

FINITE STATE MACHINES AND BIO-INSPIRED NEURAL NETWORKS

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Université Paris II, France
Playtika, Switzerland

Laboratoire DAVID, Université Versailles Saint-Quentin, France
May 16, 2022

INTRODUCTION

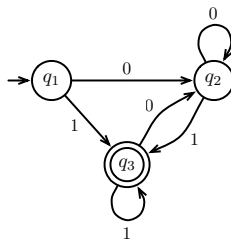
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- ▶ **Part 2:** We introduce a bio-inspired paradigm for neural computation.

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FINITE STATE AUTOMATON (FSA)

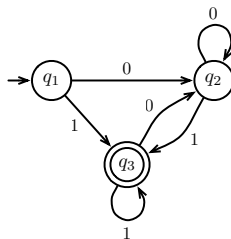
Graph composed of computational states (nodes) and transitions between those (edges).



- ▶ Input u is *accepted (rejected)* by \mathcal{A} if $\mathcal{A}(u)$ ends up in a final (non-final) state.
- ▶ **Regular languages (REG):** class of languages recognizable by finite state automata.

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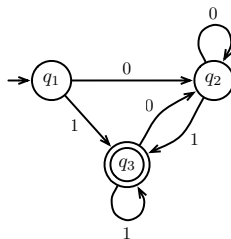
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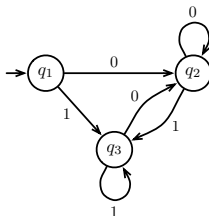
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FSA with Muller acceptance condition: collection of sets of states.

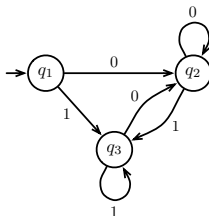


$$\mathcal{T} = \{\{q_2\}, \{q_2, q_3\}\}$$

- ▶ Infinite word u *accepted (rejected)* by \mathcal{A} if the infinite run $\mathcal{A}(u)$ satisfies $\inf(\rho_u) \in \mathcal{T}$ ($\inf(\rho_u) \notin \mathcal{T}$).
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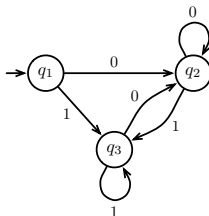


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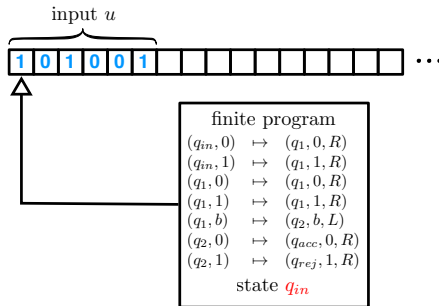


