A Memory-Based STDP Rule for Stable Attractor Dynamics in Boolean Recurrent Neural Networks

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Introduction

- *Boolean Neural Networks (BNNs)*, although relatively simple, allow for a complete analysis of their attractor dynamics.
- Local and global variations of the synaptic weights significantly influence the attractor dynamics of the networks.
- We introduce an *input-driven*, *memory-based adaptive Spike Timing-Dependent Plasticity (STDP) rule* which stabilizes the attractor dynamics of the network.
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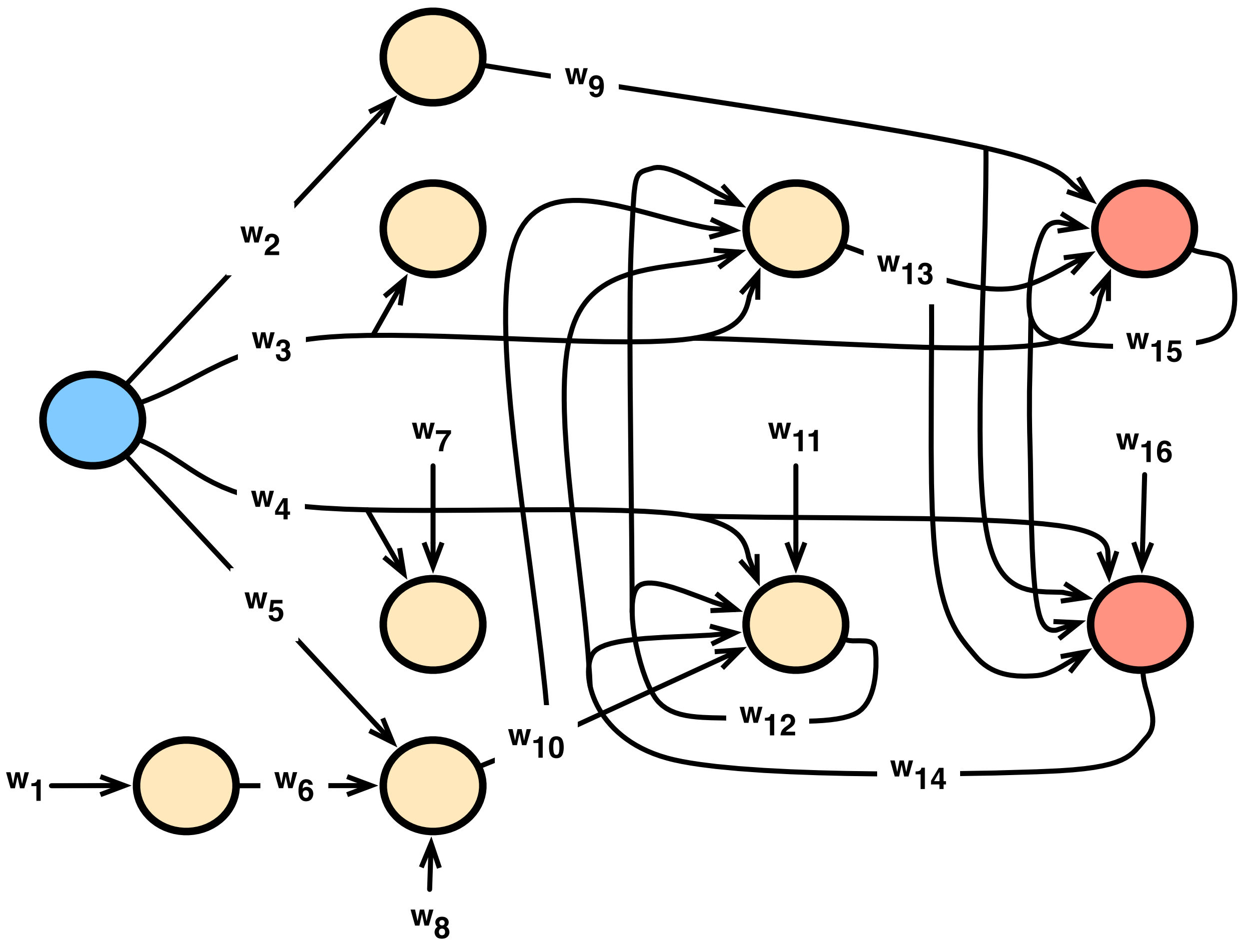
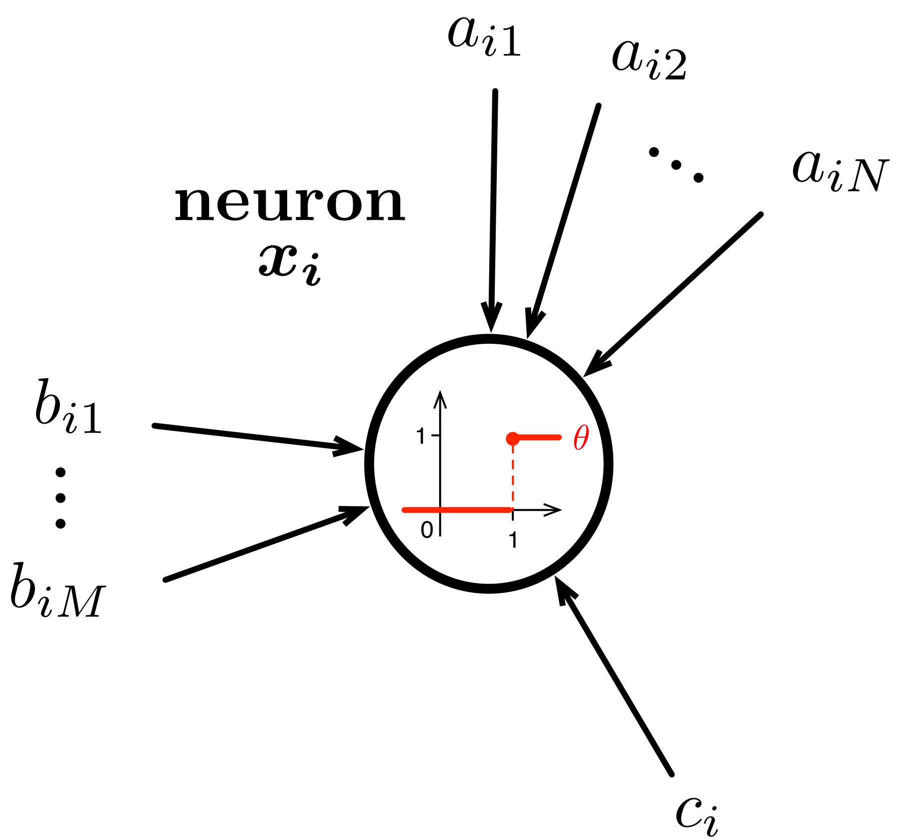
Recurrent Neural Network



Boolean Neural Networks – Dynamics



$$x_i(t+1) = \theta\left(\sum_{j=1}^{N} a_{ij} \cdot x_j(t) + \sum_{j=1}^{M} b_{ij} \cdot u_j(t) + c_i\right)$$

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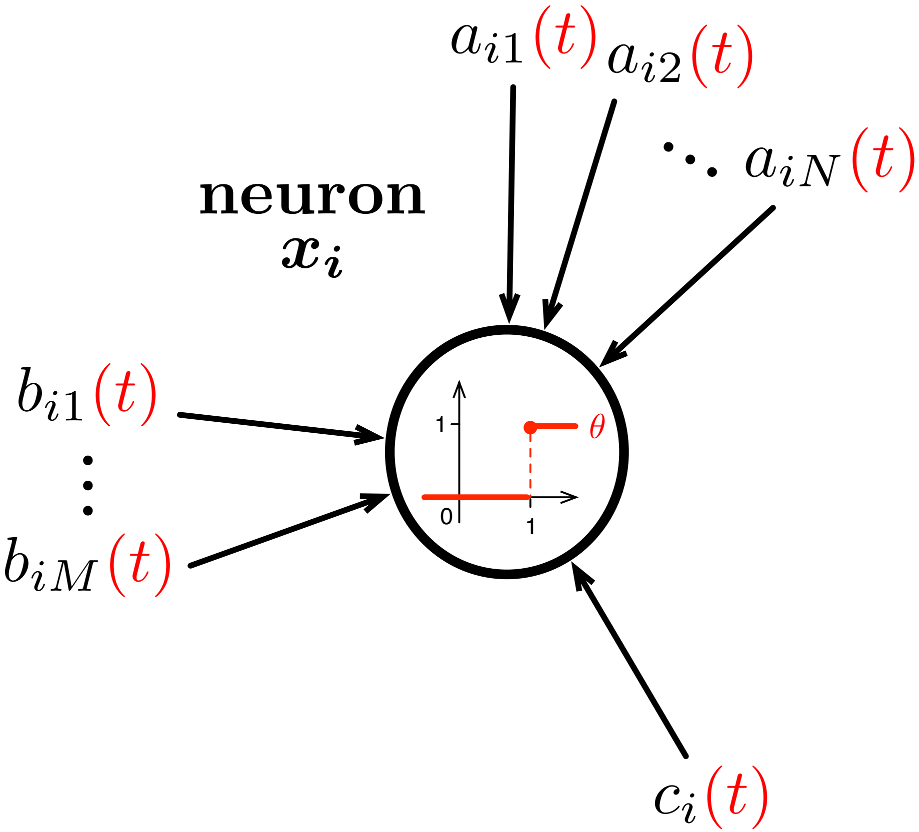

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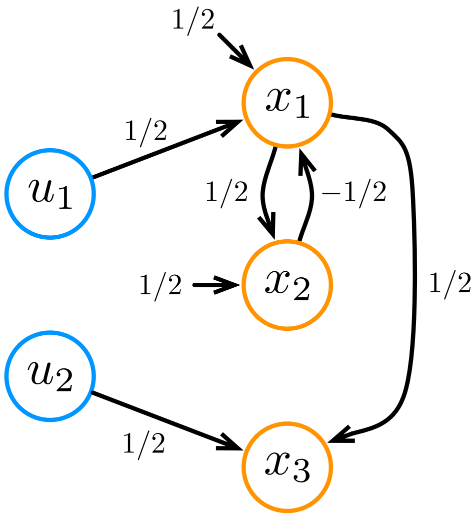
Boolean Neural Networks – Attractors

Boolean Neural Network **Finite State Automaton**



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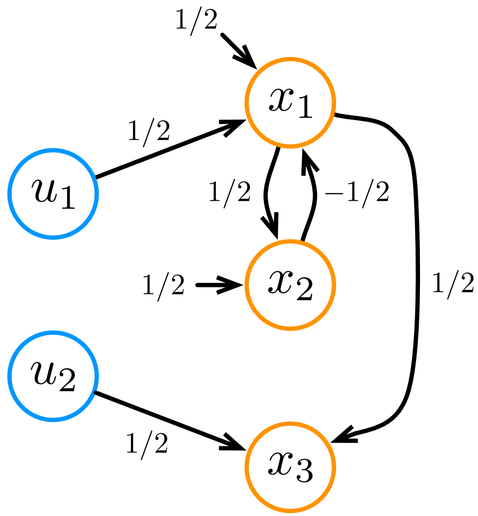
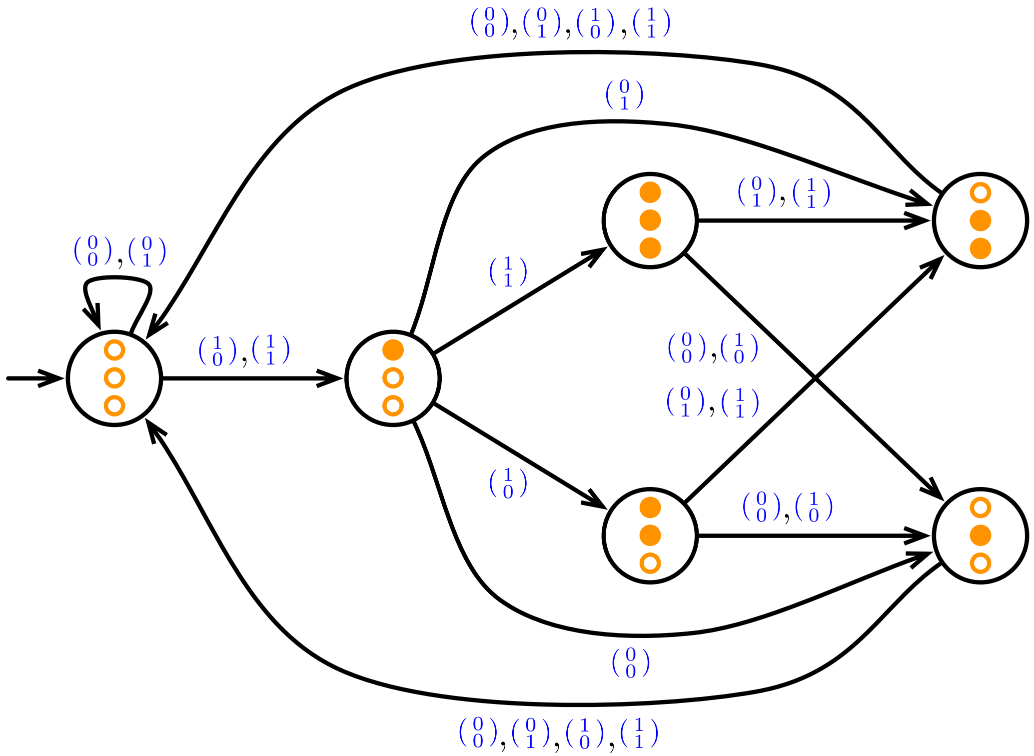


Finite State Automaton



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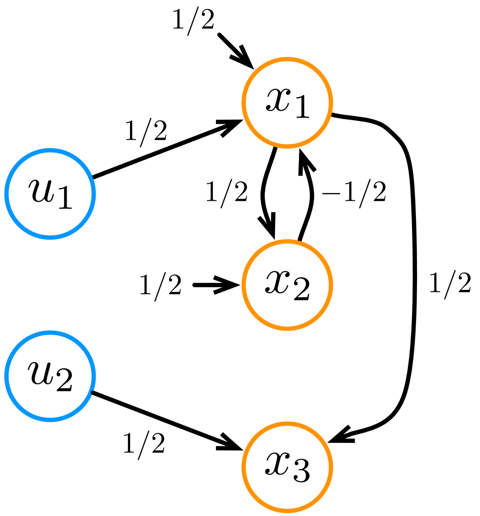
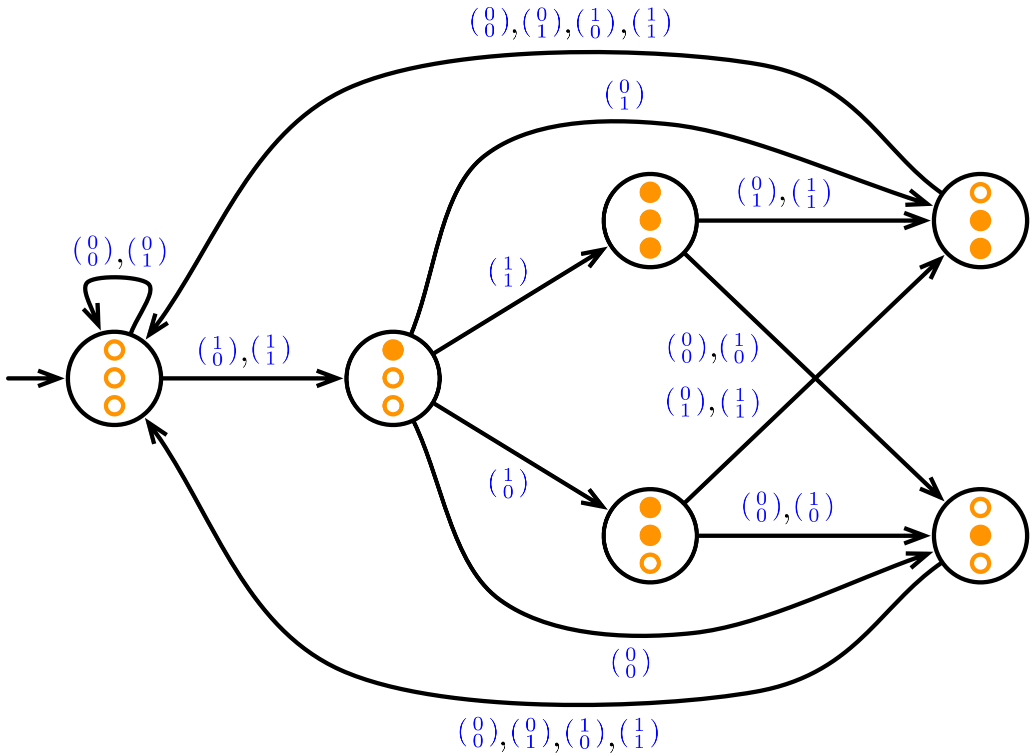
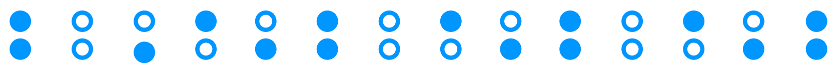




Input stream  . . .

