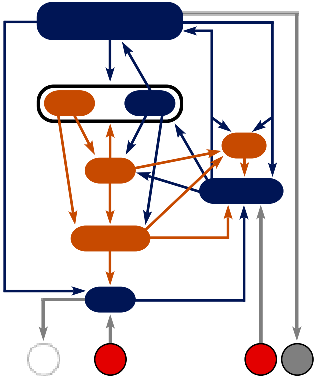

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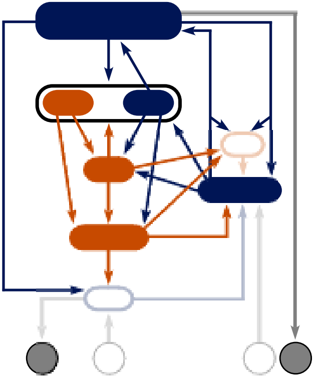

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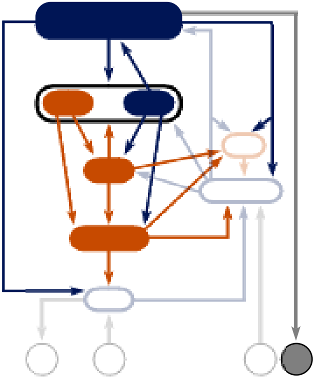

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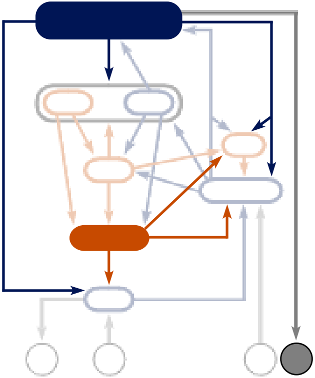

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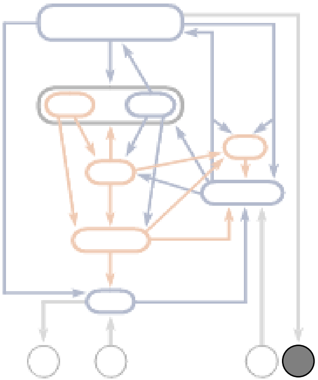

Attractor-Based Complexity

Meaningfulness criterion

- A constitutive cycle – i.e., a basic attractor – is *spurious* if it is characterised either by active SC and quiet Thalamus at the same time step, or by a quiet GPi/SNr during the majority of the duration of the cycle.
- A constitutive cycle is *meaningful* otherwise.
- A non-constitutive cycle – i.e., a composed attractor – is considered *meaningful* if it contains a majority of meaningful constitutive cycles.
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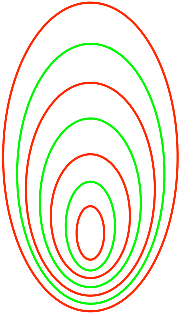
Attractor-Based Complexity

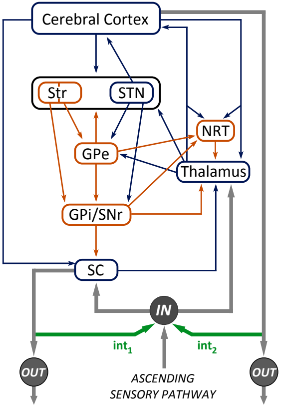
- In the corresponding automaton, we have a maximal 'growing" and "alternating" sequence of 7 cycles (attractors).