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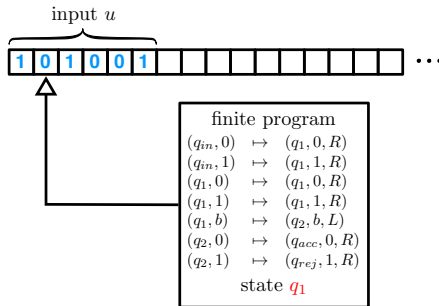
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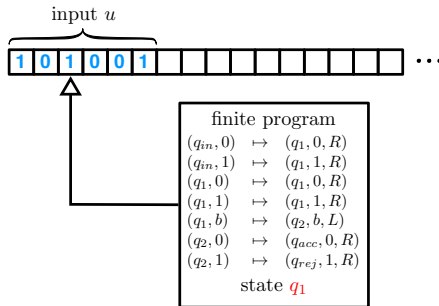
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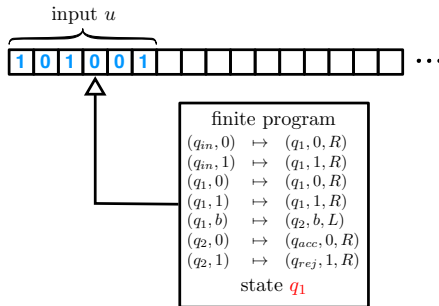
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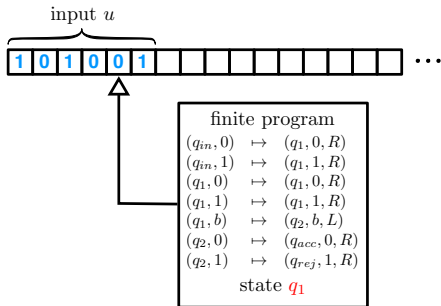
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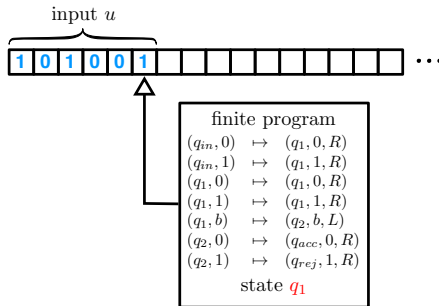
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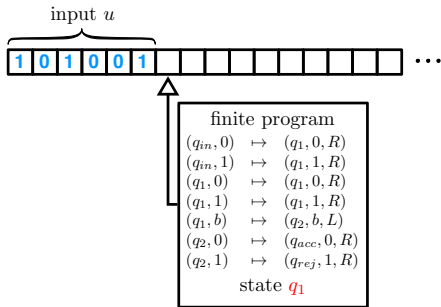
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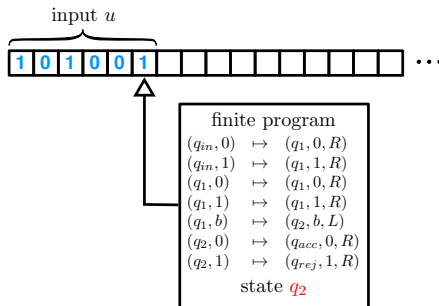
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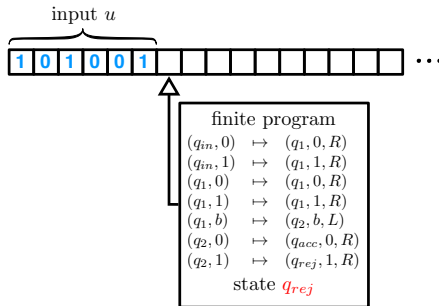
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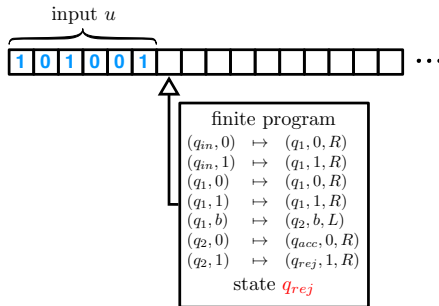
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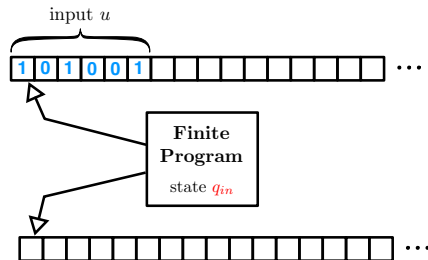
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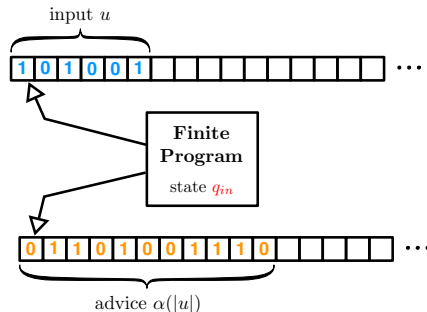
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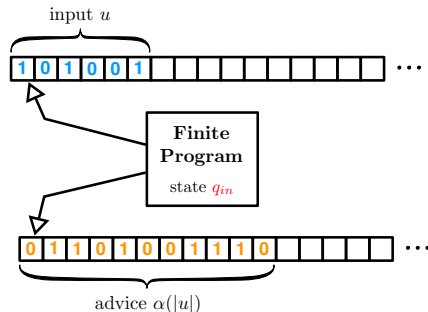
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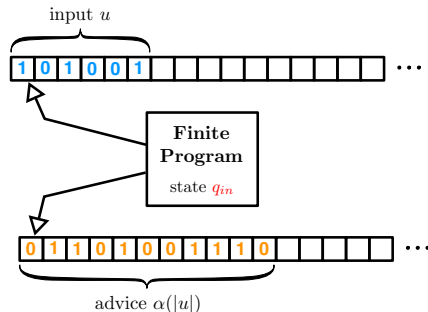
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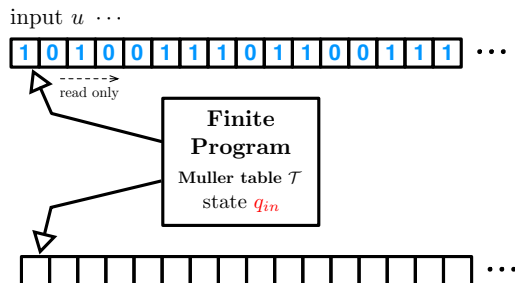
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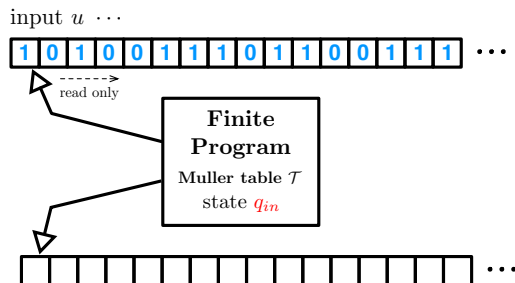
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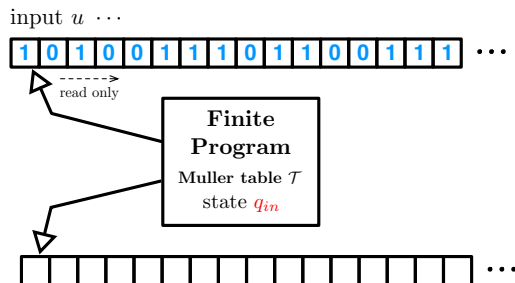
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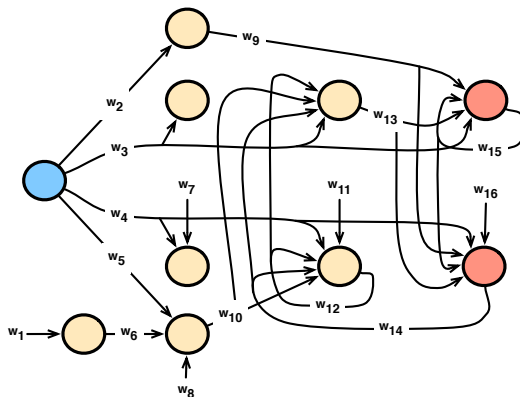
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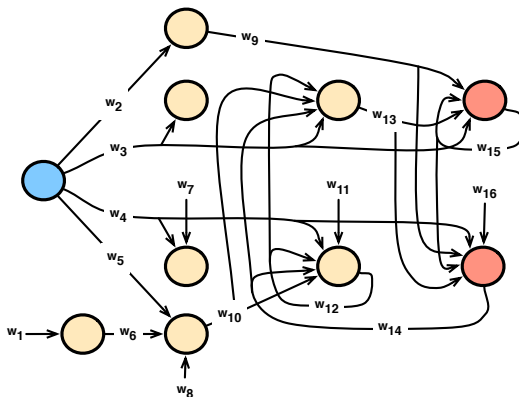
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RECURRENT NEURAL NETWORK (RNN)



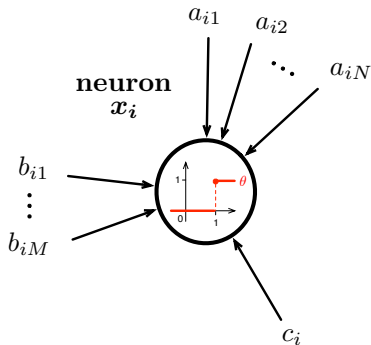
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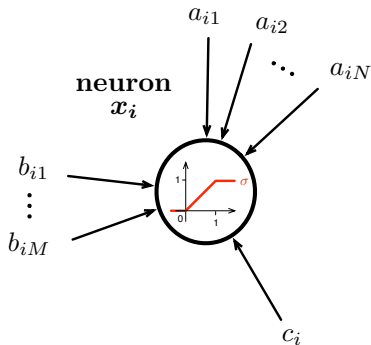
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BOOLEAN RECURRENT NEURAL NETWORK



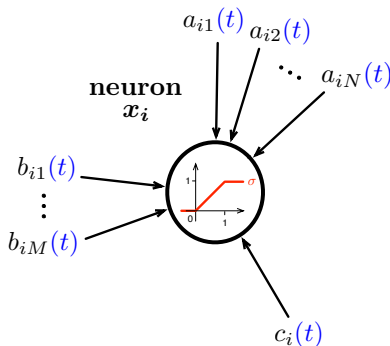
$$x_i(t+1) = \theta \left(\sum_{j=1}^N a_{ij} \cdot x_j(t) + \sum_{j=1}^M b_{ij} \cdot u_j(t) + c_i \right)$$

SIGMOIDAL RECURRENT NEURAL NETWORK



$$x_i(t+1) = \sigma \left(\sum_{j=1}^N a_{ij} \cdot x_j(t) + \sum_{j=1}^M b_{ij} \cdot u_j(t) + c_i \right)$$

EVOLVING RECURRENT NEURAL NETWORK



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RESULTS: CLASSICAL COMPUTATION

	BOOLEAN	SIGMOID		
		STATIC	BI-VALUED EVOLVING	EVOLVING
Q	FSA	TM	TM/poly(A)	TM/poly(A)
	REG	P	P/poly	P/poly
	KI 56, Mi 67	Si & So 95	Ca & Si 11,14	Ca & Si 11,14
R	FSA	TM/poly(A)	TM/poly(A)	TM/poly(A)
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RESULTS: INFINITE COMPUTATION / DET.