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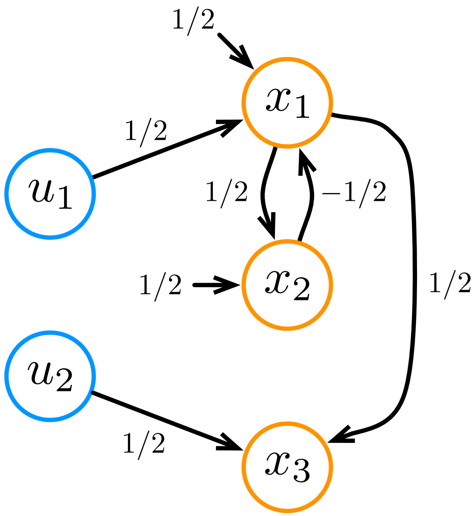

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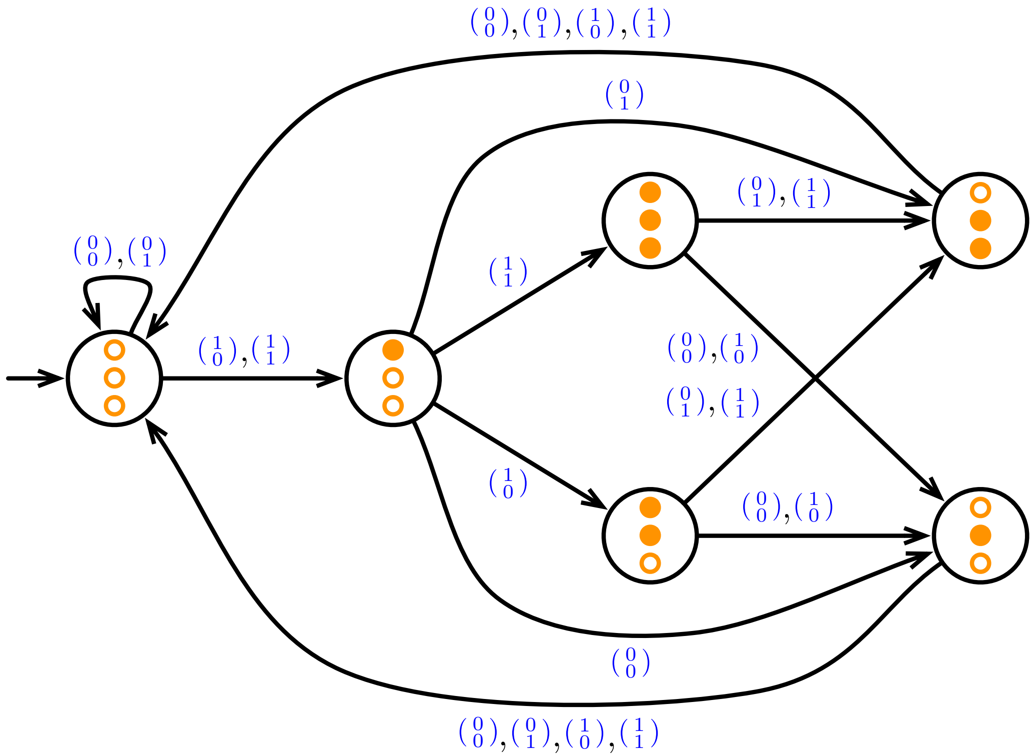
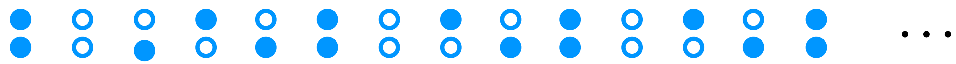
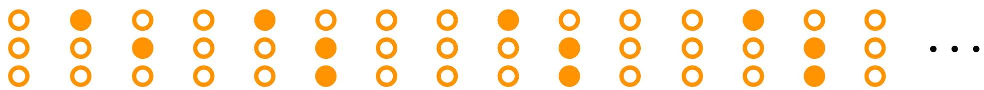


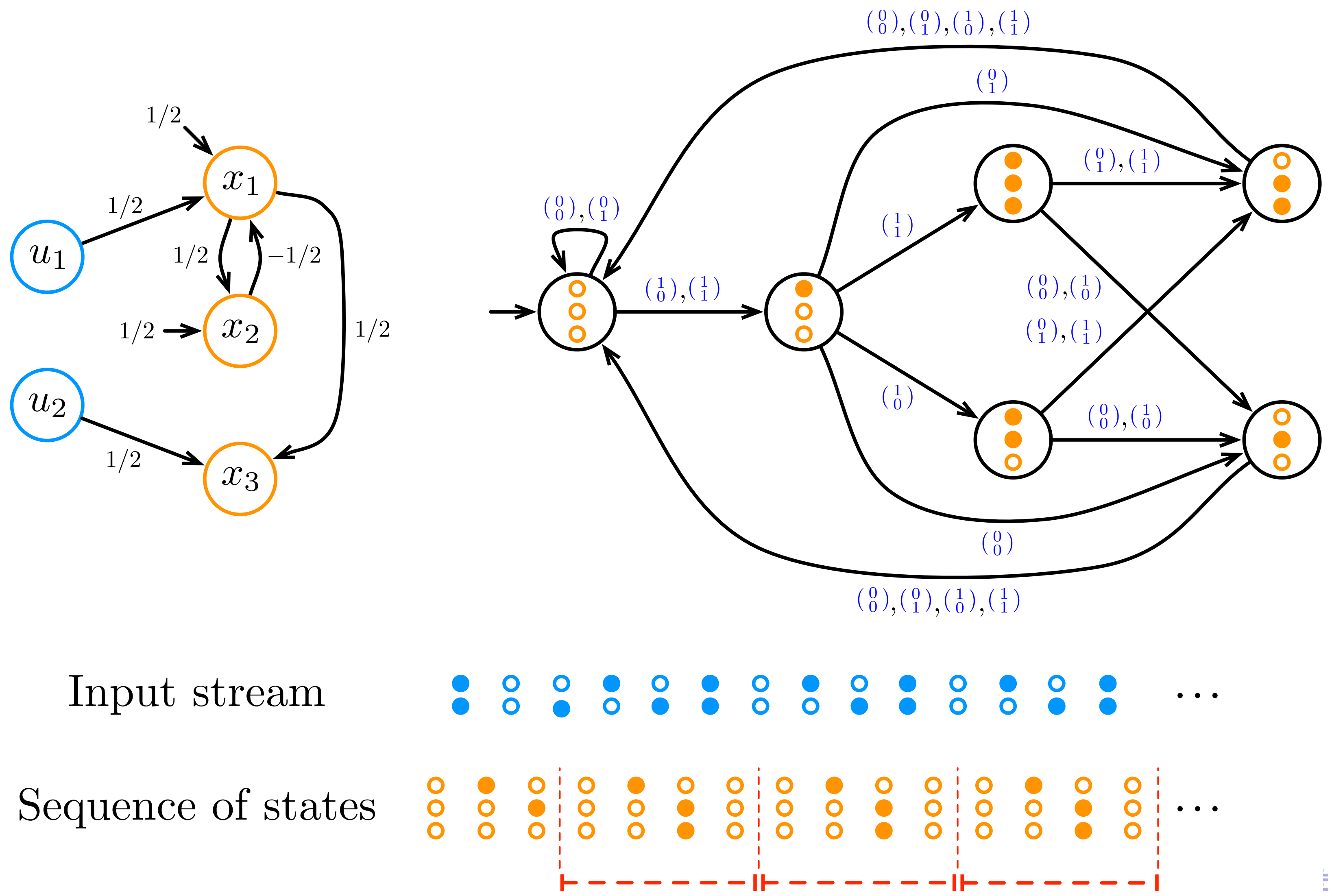
Input stream  …

Sequence of states  …

Boolean Neural Networks – Attractors

Boolean Neural Network **Finite State Automaton**

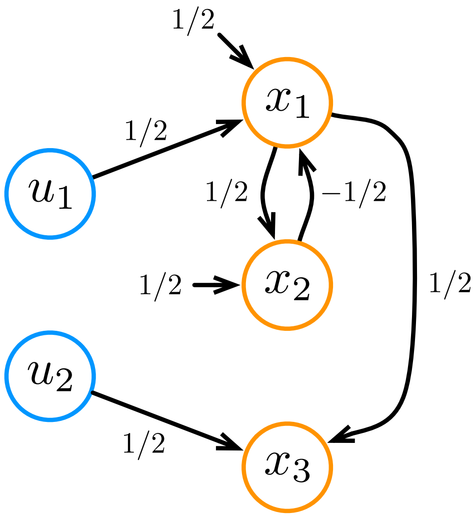
ATTRACTOR

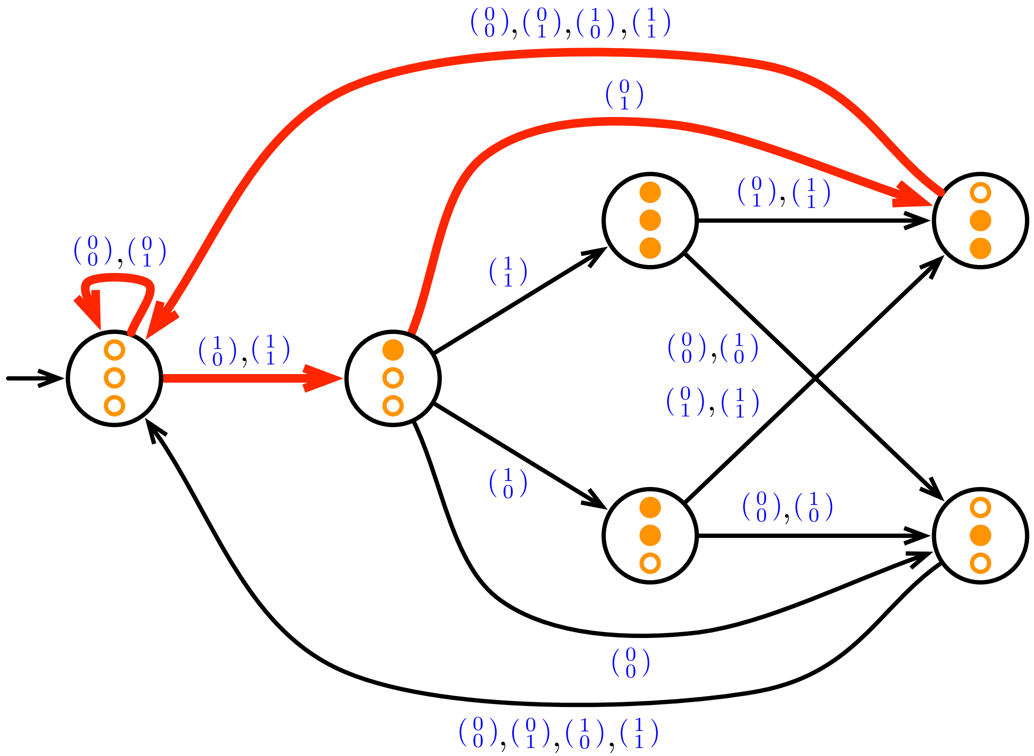


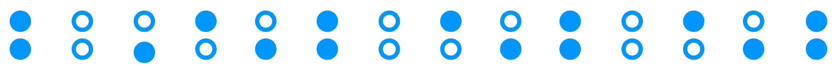
Boolean Neural Networks – Attractors

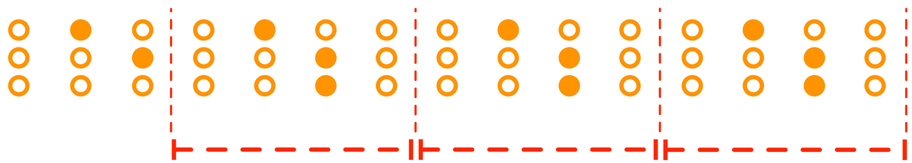
Boolean Neural Network ⟺ **Finite State Automaton**

ATTRACTOR ⟺ **CYCLE**





Input stream  …

Sequence of states  …

Boolean Neural Networks – Attractors

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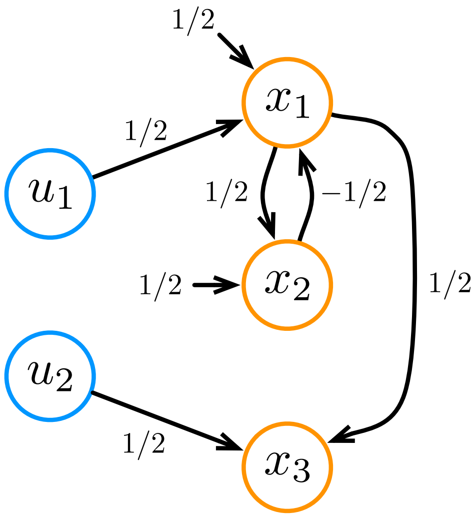
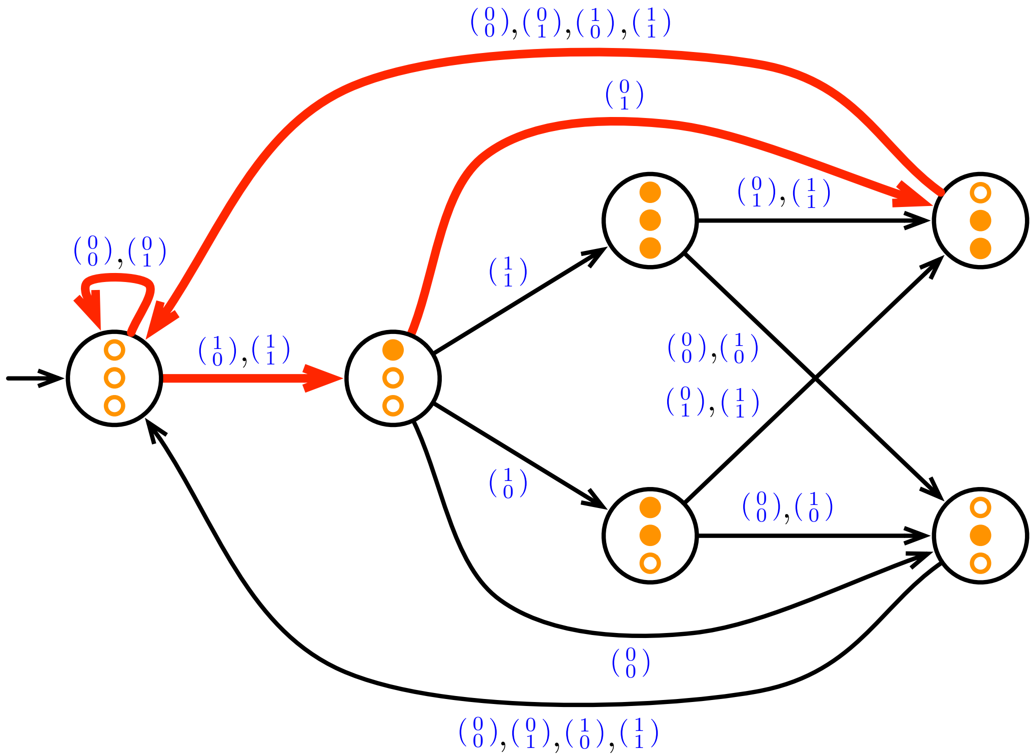
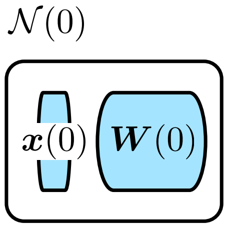