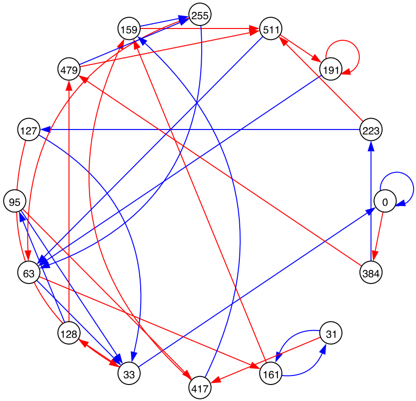
Attractor-based complexity



degree 7

Synaptic Plasticity's Influence on the Network's Attractors





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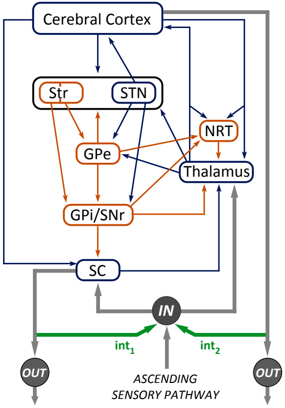
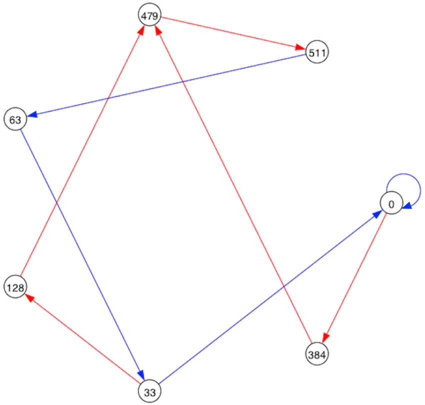
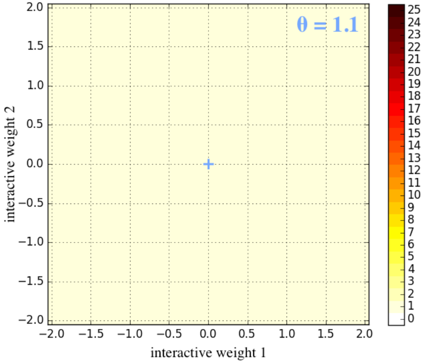




Number of attractors (fct of int. weights): global threshold (or weights) modifications



Lowering the global threshold, i.e., potentiating the global synaptic level, increases the maximal numbers of attractors and improves the robustness of the "no interactivity" configuration.

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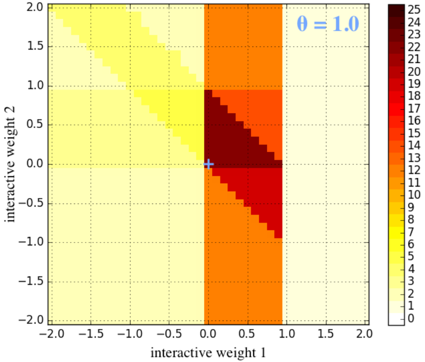

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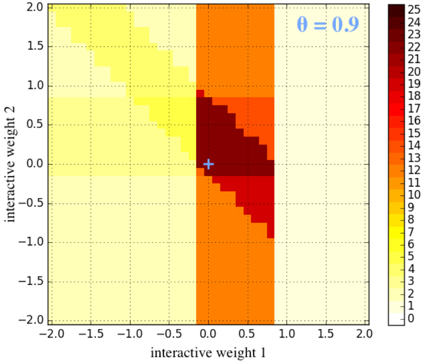

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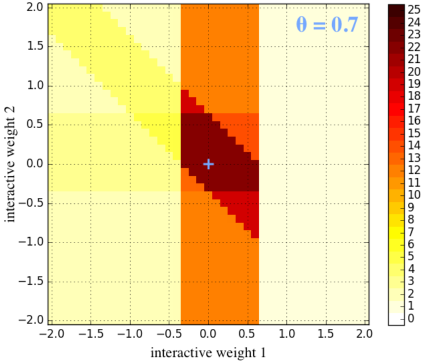
Number of Attractors (fct of int. weights): global threshold (or weights) modifications



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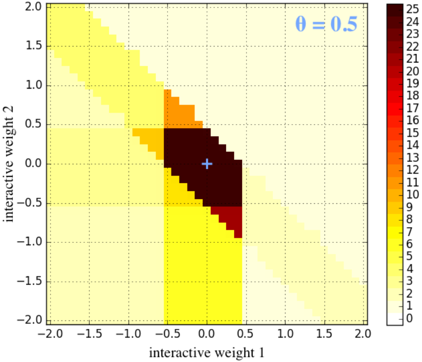

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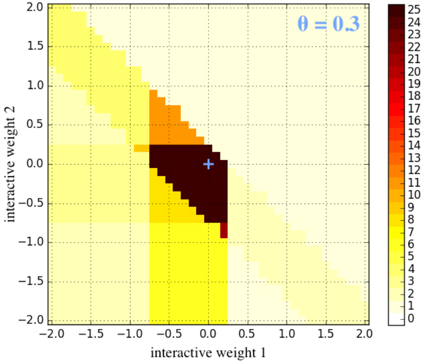