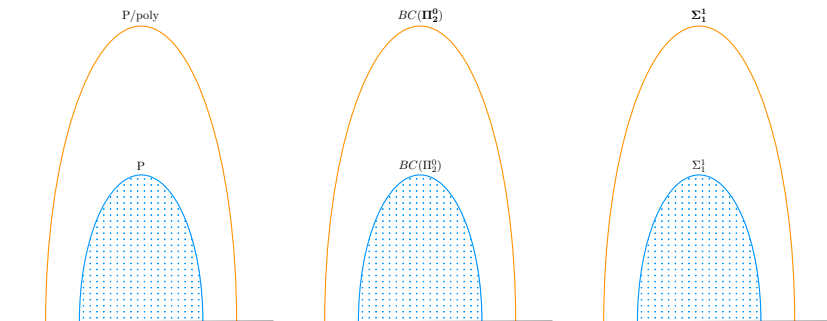
	BOOLEAN	SIGMOID		
		STATIC	BI-VALUED EVOLVING	EVOLVING
\mathbb{Q}	Muller FSA $\in BC(\Pi_2^0)$	Muller TM $= BC(\Pi_2^0)$	super-Turing $= BC(\Pi_2^0)$	super-Turing $= BC(\Pi_2^0)$
\mathbb{R}	Muller FSA $\in BC(\Pi_2^0)$	super-Turing $= BC(\Pi_2^0)$	super-Turing $= BC(\Pi_2^0)$	super-Turing $= BC(\Pi_2^0)$

RESULTS: INFINITE COMPUTATION / NONDET.

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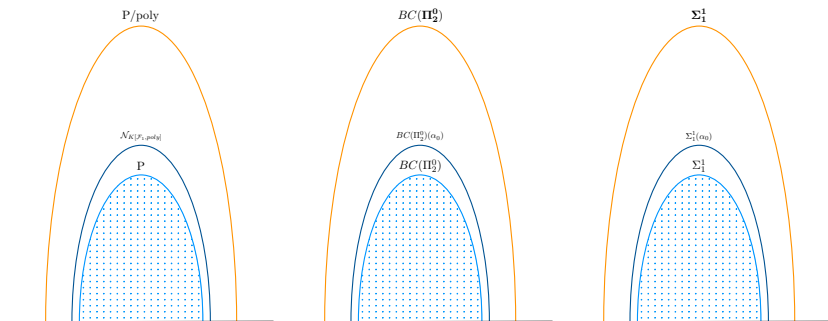
HIERARCHY THEOREMS

- ▶ Stratifying the “super-Turing world”.
- ▶ Finite and infinite computation: we can define infinitely many complexity classes between the Turing and super-Turing levels.



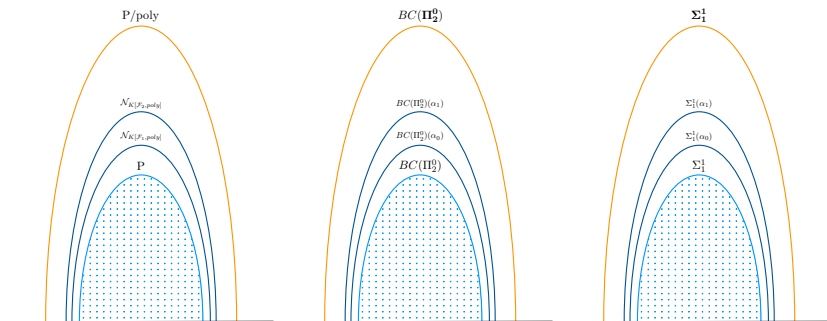
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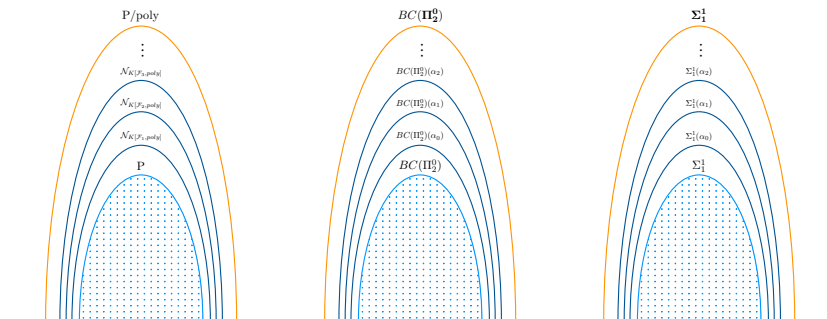
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- ▶ Recurrent neural networks is a natural model for *oracle-based (super-Turing) computation*.
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LIMITATIONS

- ▶ Computational states of the machines are represented as Boolean states, i.e., spiking configurations of the network.
- ★ Computational states should rather be represented by *sustained activities of neural assemblies*, e.g., by *cyclic attractors*.
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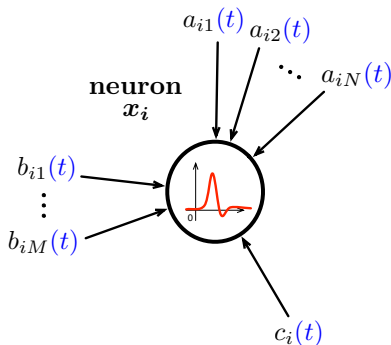
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BIO-INSPIRED RECURRENT NEURAL NETWORK



- Dynamics governed by the Hodgkin-Huxley equations.

HODGKIN-HUXLEY MODEL

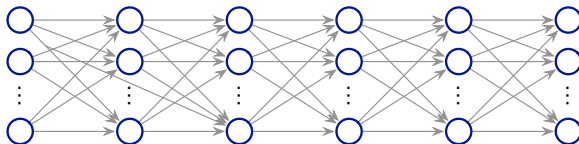
$$\begin{aligned}
 C \cdot \frac{dV}{dt} &= -I_L - I_{Na} - I_K - I_C - I_{input} \quad \text{where} \\
 I_L &= g_L \cdot (V - V_L) \\
 I_{Na} &= g_{Na} \cdot m \cdot h \cdot (V - V_{Na}) \\
 \frac{dm}{dt} &= \frac{m_\infty - m}{\tau_m} & m_\infty &= \frac{1}{1 + e^{-s_m \cdot (V - V_{hm})}} \\
 \frac{dh}{dt} &= \frac{h_\infty - h}{\tau_h} & h_\infty &= 1 - \frac{1}{1 + e^{-s_h \cdot (V - V_{hh})}} \\
 I_K &= g_K \cdot n \cdot (V - V_K) \\
 \frac{dn}{dt} &= \frac{n_\infty - n}{\tau_n} & n_\infty &= \frac{1}{1 + e^{-s_n \cdot (V - V_{hn})}} \\
 I_C &= w_{intra}^{exc} + w_{inter}^{exc} + w_{inter}^{inh} + w_{output}^{exc} + w_{output}^{inh} \\
 I_{input} &= a_{input}^{exc} \cdot \chi_{t_{input}}(t)
 \end{aligned}$$

V : membrane potential; C membrane capacitance; I_L : leakage current; I_{Na}, I_K : sodium and the potassium fast currents; I_C : synaptic currents coming from the neighboring neurons; I_{input} : pulse-like input current.