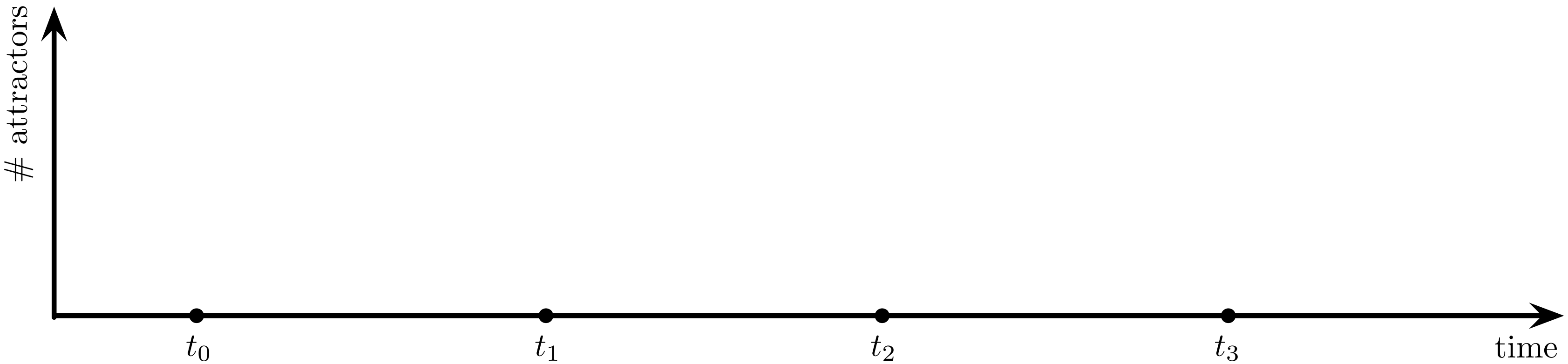
7 attractors **7 simple cycles**

Attractor Dynamics

networks



automata



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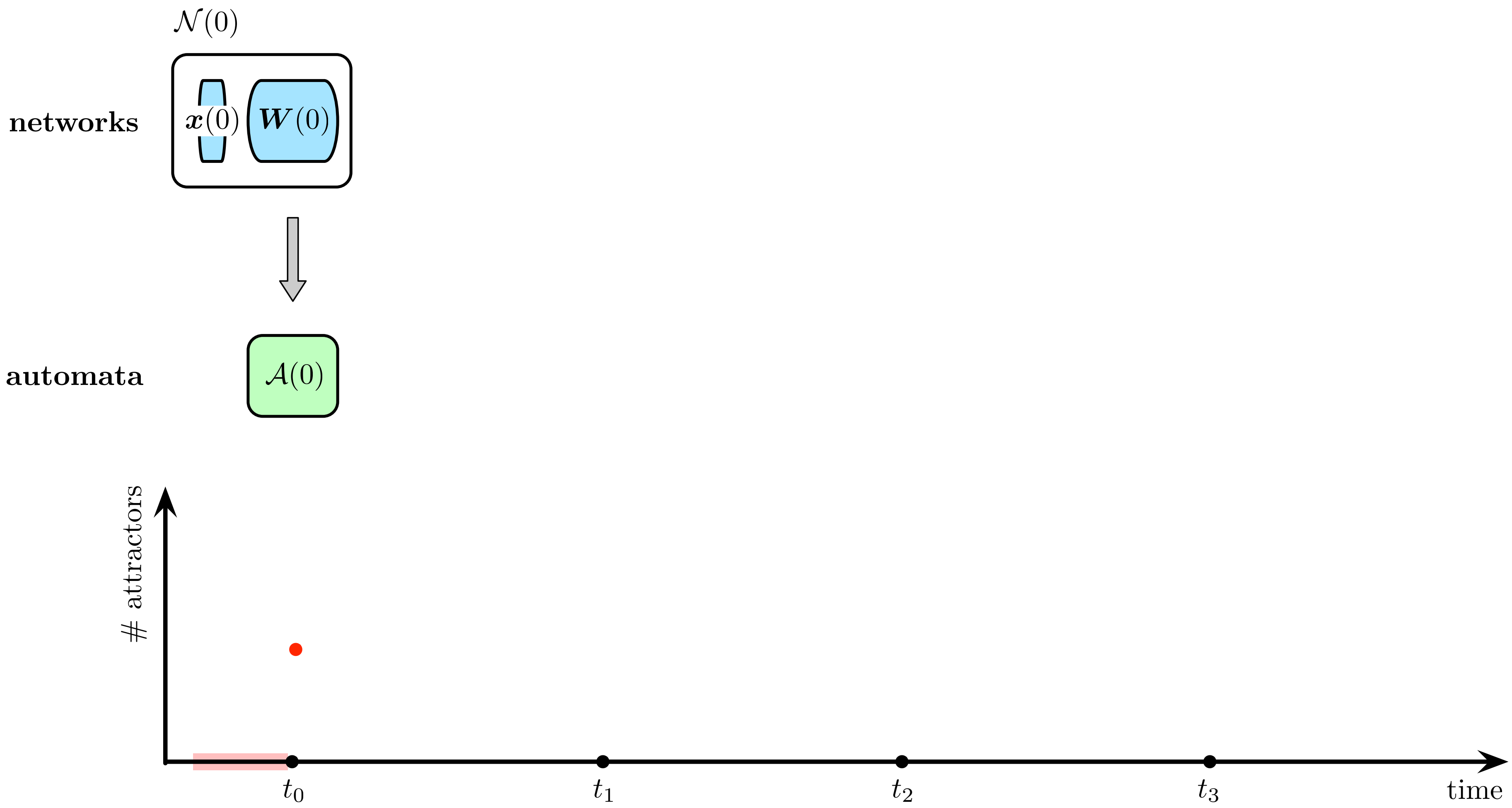

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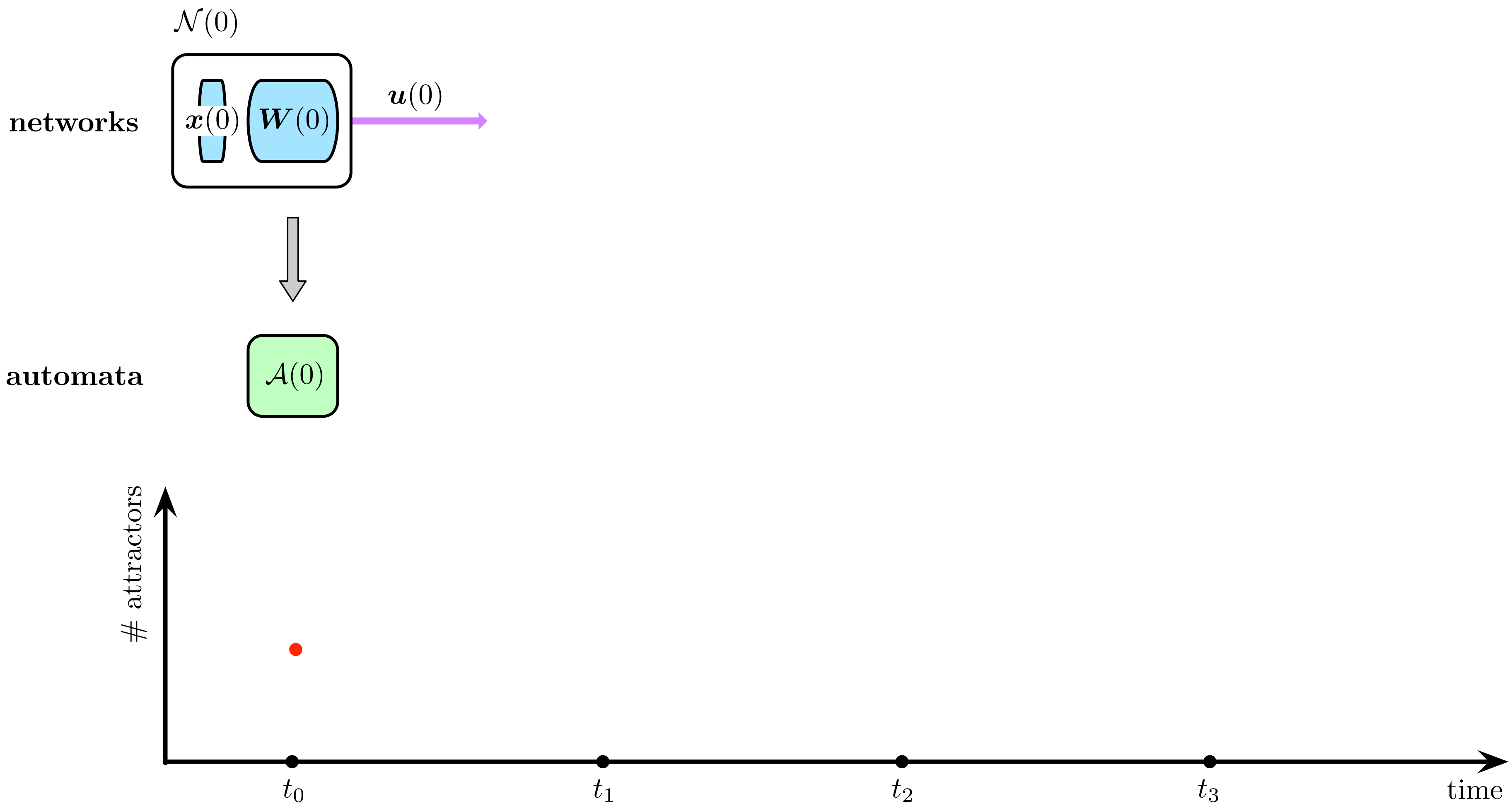

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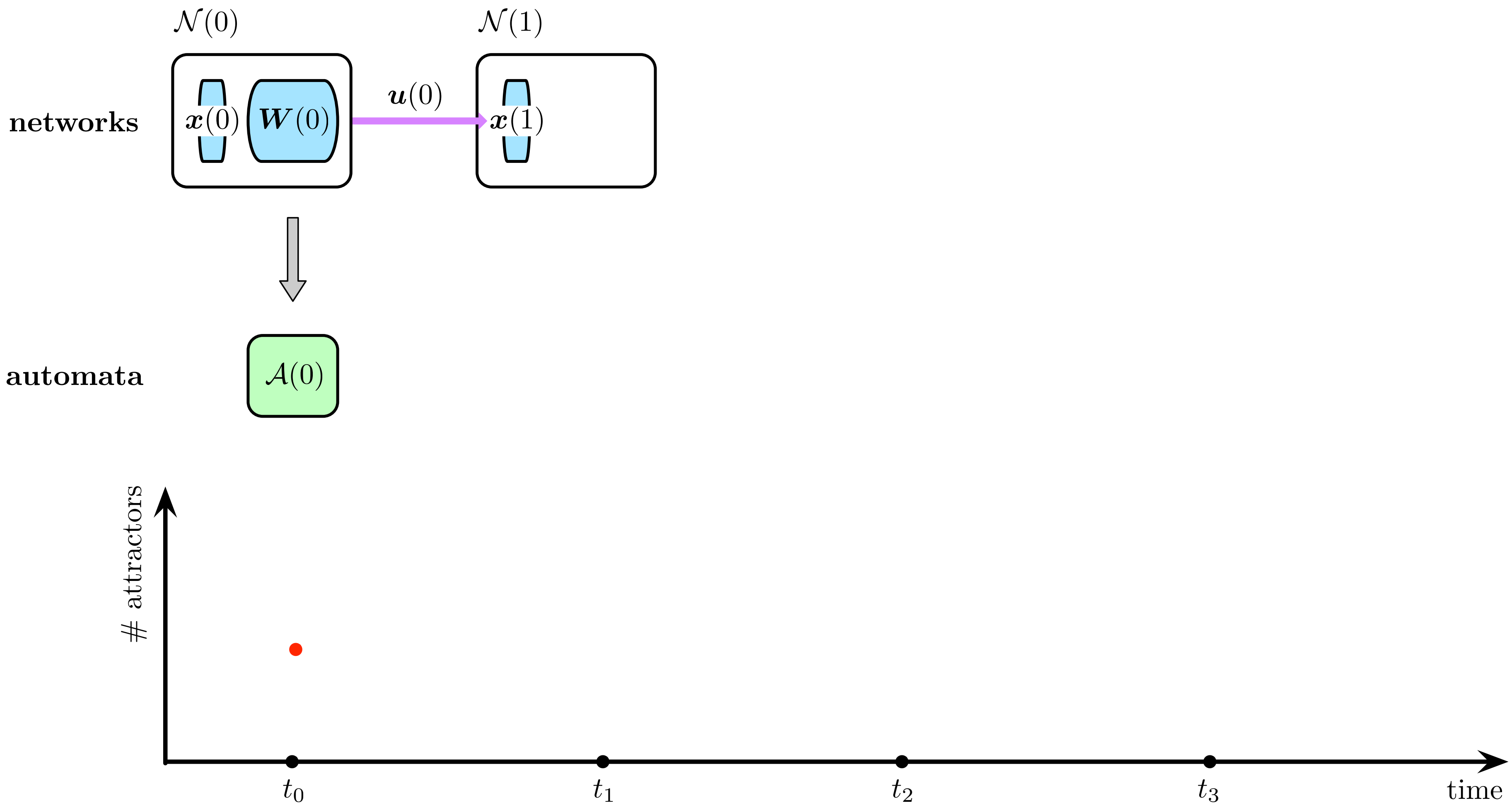