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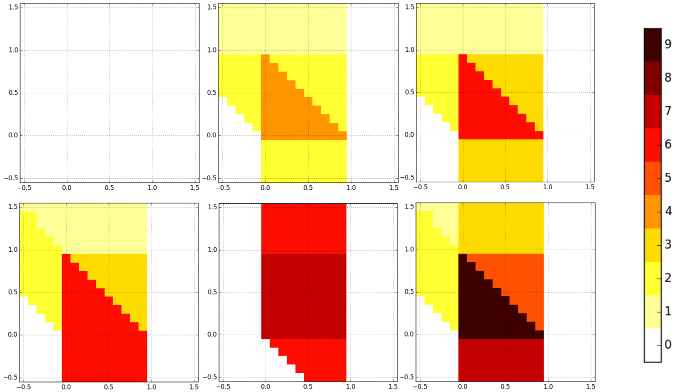
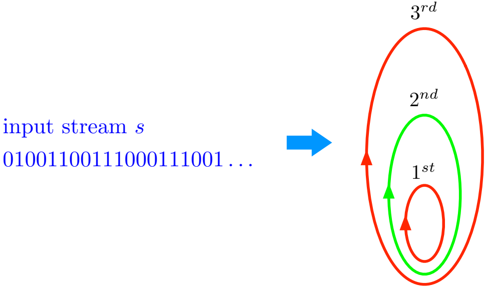
Attractor Complexity (fct of int. weights): local weights modifications



Even single weight variations of ± 0.1 can significantly affect the complexity pattern.

Input discrimination

- An input stream s is *discriminated* by a sequence of attractors $\mathcal{C} = (C_0, \ldots, C_n)$ if the network, when receiving input s stream s, visits the successive attractors $C_0, \ldots, C_n$ of $\mathcal{C}$.
- The *discriminability degree* of s, $d^*(s)$, is the length of a maximal sequence of attractors $\mathcal{C}$ that discriminates s.



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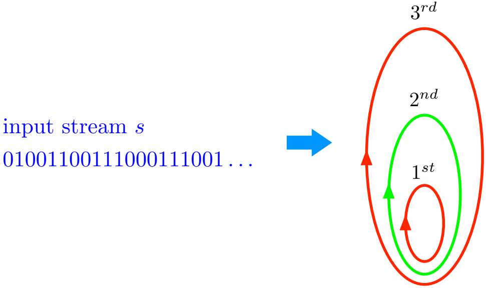


Input discrimination

- We introduce an *attractor-based learning procedure* which updates the (interactive) network's weights in order to achieve a targeted discriminability degree of some input stream s.

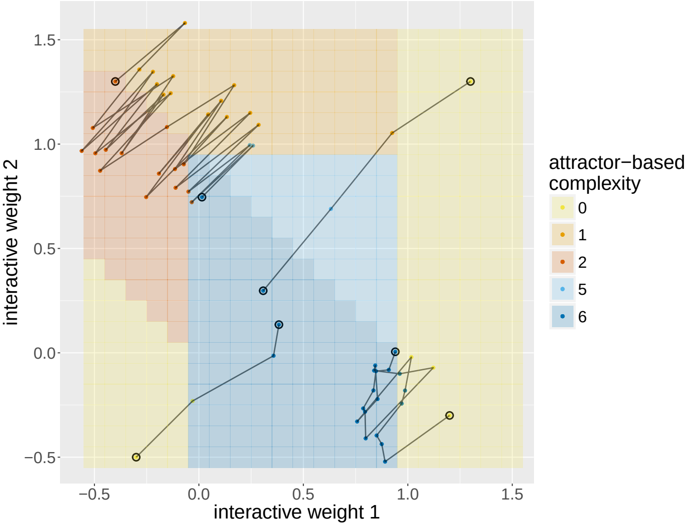
Algorithm 1 Attractor-based learning procedure

Require: input stream s; initial weights w_1, w_2; target discriminability degree N^*

compute $d^*(s)$
while $d^*(s) < N^*$ **do**
  $w_k \leftarrow f(w_k)$, for $k = 1, \ldots, N$
  compute $d^*(s)$ for the network with updated weights w_k, for $k = 1, \ldots, N$
end while
return w_k, for $k = 1, \ldots, N$

$$f(w_k) \quad = \quad w_k + step \cdot \frac{-sum(m_s)}{len(m_s)} \cdot \left(1 + \frac{len(m_s) - d^*(s)}{len(m_s)}\right) + \epsilon$$

Input discrimination



Conclusions

- Global and local modifications of the *synaptic weights* significantly affect the attractor complexity of the networks.
- The values of the *interactive connections* also play a significant role in the maintenance and robustness of an optimal level of attractor-based complexity.
- We have an *attractor-based learning procedure* which modfiies the interactive weight in order to reach a targeted discriminability degree of a certain input stream.
- These considerations support the rationale that *synaptic plasticity* might be crucially involved in the computational capabilities of neural networks.

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