(show simulator)

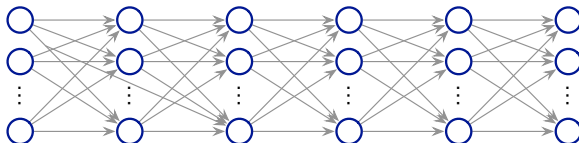
SYNFIRE CHAINS

- ▶ *Synfire chains* have been theorized as fundamental neuronal structures (ABELES 82).
- ▶ They allow for robust and highly precise transmission of information in neural networks.



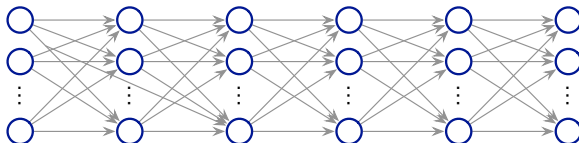
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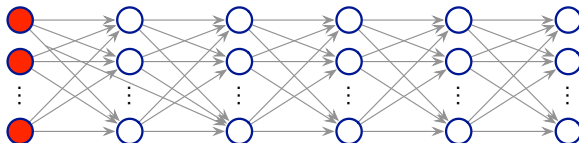
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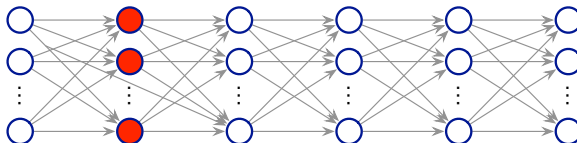
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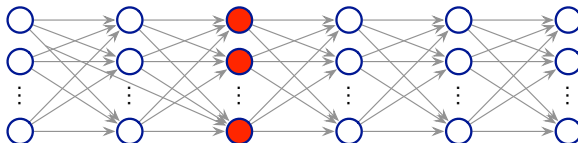
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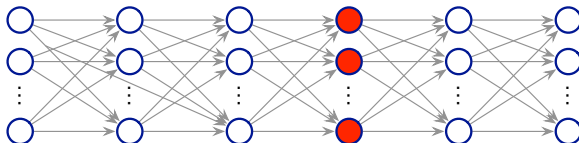
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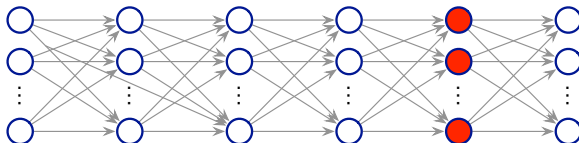
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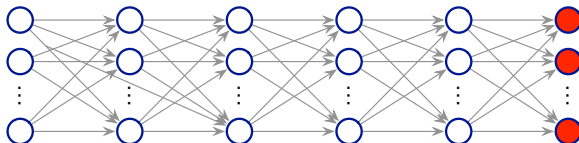
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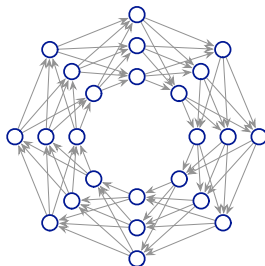
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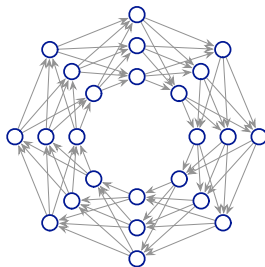
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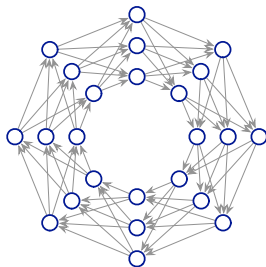
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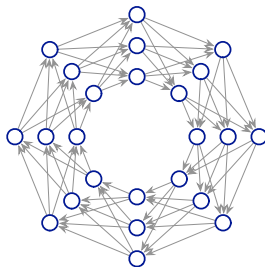
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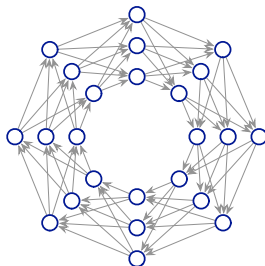
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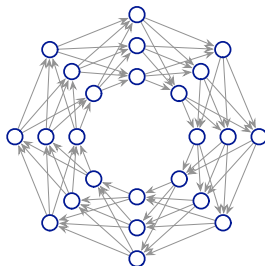
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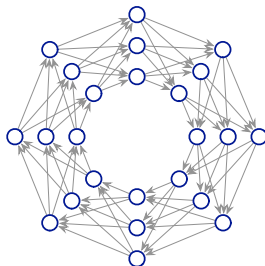
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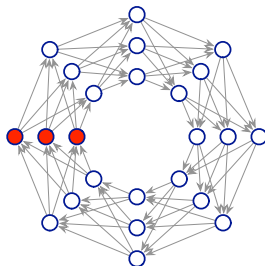
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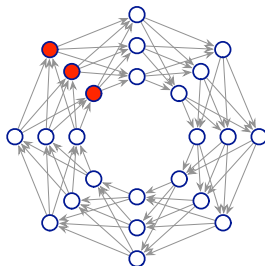
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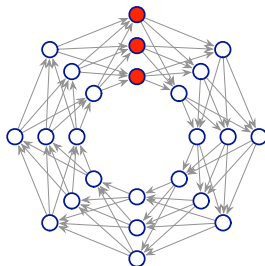
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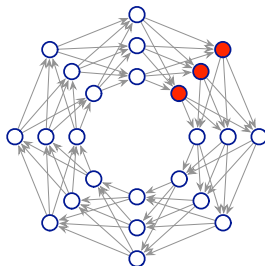
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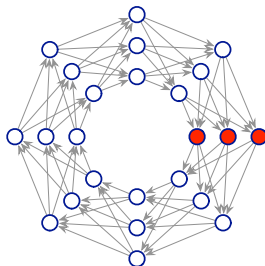
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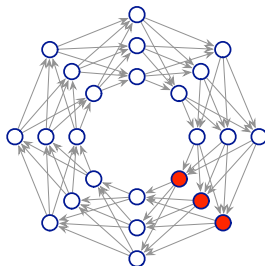
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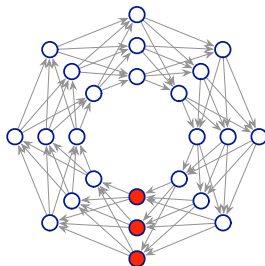
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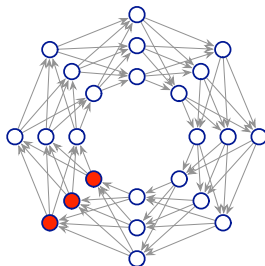
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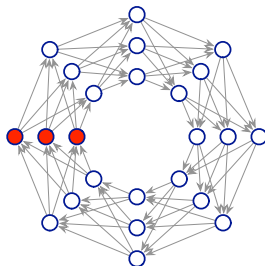
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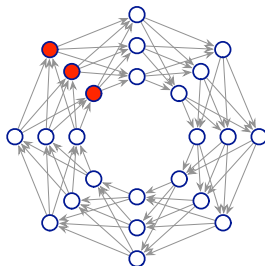
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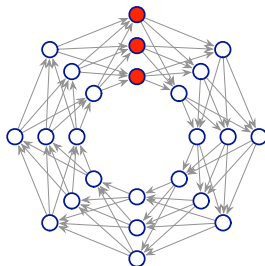
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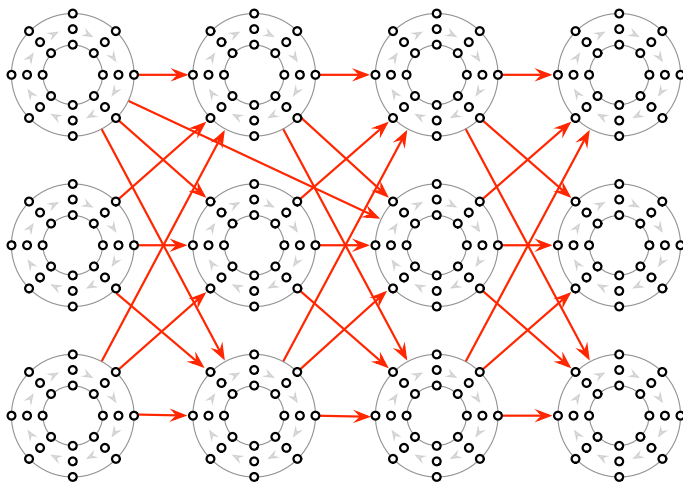