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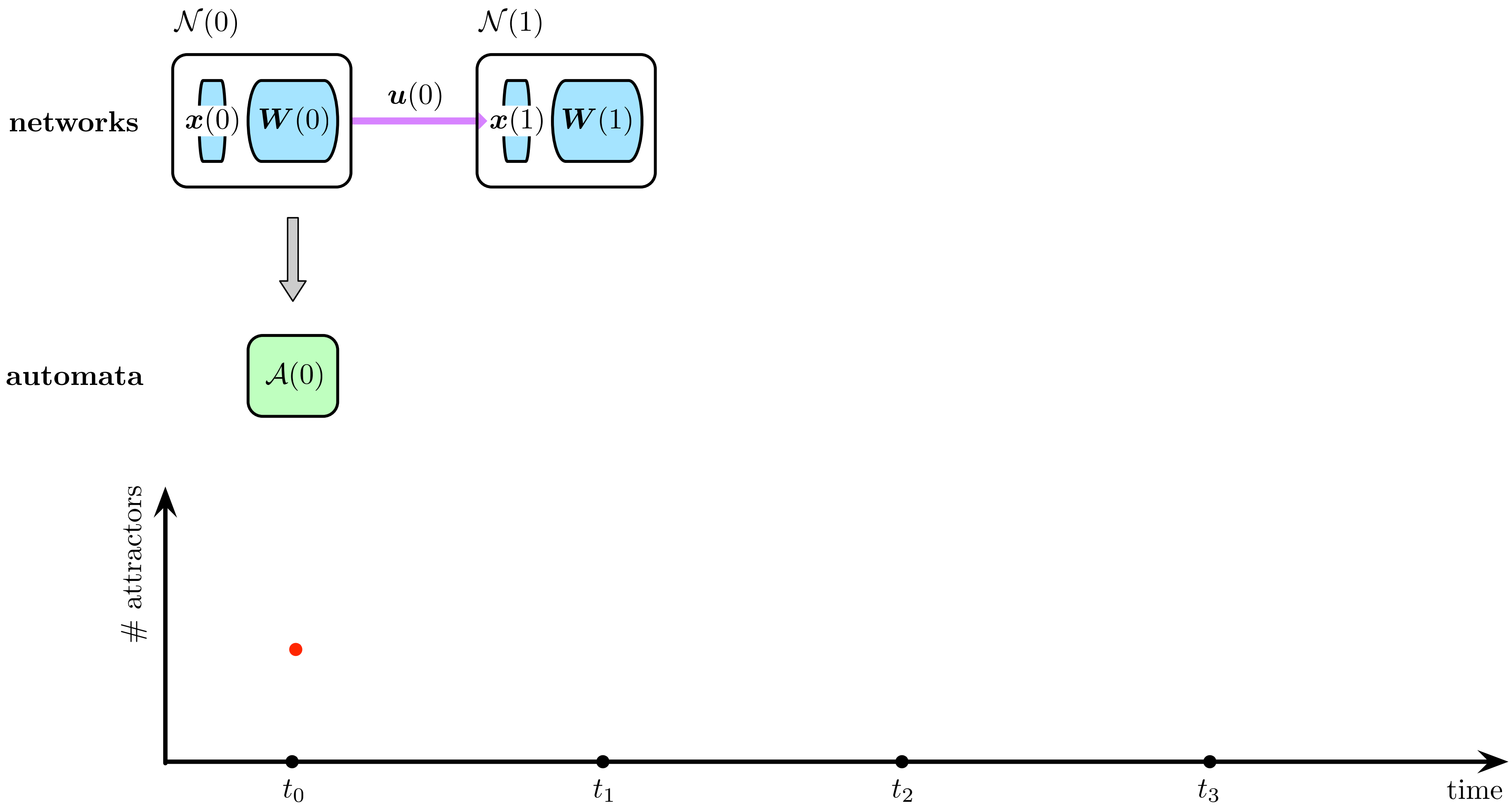

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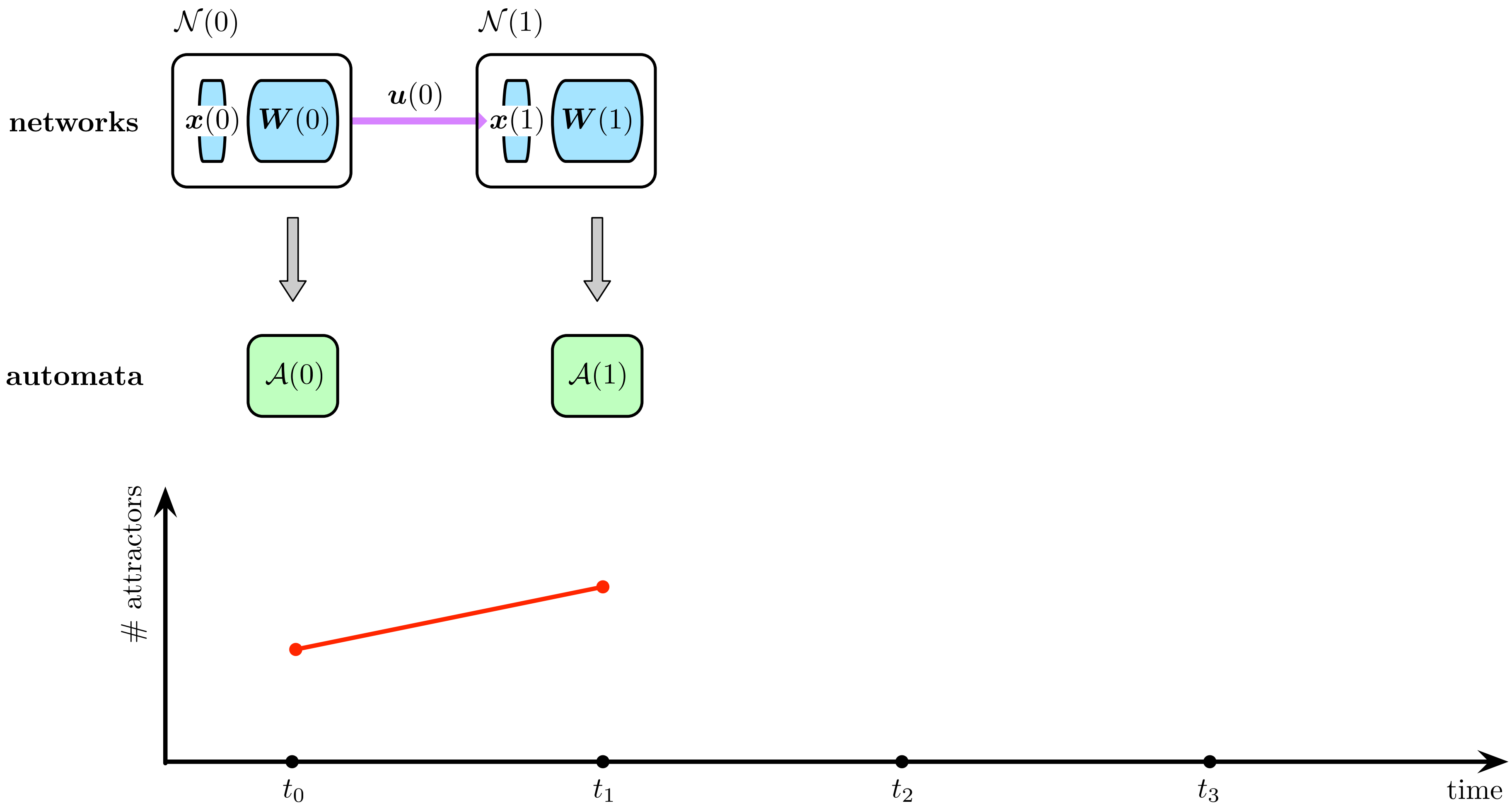

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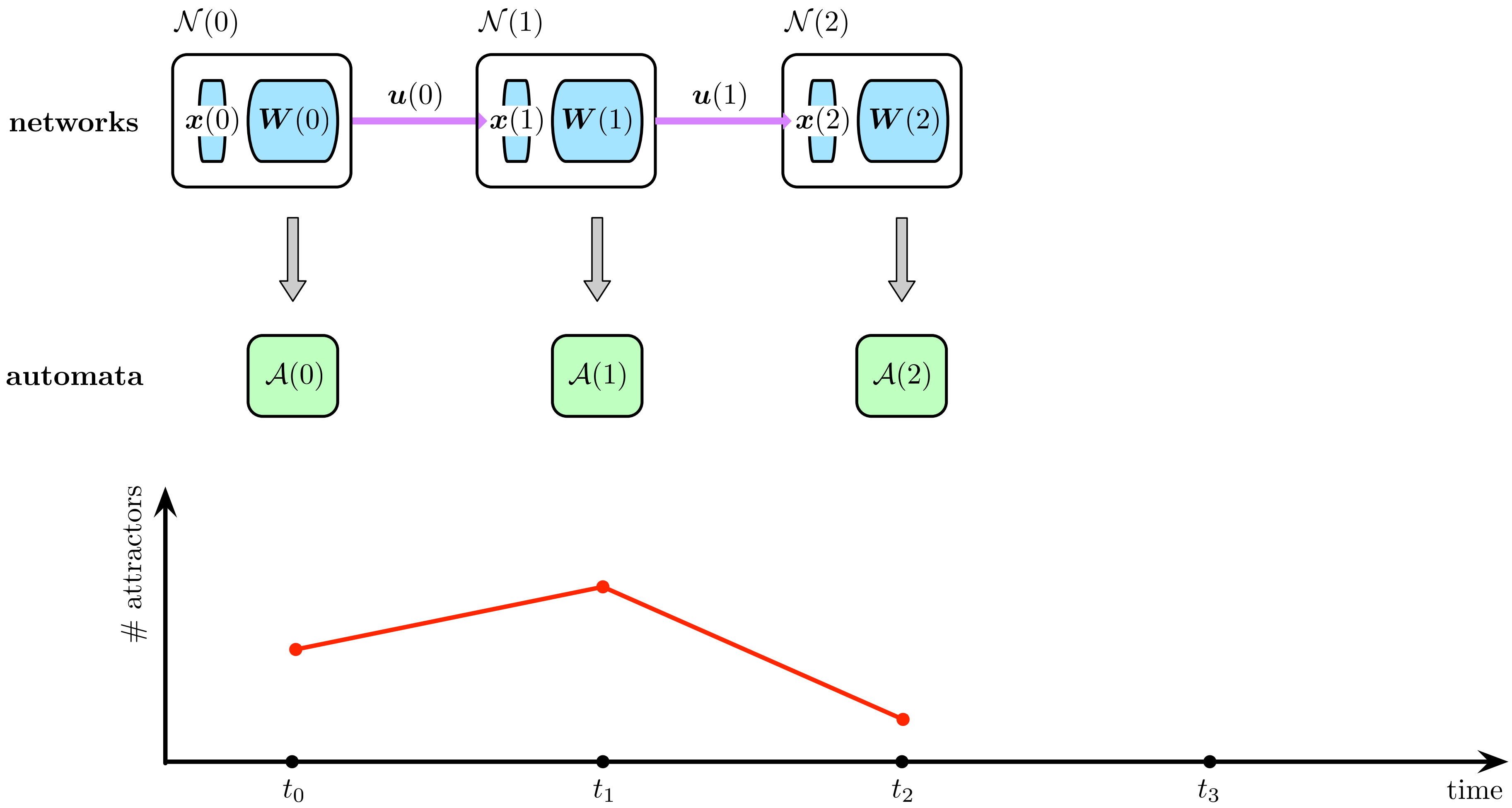

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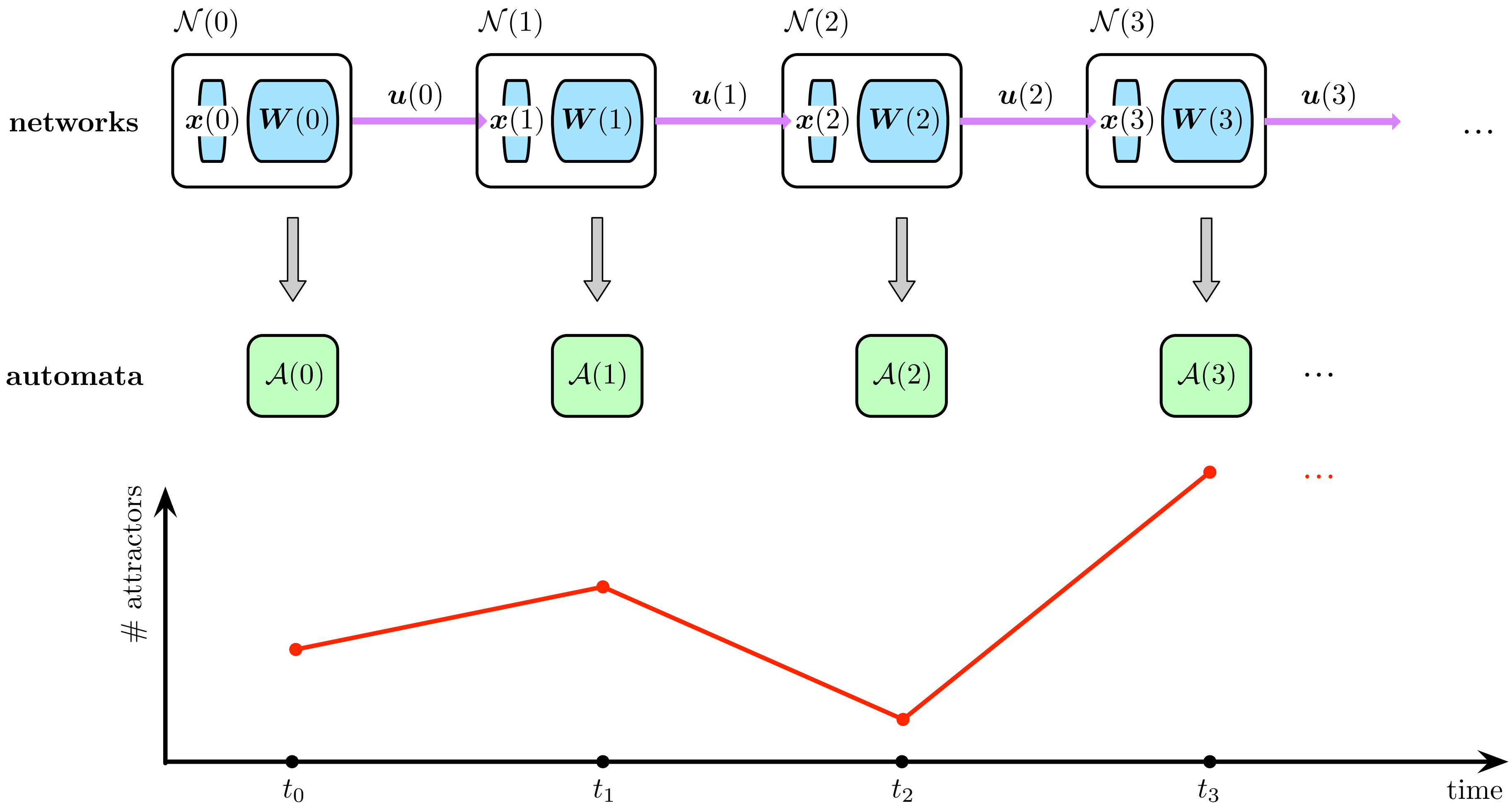

Adaptive STDP Rule

- **Fixed STDP rule:**
  $a_{ji}(t)$: synaptic weight between x_i and x_j at time t

$$a_{ji}(t+1) = a_{ji}(t) + \lambda \Big[x_i(t) x_j(t+1) \, \mathcal{A} \, x_j(t) x_i(t+1) \Big]$$

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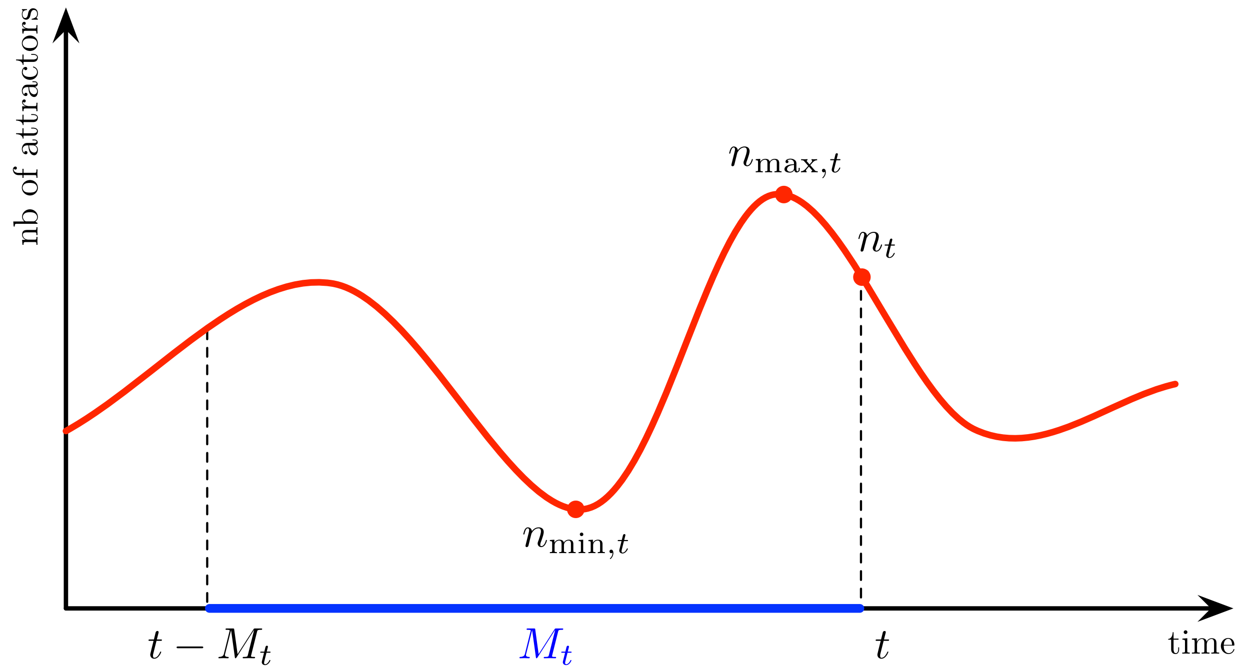
Dynamic Memory – Input Driven

$$\begin{array}{rcl} \color{blue}{M_t} & = & \textit{memory} \text{ of network } \mathcal{N} \text{ at time } t \text{ (past time window)} \\ n_t & = & \text{number of attractors of } \mathcal{N} \text{ at time } t \\ n_{\min,t} & = & \text{minimal nb of attractors remembered by } \mathcal{N} \text{ at time } t \\ n_{\max,t} & = & \text{maximal nb of attractors remembered by } \mathcal{N} \text{ at time } t \end{array}$$



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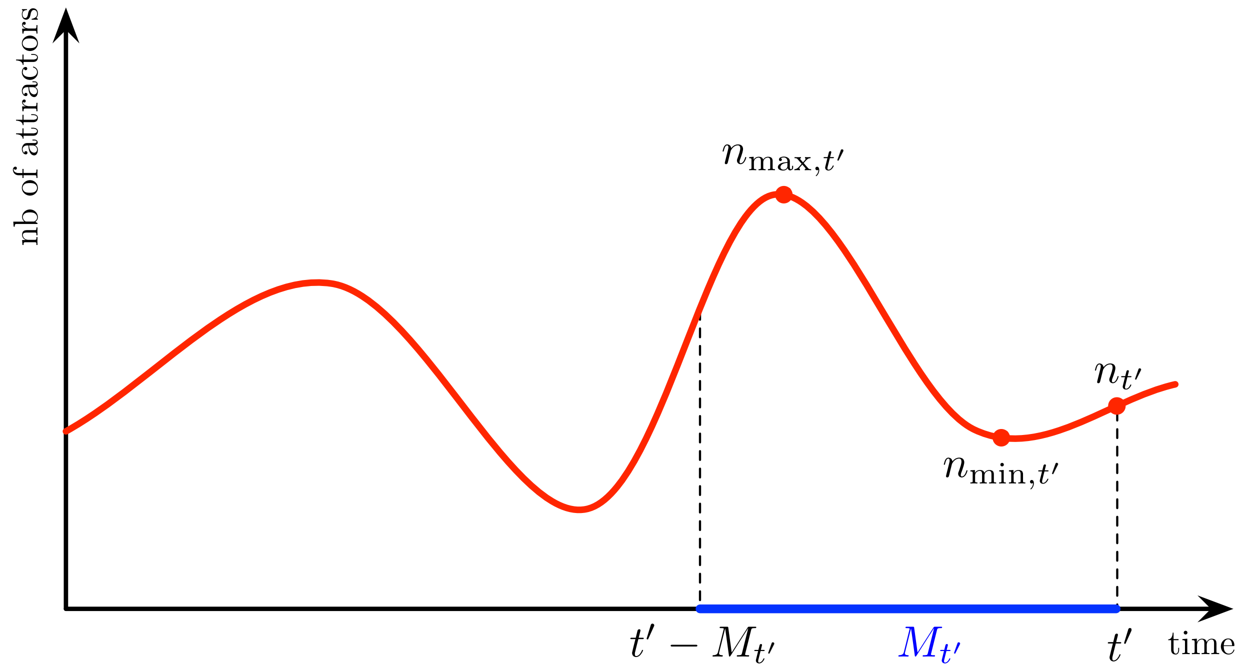

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$$
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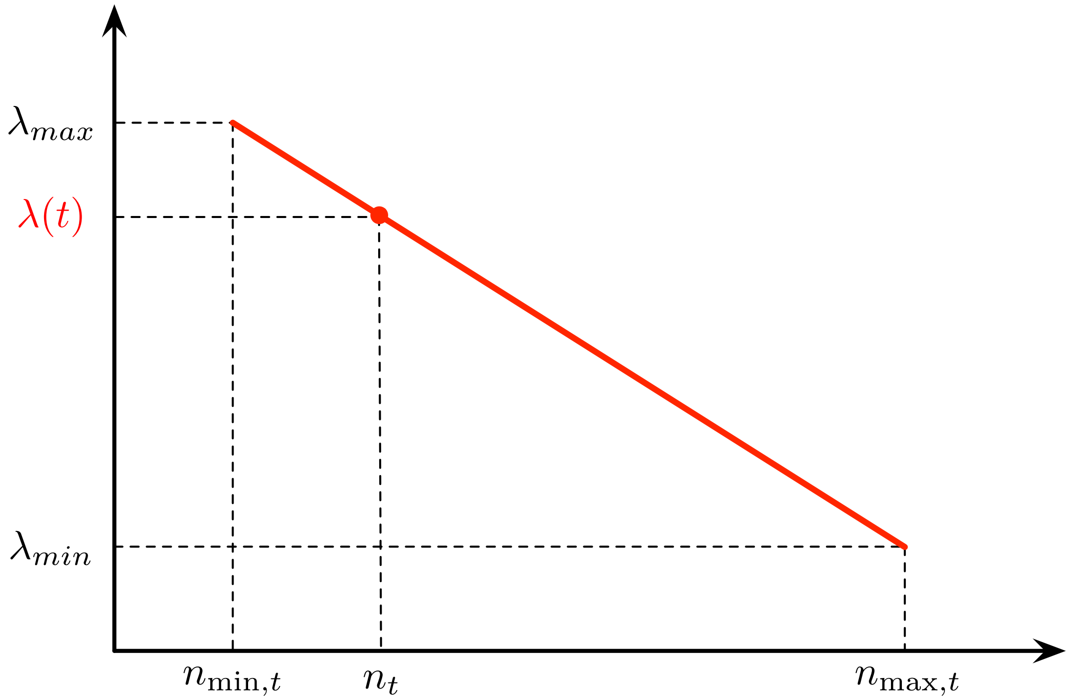
M_0 &= 0 \\
M_{t+1} &= \begin{cases} M_t + K & \text{if the } \textbf{input trigger pattern} \\ & \text{is detected at time } t \\ \max(M_t - 1, 0) & \text{otherwise} \end{cases}
\end{aligned}
$$

Adaptive STDP Rule: learning rate

- The *learning rate* $\lambda(t)$ depends on the current, min and max number of attractors seen during the last M_t time steps.

$$\lambda(t) = \begin{cases} \lambda_{\max} + \dfrac{(n_t - n_{\min,t})(\lambda_{\min} - \lambda_{\max})}{n_{\max\,.t} - n_{\min,t}} & \text{if } n_{\min,t} \neq n_{\max,t} \\ \lambda_{\max} & \text{otherwise} \end{cases}$$



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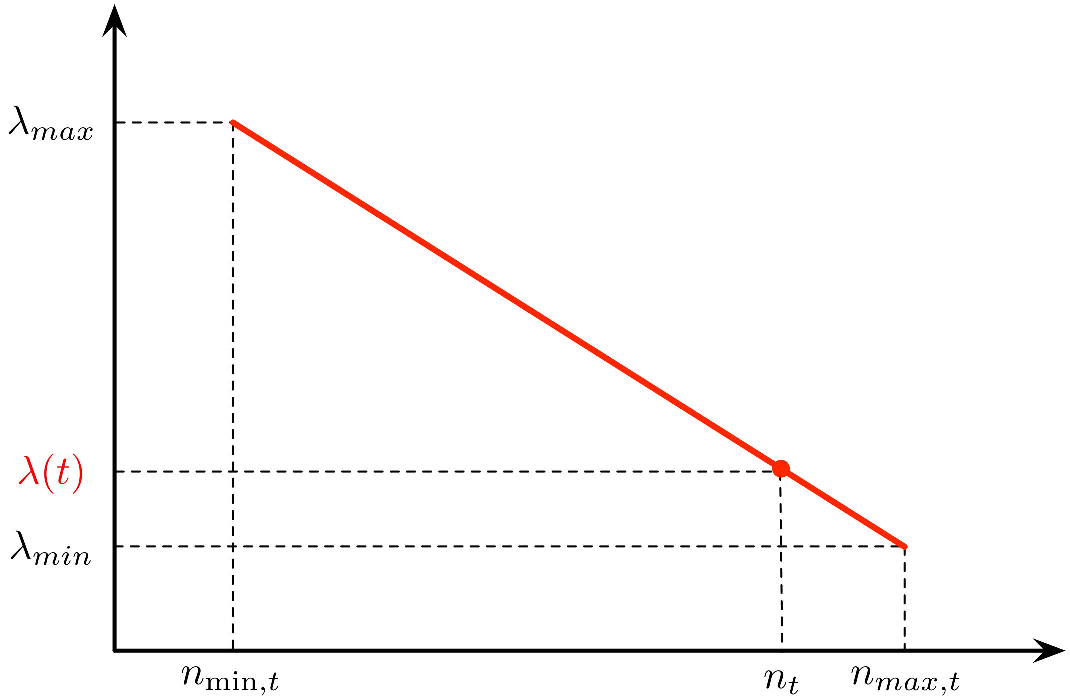
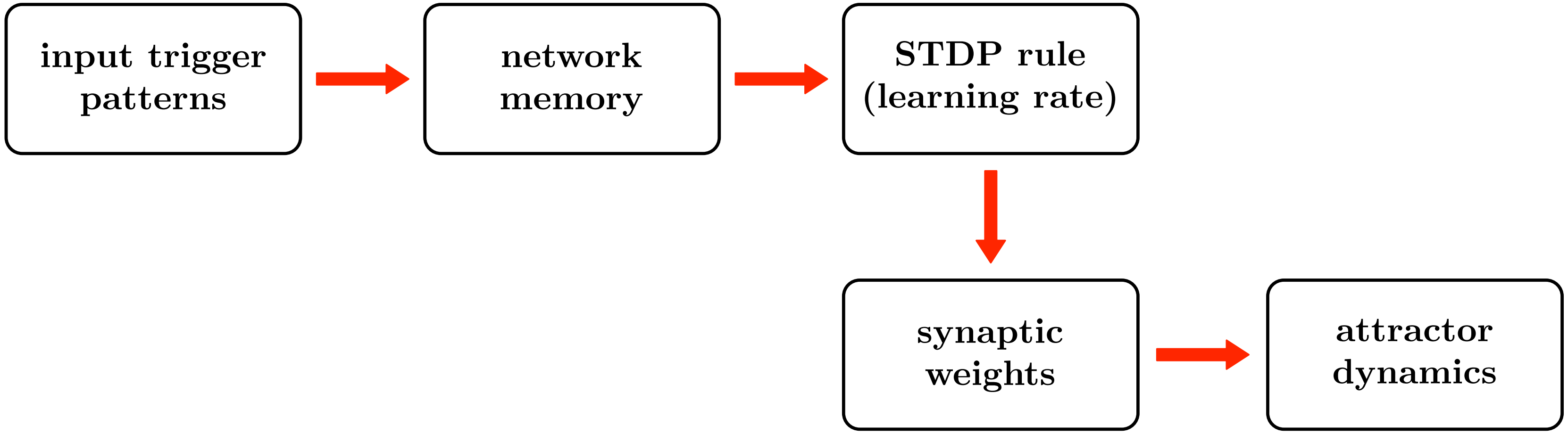


Network Dynamics

Multi-level evolving process:

1. **Input trigger patterns** influence the **network memory**.
2. The **network memory** influence the **STDP rule** (learning rate).
3. The **STDP rule** influence the **synaptic weights**.
4. The **synaptic weights** influence the **attractor dynamics**.



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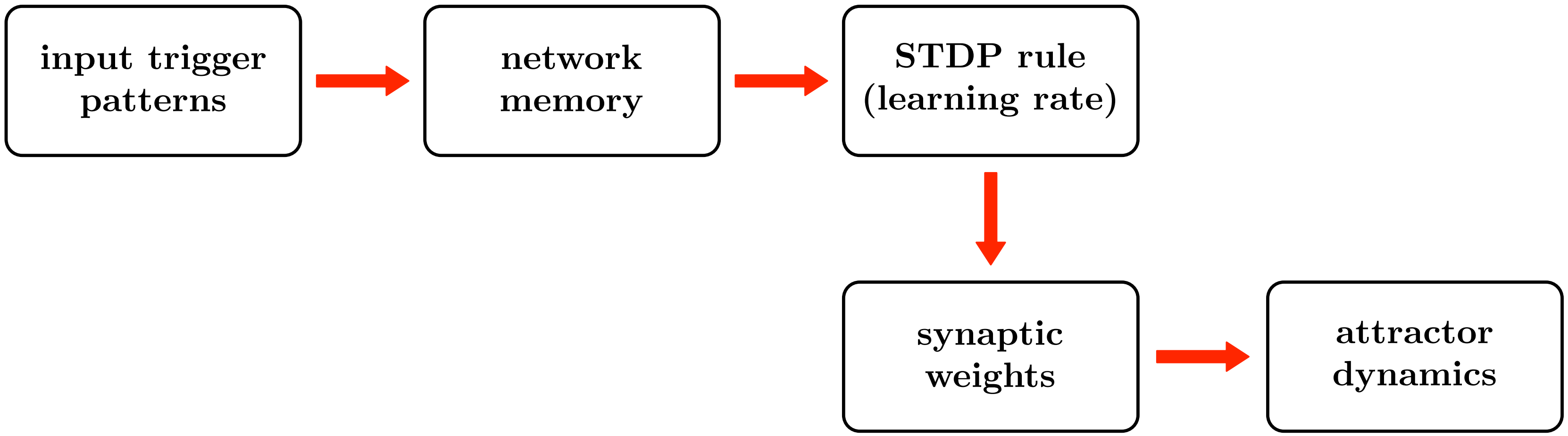

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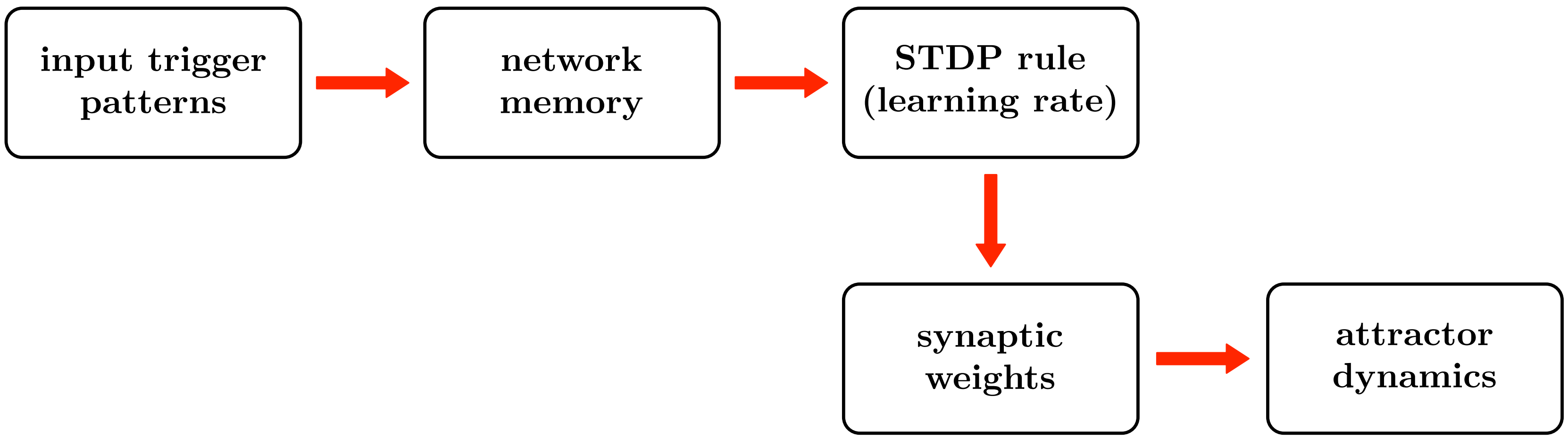