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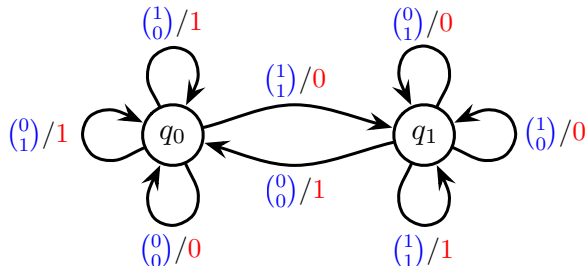
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SYNFIRE RING ARCHITECTURE

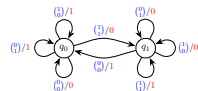


BINARY ADDER AUTOMATON



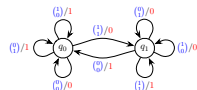
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BINARY ADDER BOOLEAN NEURAL NETWORK

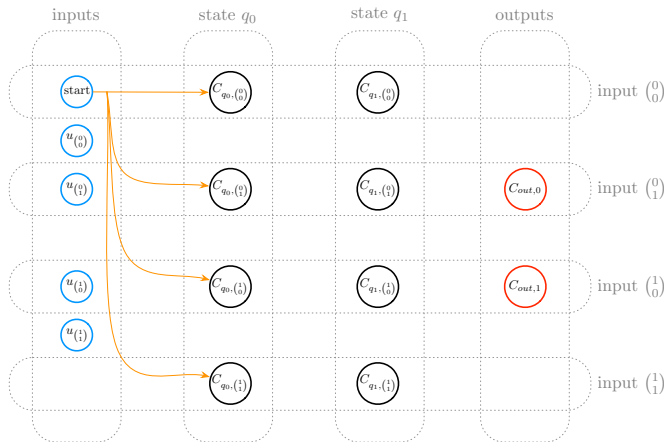
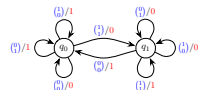


inputs	state q_0	state q_1	outputs
			input $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
			input $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
			input $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
			input $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

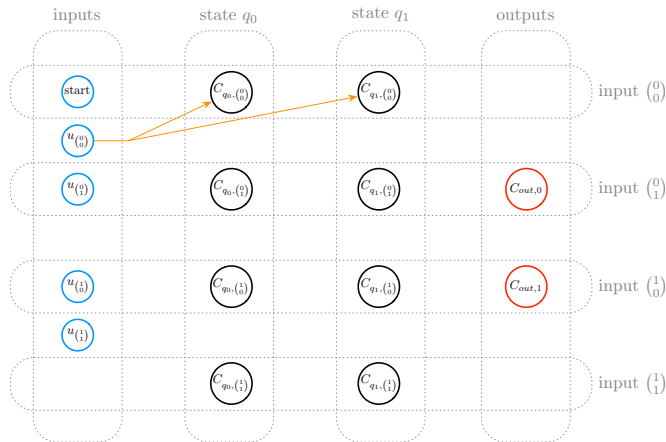
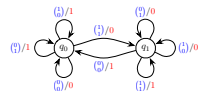
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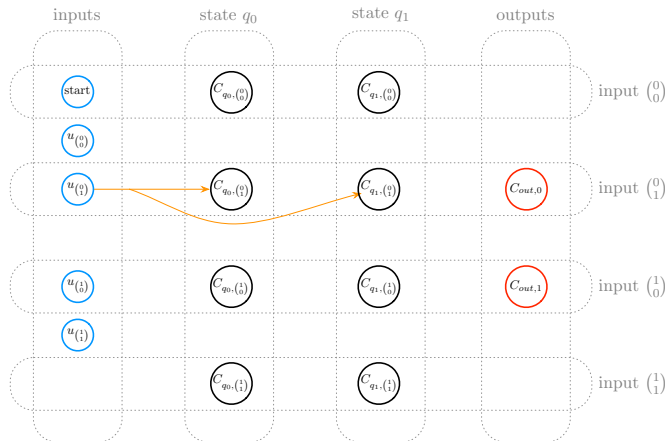
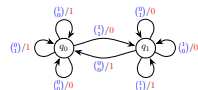
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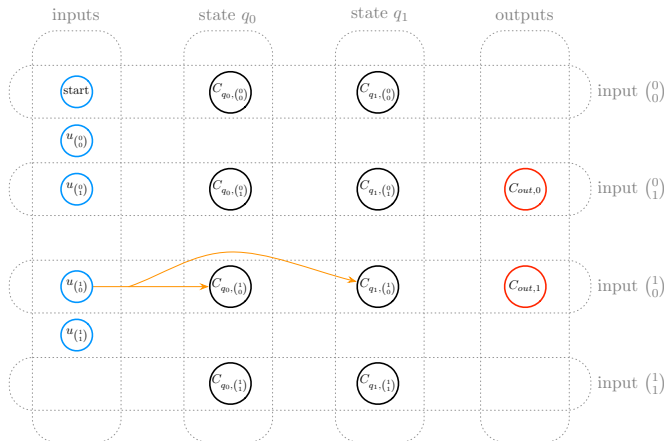
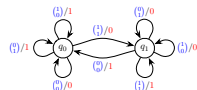
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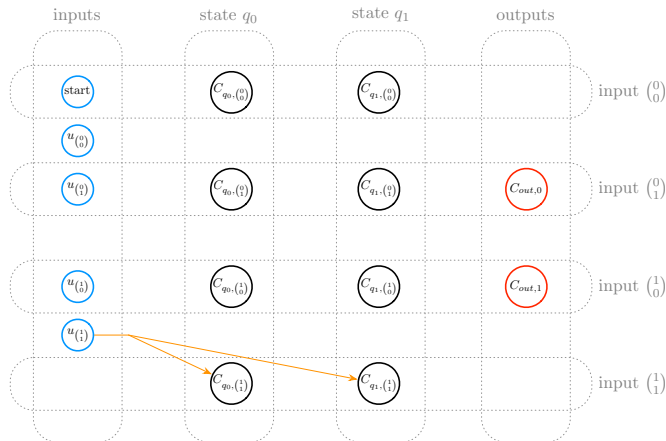
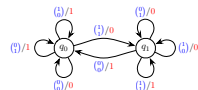
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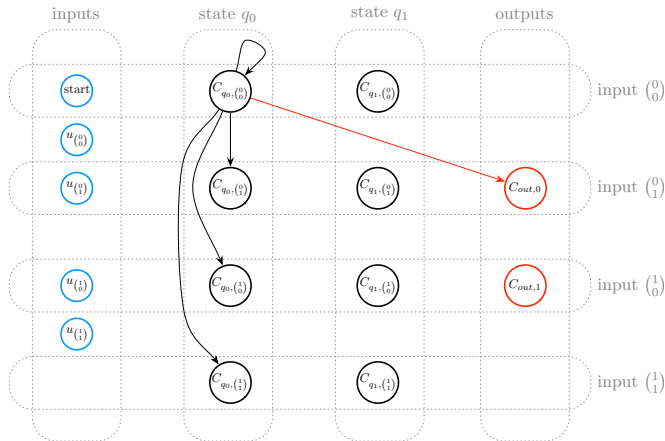
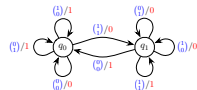
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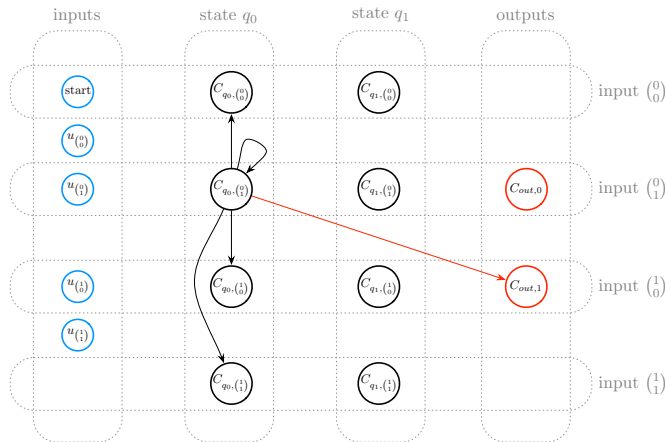
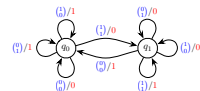
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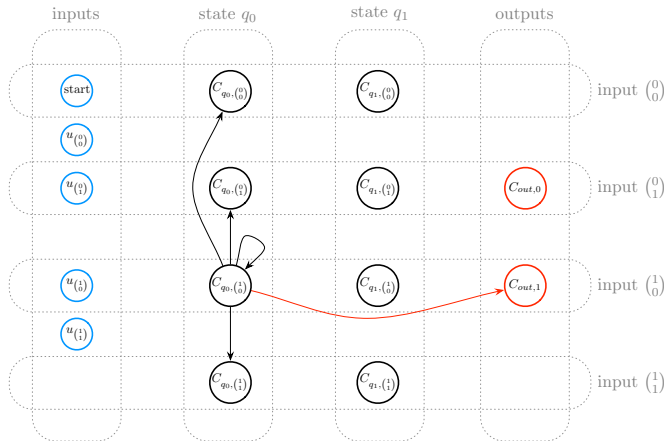
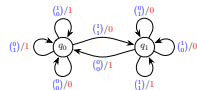
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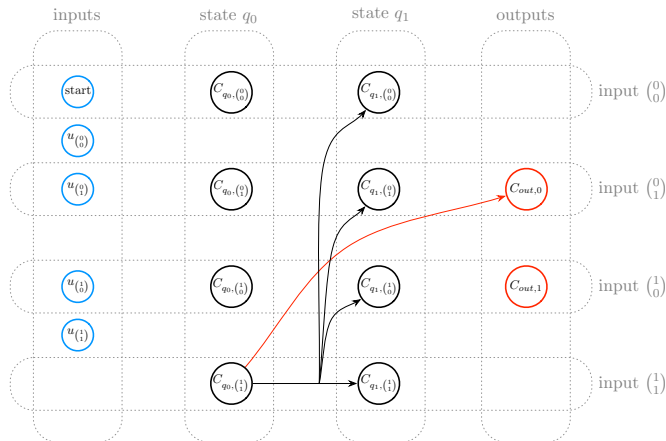
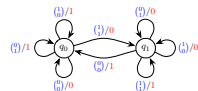
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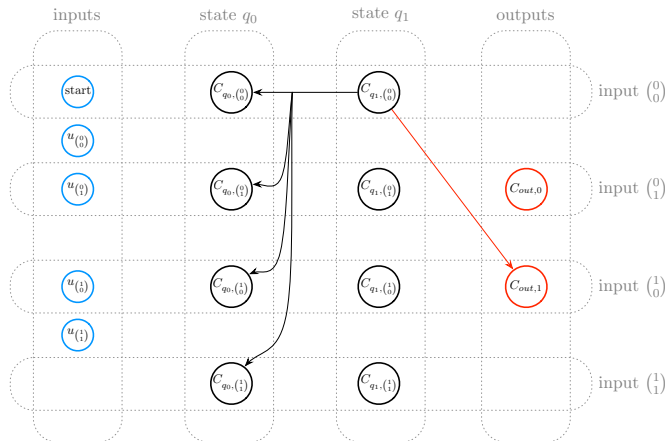
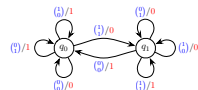
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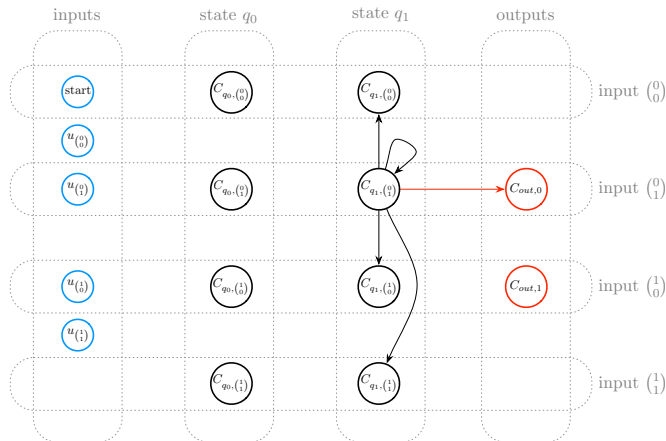
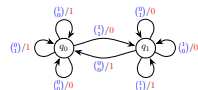
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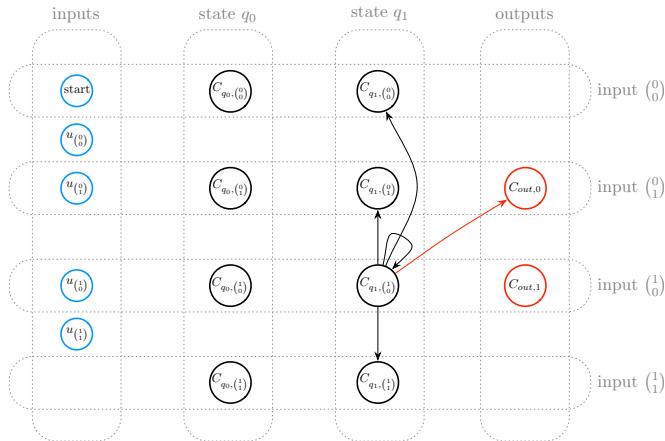
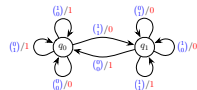
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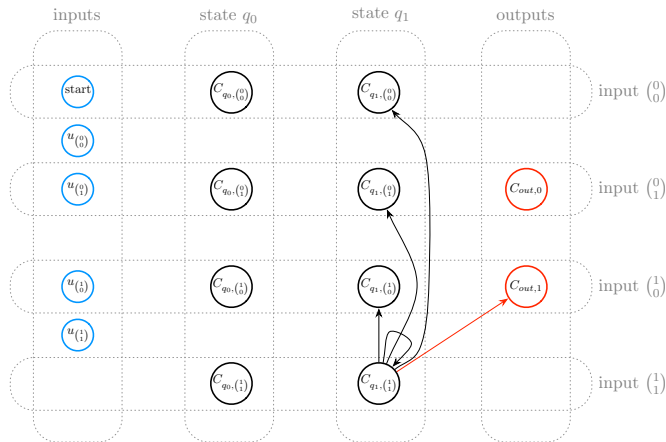
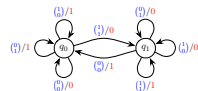
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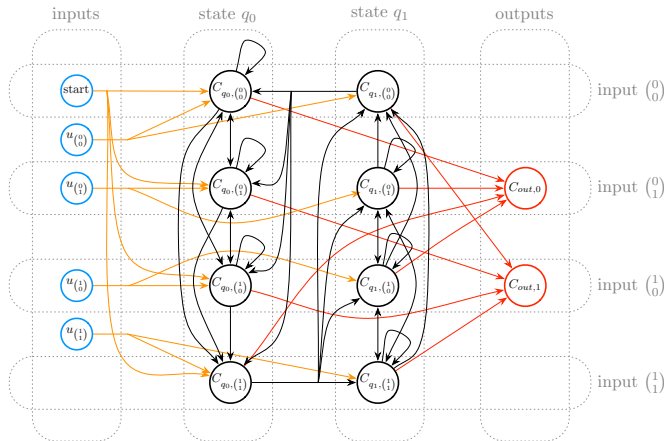
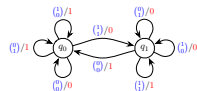
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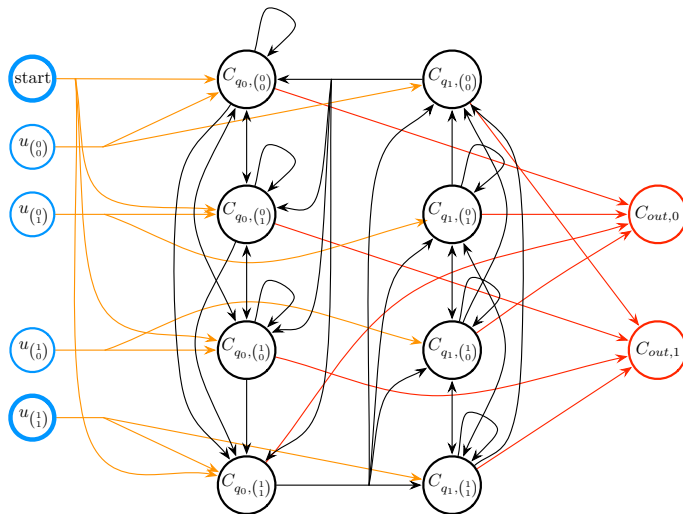
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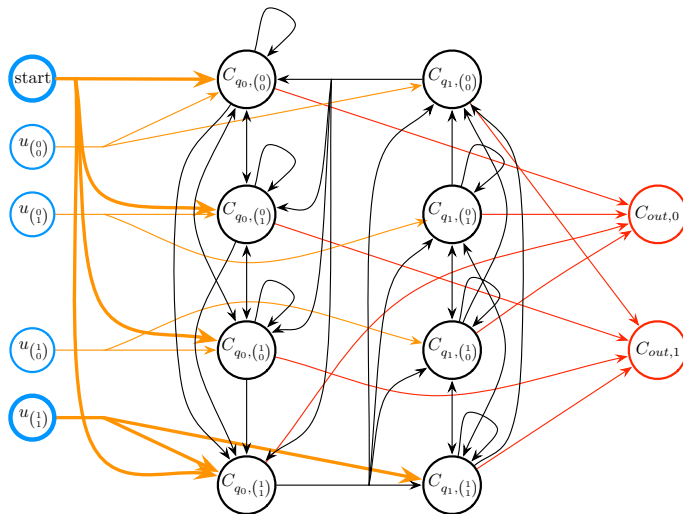
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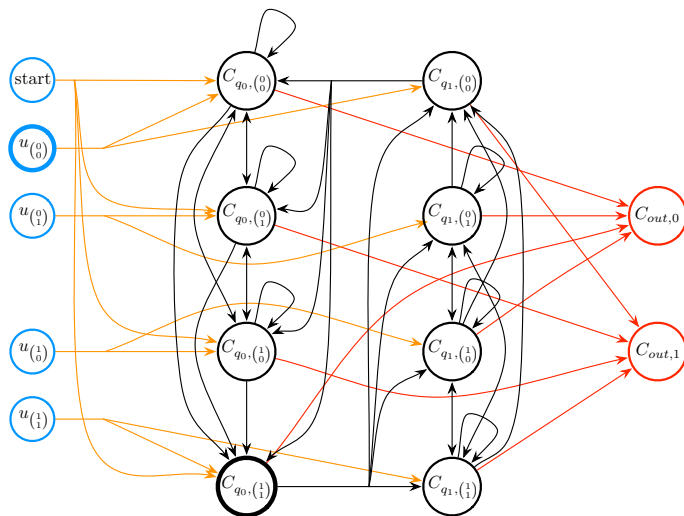
SIMULATION