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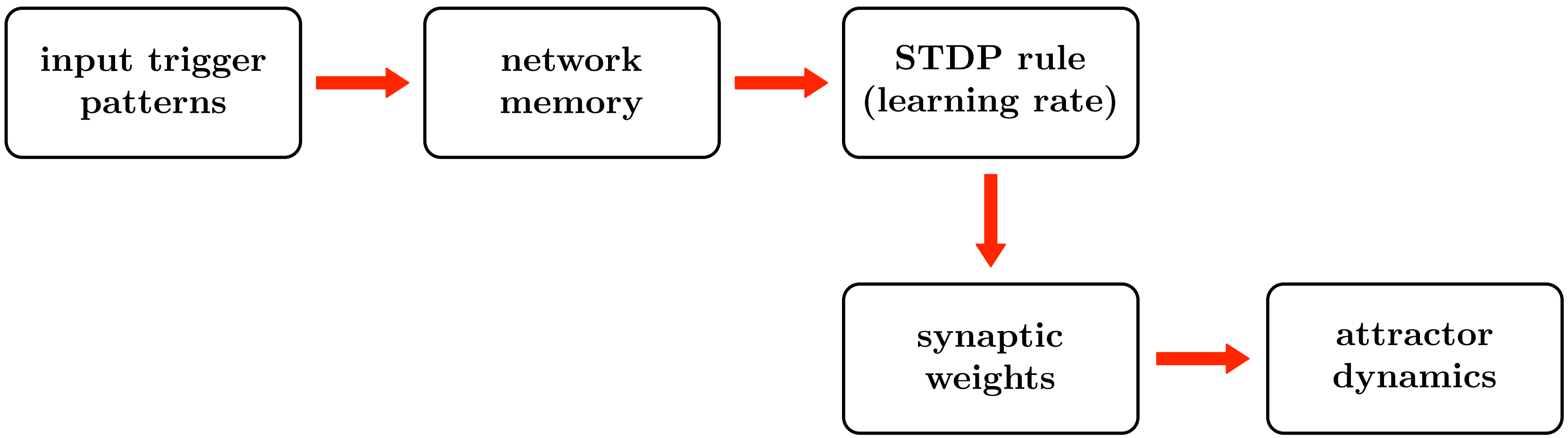

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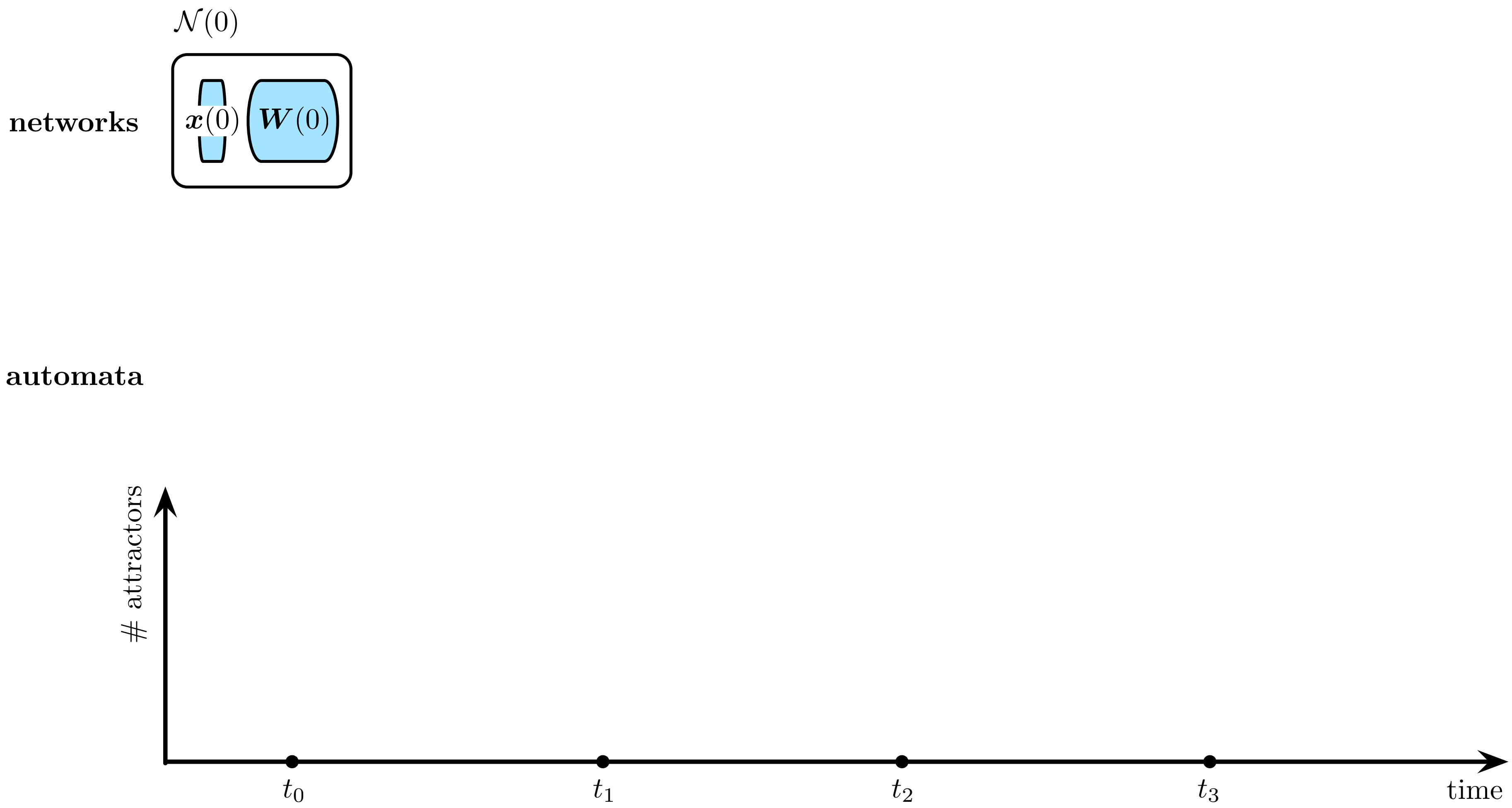

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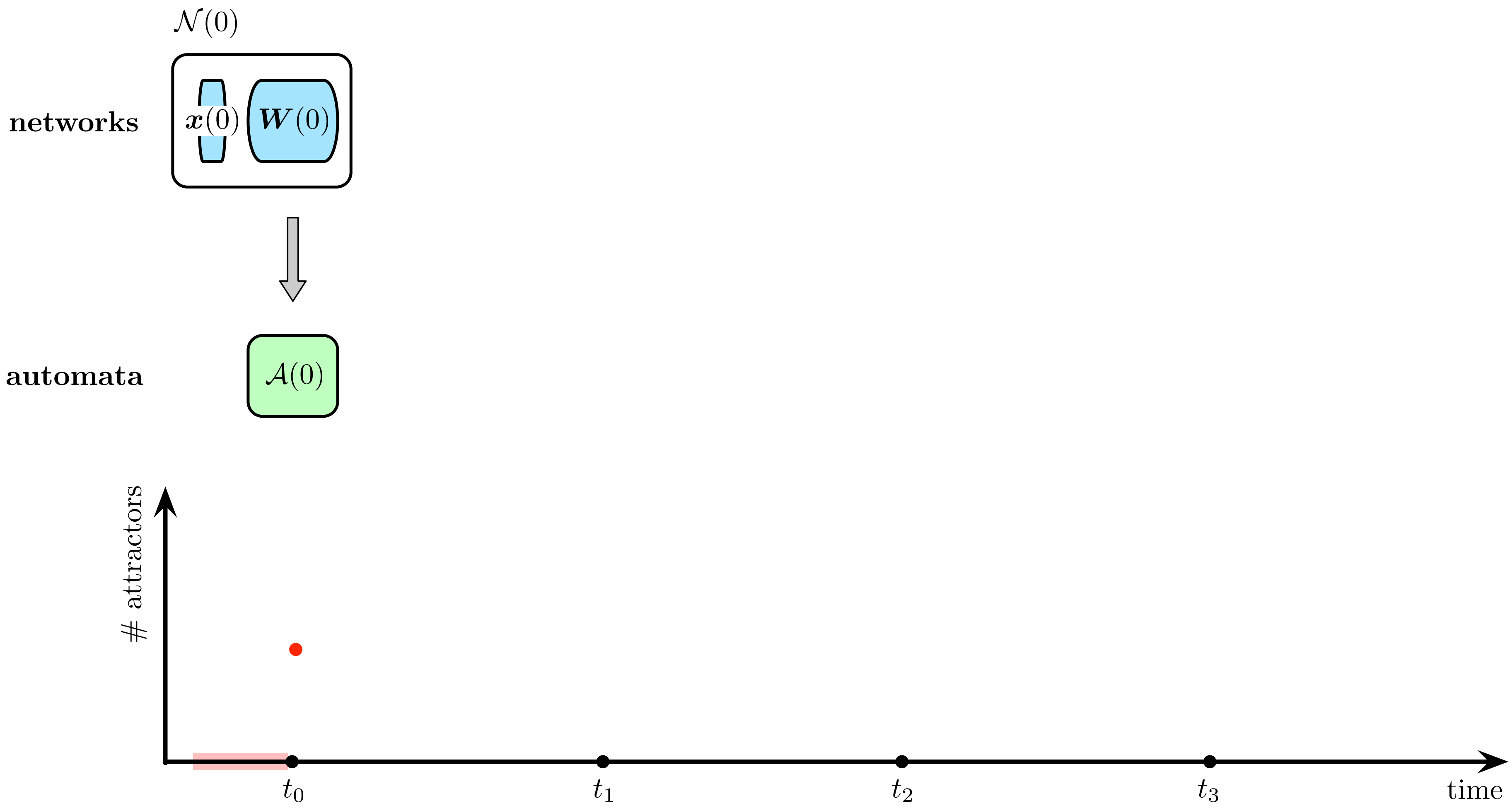

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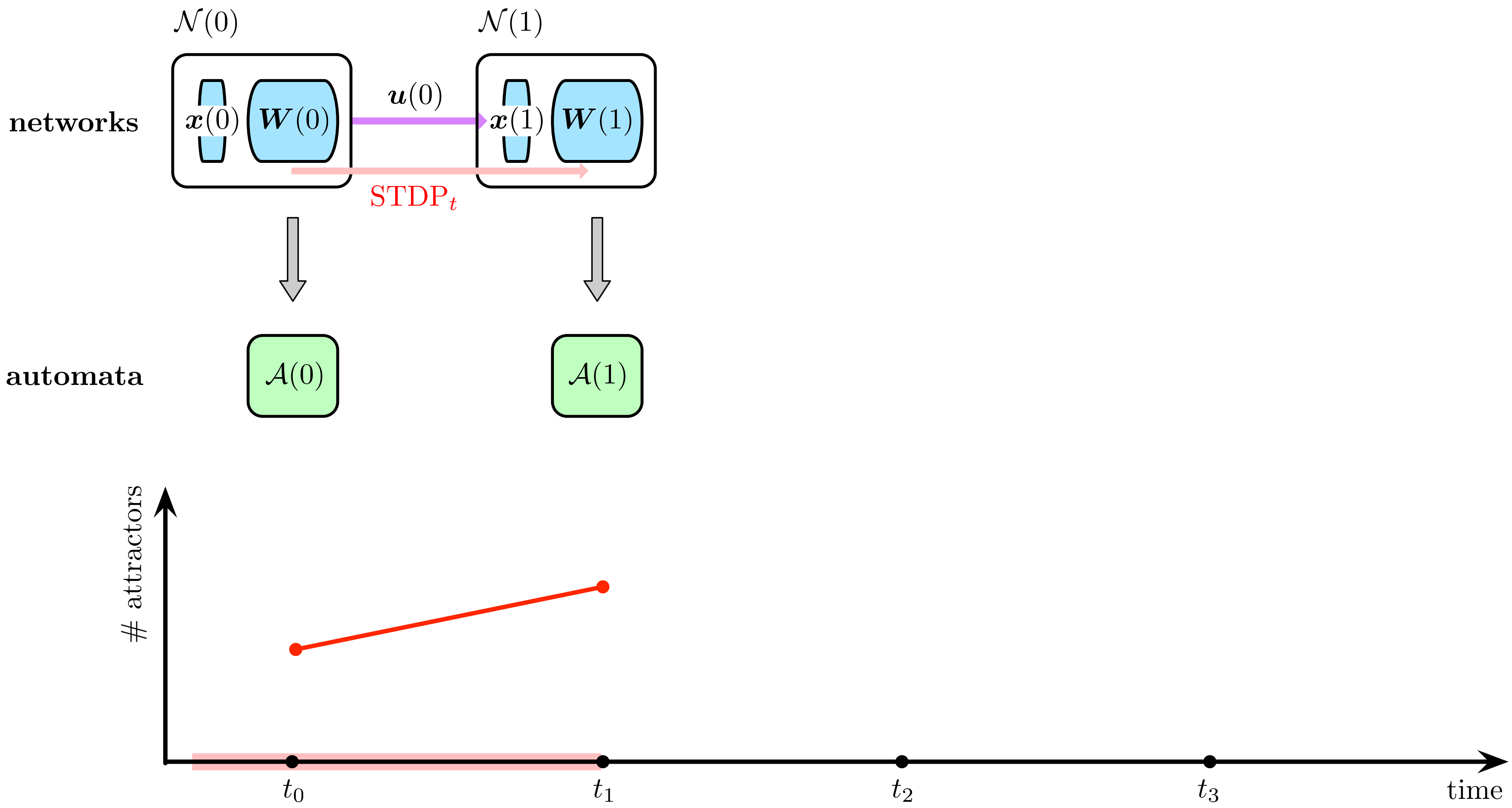

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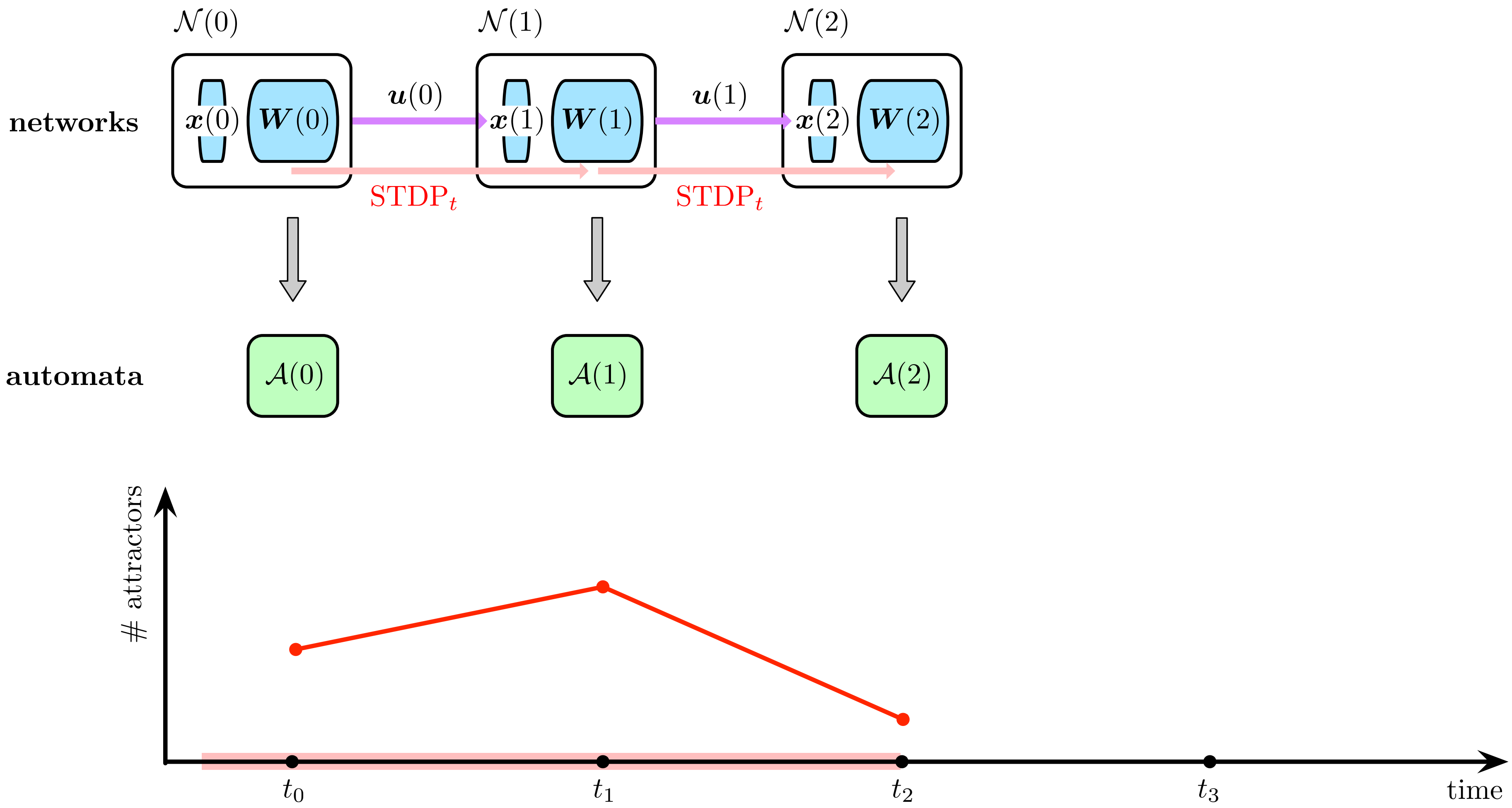

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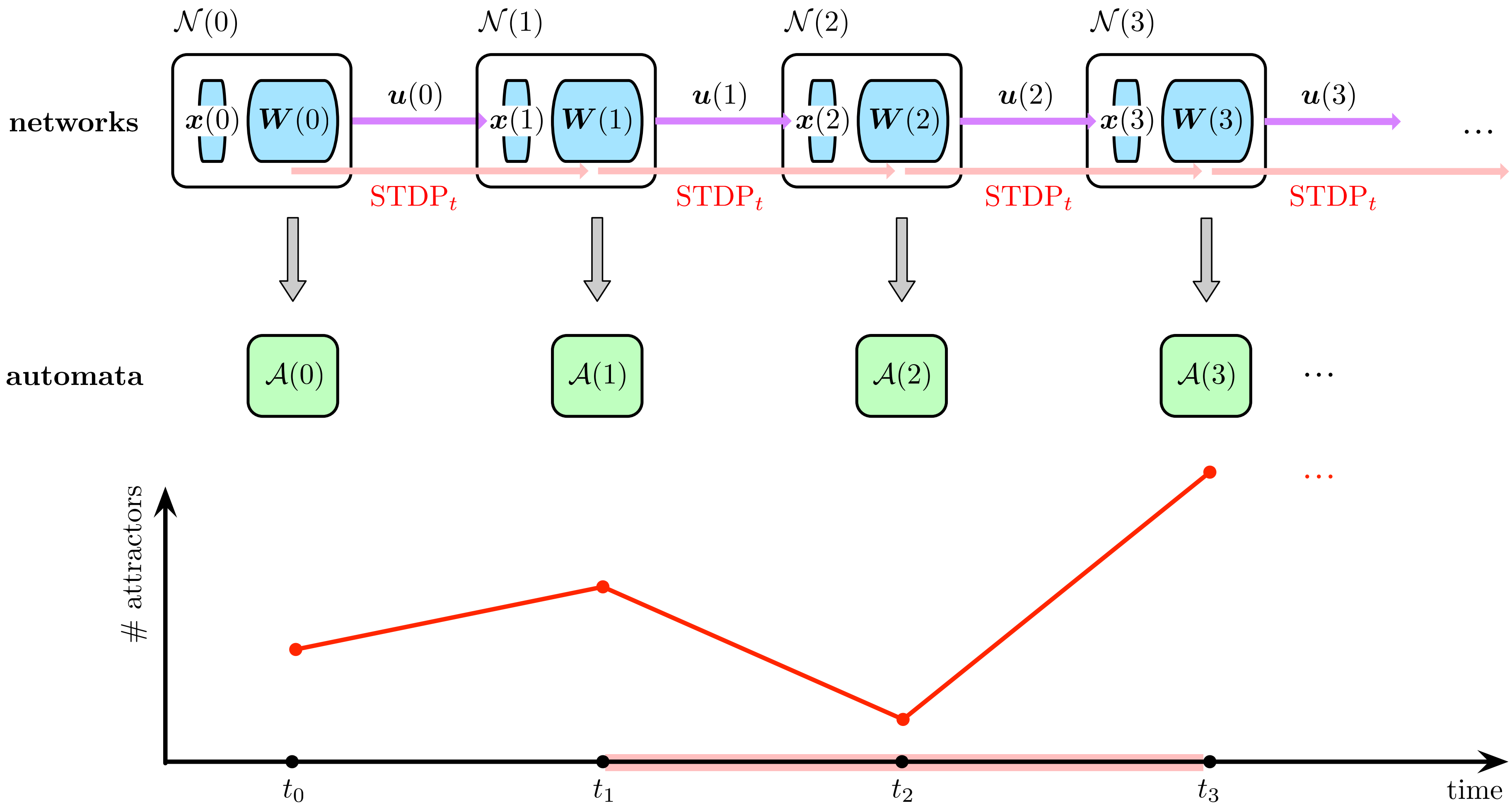
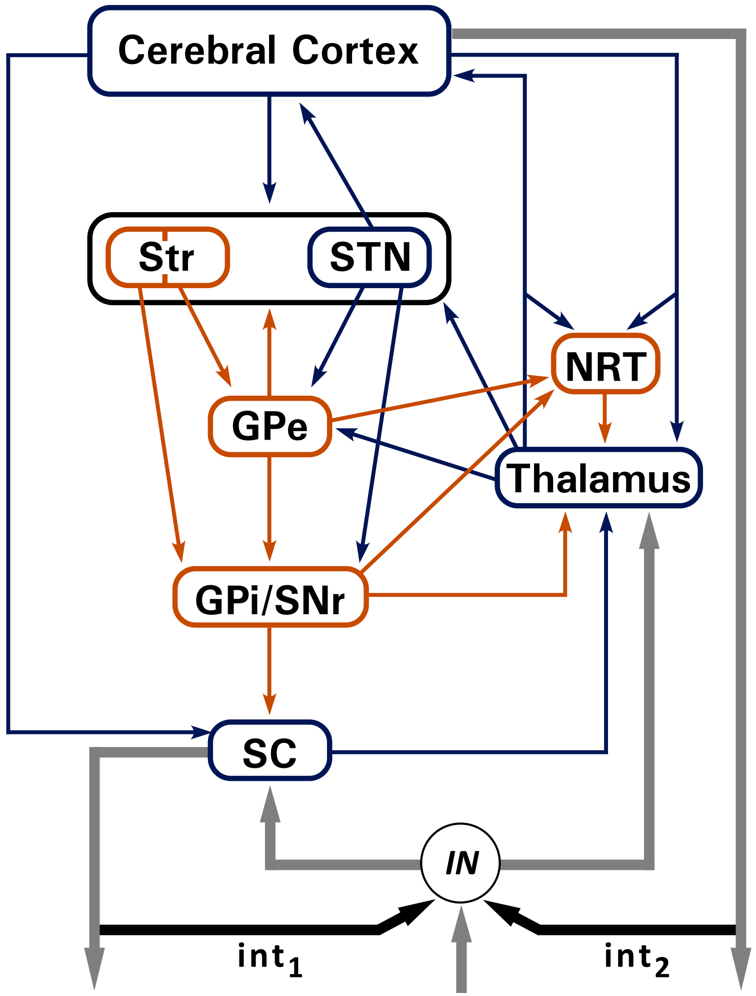


Boolean Model of the Basal Ganglia-Thalamocortical Network



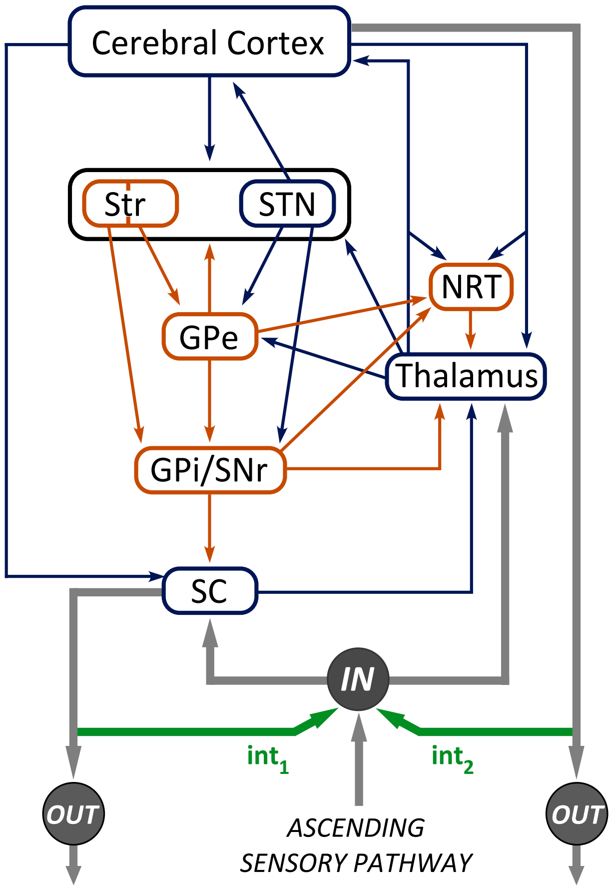
IN	input node
SC	superior colliculus
GPi/SNr	output nuclei of the basal ganglia formed by the GABAergic projection neurons of the intermediate part of the pallidum and of the substantia nigra pars reticulata
Thalamus	thalamus
GPe	external part of the pallidum
NRT	thalamic reticular nucleus
Str-D1	striatopallidal component of the striatum
Str-D2	striatonigral component of the striatum
STN	subthalamic nucleus
Cerebral Cortex	cerebral cortex

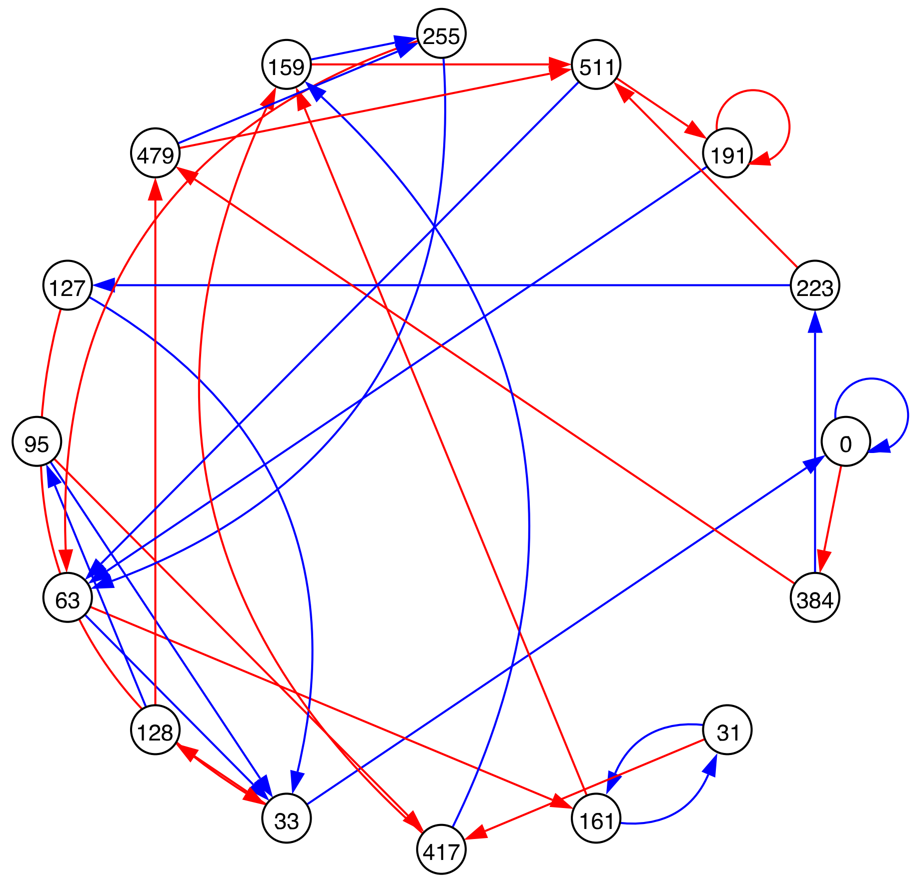
Boolean Model of the Basal Ganglia-Thalamocortical Network

Source	Target (Node #)									
Node # (Name)	0	1	2	3	4	5	6	7	8	9
0 (IN)	·	1	1	·	·	·	·	·	·	·
1 (SC)	int_1	·	1	·	·	·	·	·	·	·
2 (Thalamus)	·	·	·	1	·	1	1	1	1	1
3 (RTN)	·	·	-1	·	·	·	·	·	·	·
4 (GPi/SNr)	·	-1	-1	-1	·	·	·	·	·	·
5 (STN)	·	·	·	·	2	·	2	·	·	2
6 (GPe)	·	·	·	-1/2	-1/2	-1/2	·	-1/2	-1/2	·
7 (Str-D2)	·	·	·	·	·	·	-1	·	·	·
8 (Str-D1)	·	·	·	·	-1/2	·	-1/2	·	·	·
9 (CCortex)	int_2	1/2	1/2	1/2	·	1/2	·	1/2	1/2	·