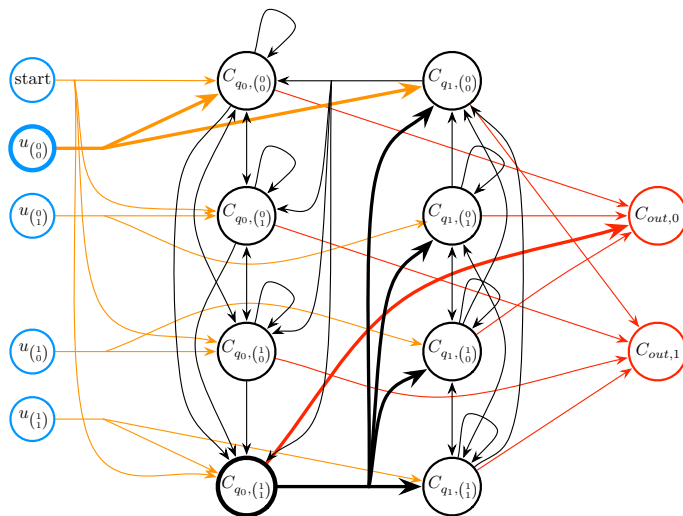
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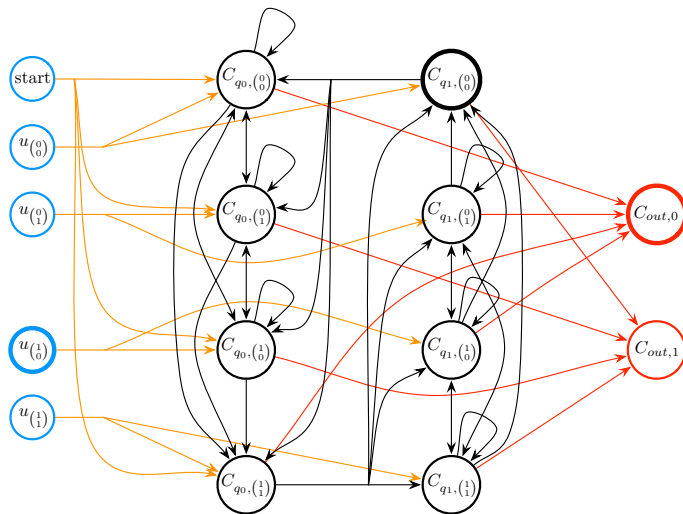
SIMULATION