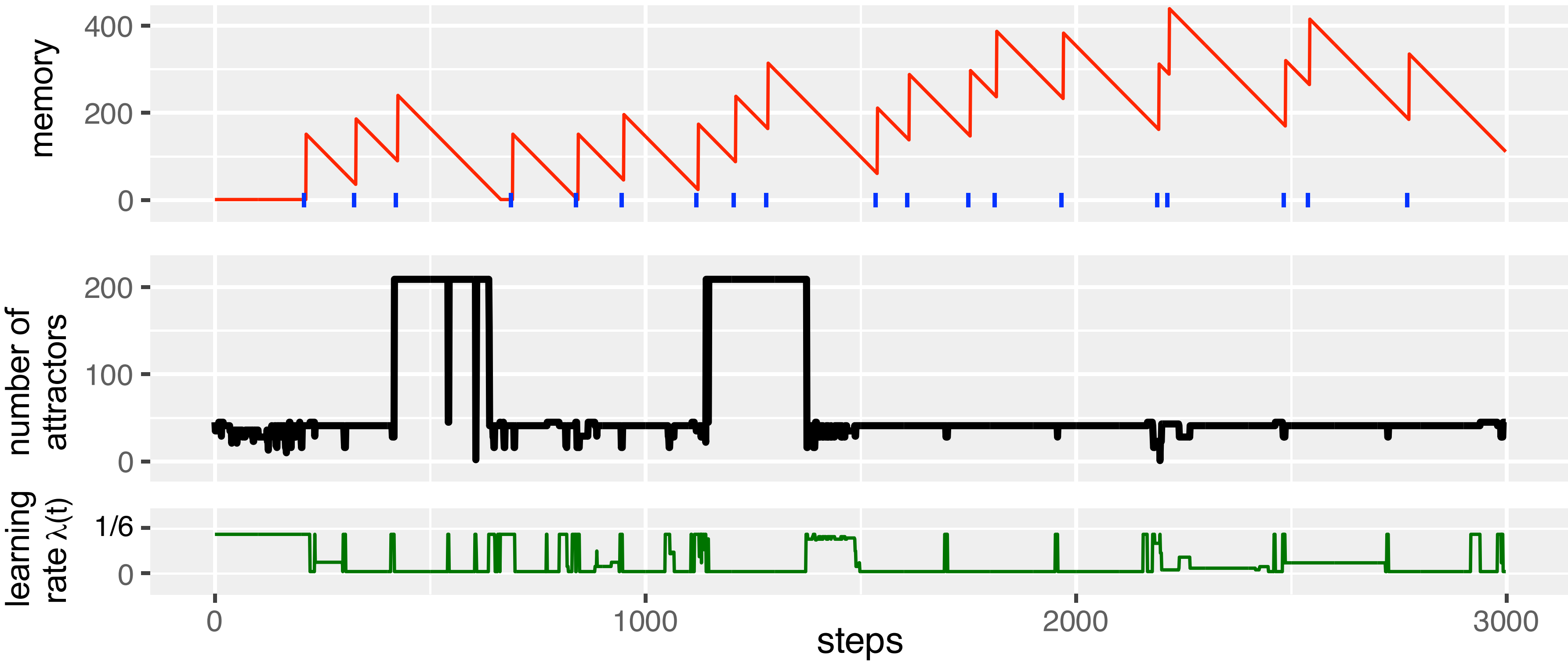
Table: Adjancency matrix

BGT Network and Corresponding FSA





Stabilization of Attractor Dynamics



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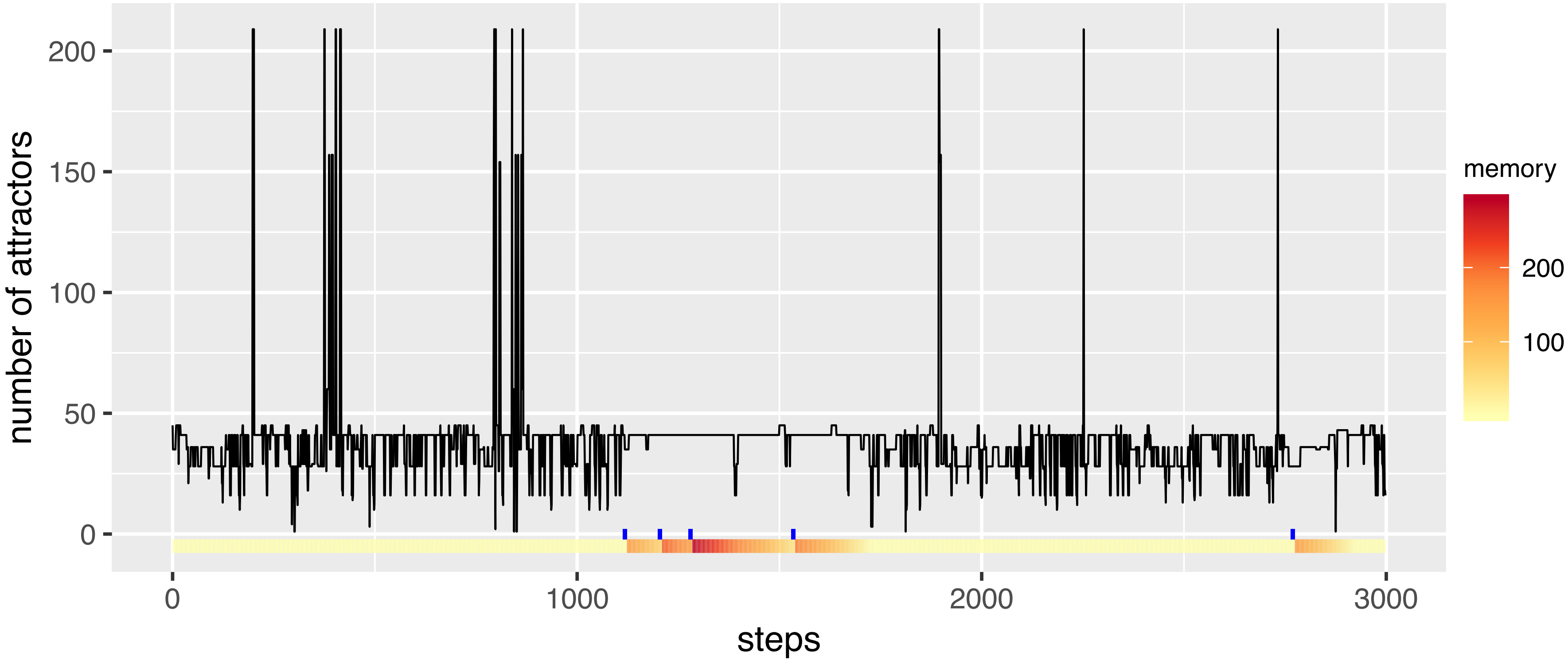

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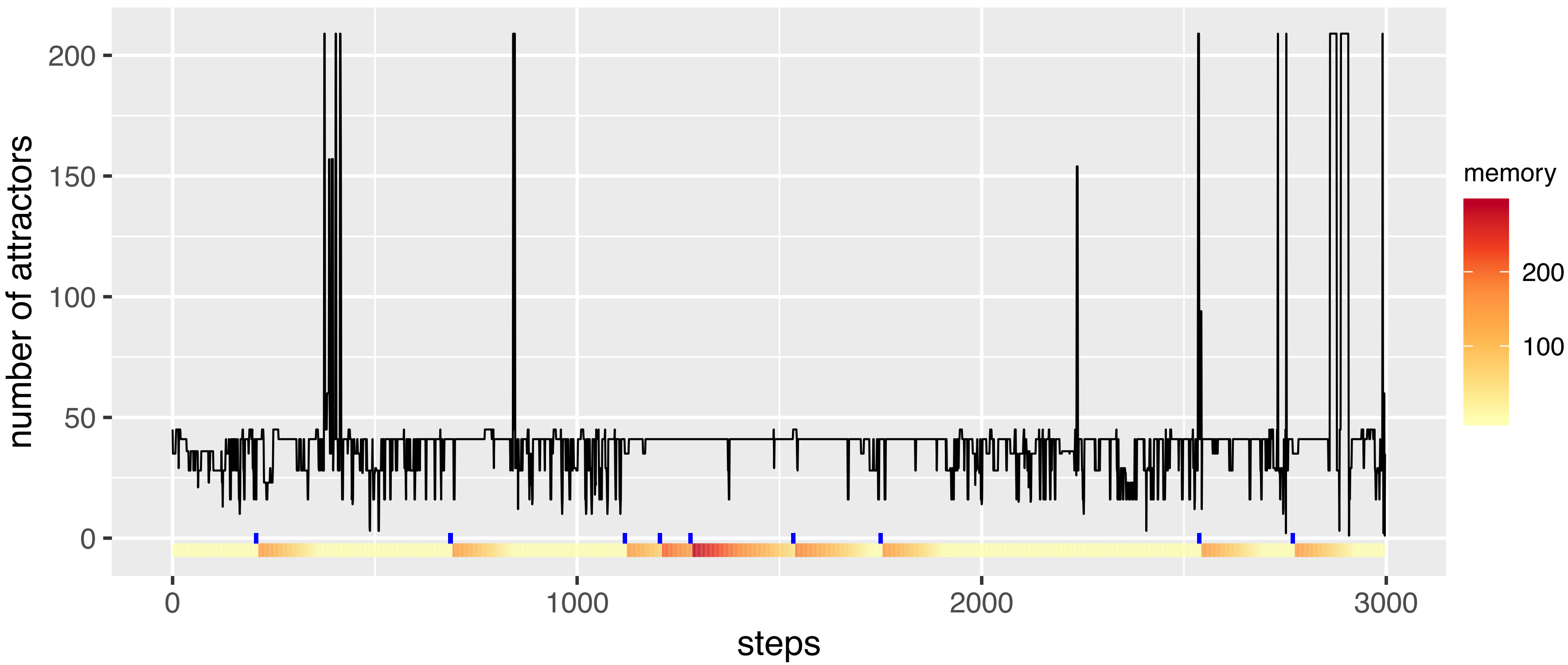
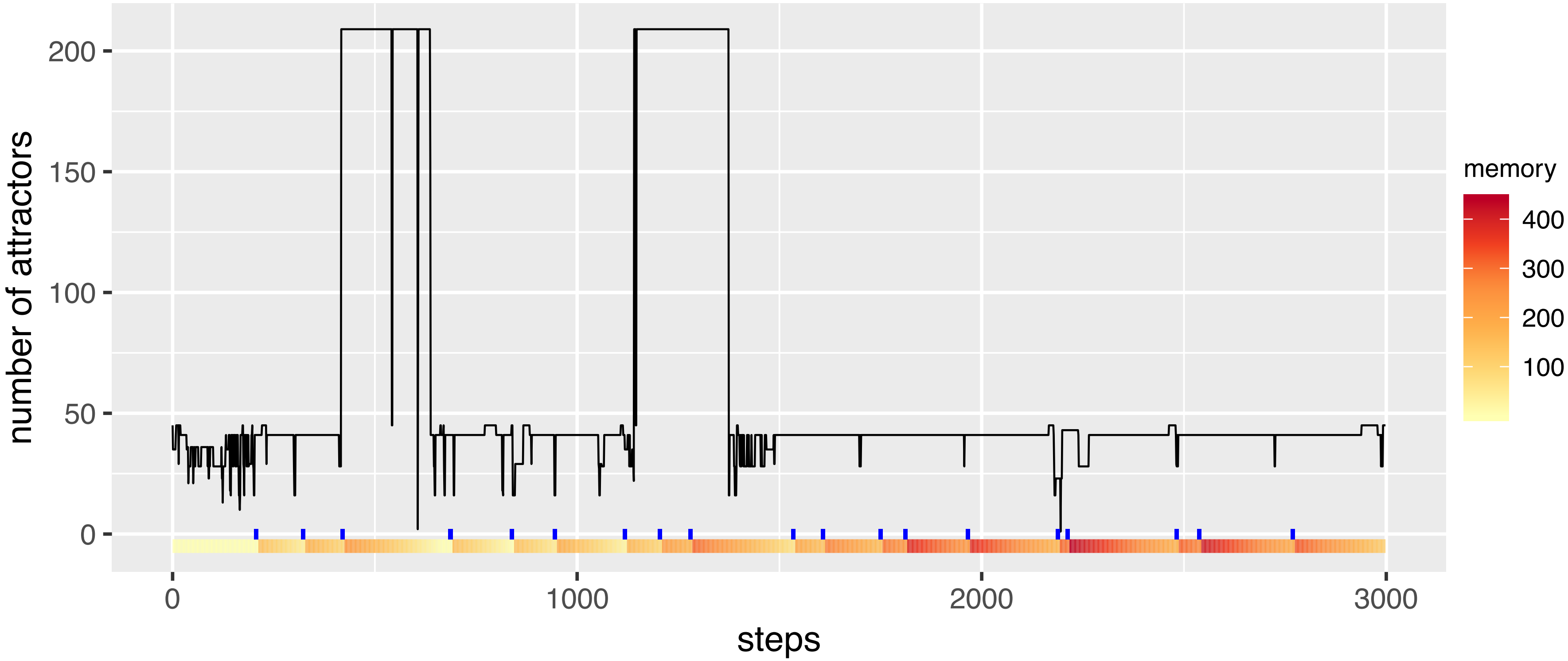
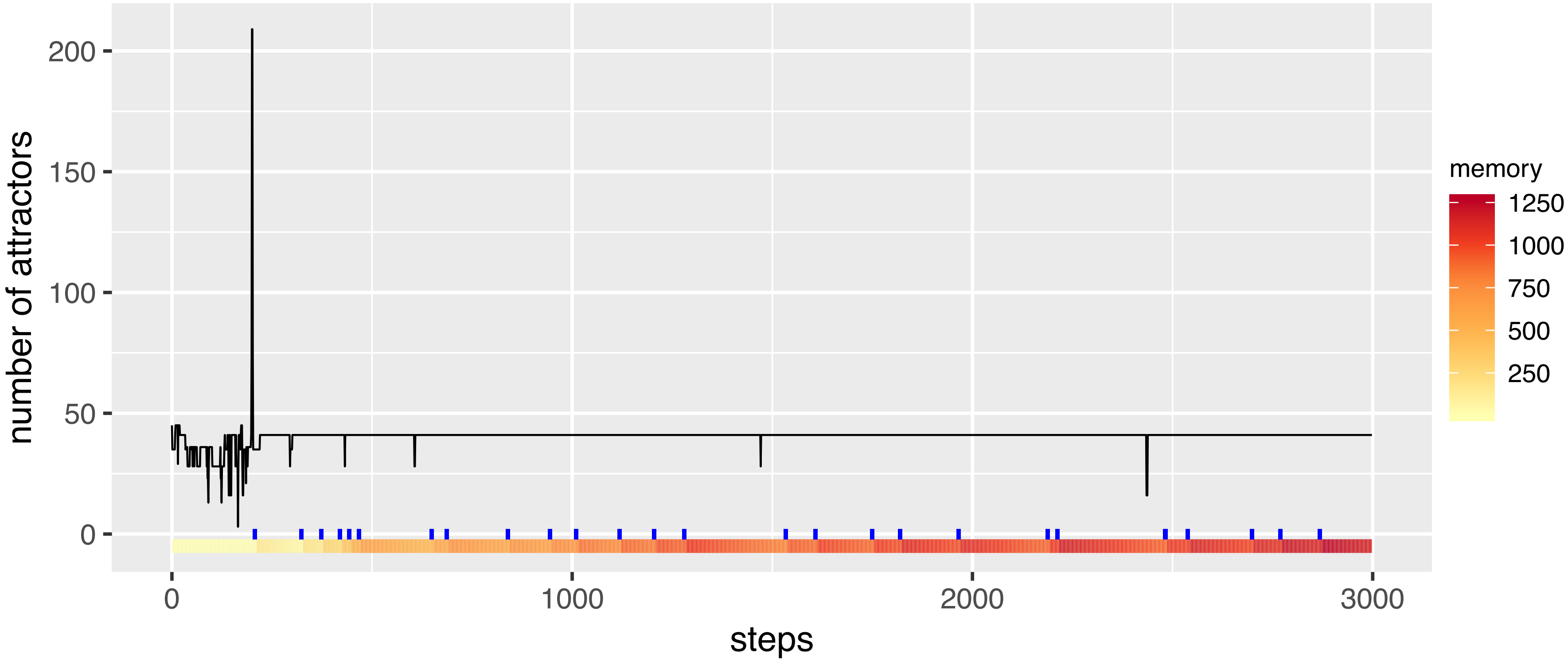


Stabilization of Attractor Dynamics



Stabilization of Attractor Dynamics



Conclusions

- The rationale underlying this study is that the **attractor dynamics** of the networks would be significantly related to their **computational complexity**.
- We showed that the attractor dynamics of the BGT network can be stabilized by means of an **adaptive STDP rule**.
- The general idea behind the adaptive STDP rule is the combination of **reinforcement learning** (input trigger pattern) and **self-organizing** (STDP) processes.
- These considerations support the rationale that **synaptic plasticity** might be crucially involved in the computational capabilities of neural networks.

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Thank you