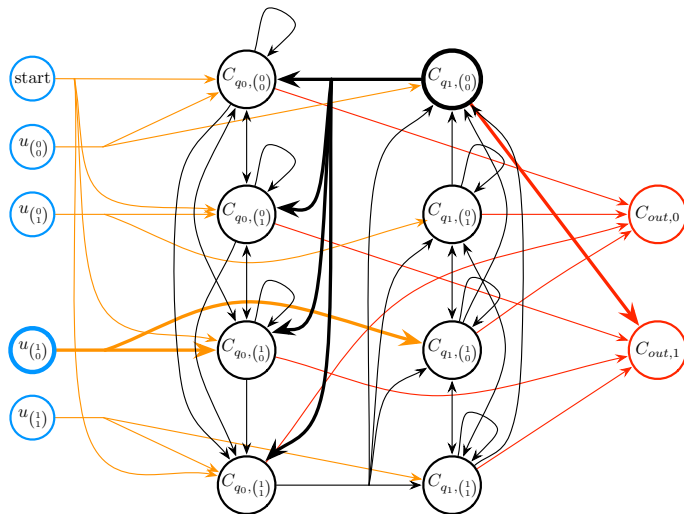
SIMULATION



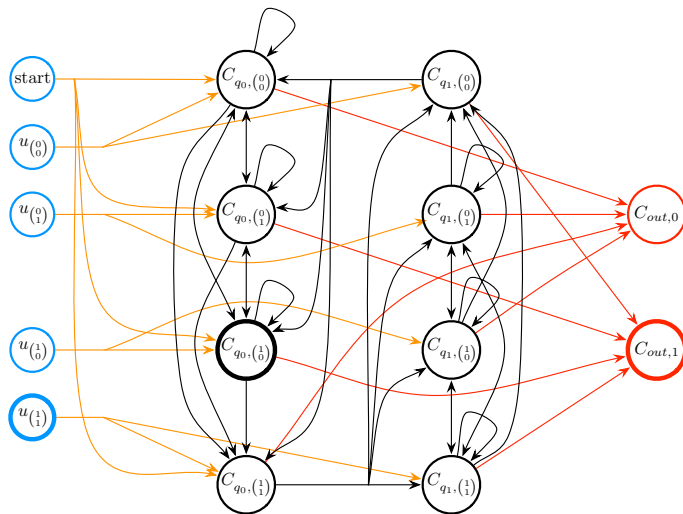
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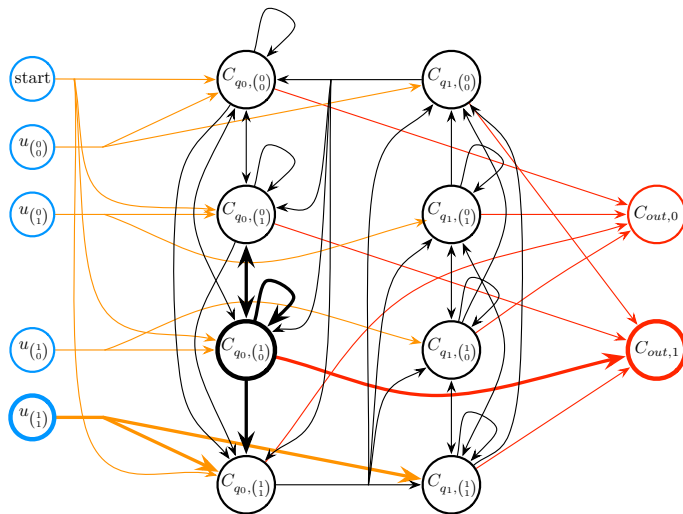
SIMULATION



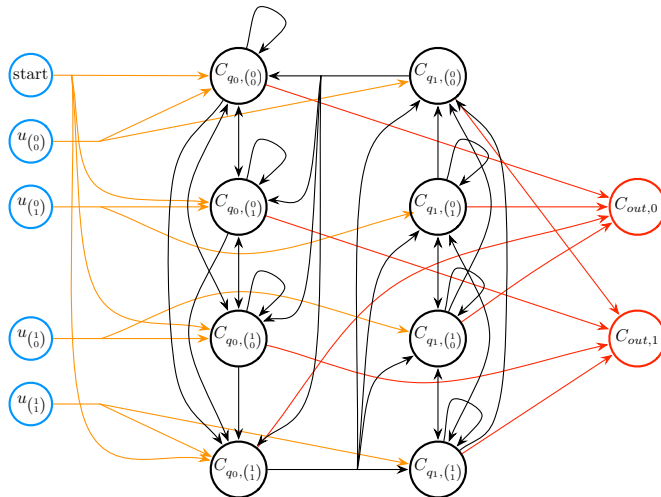
SIMULATION



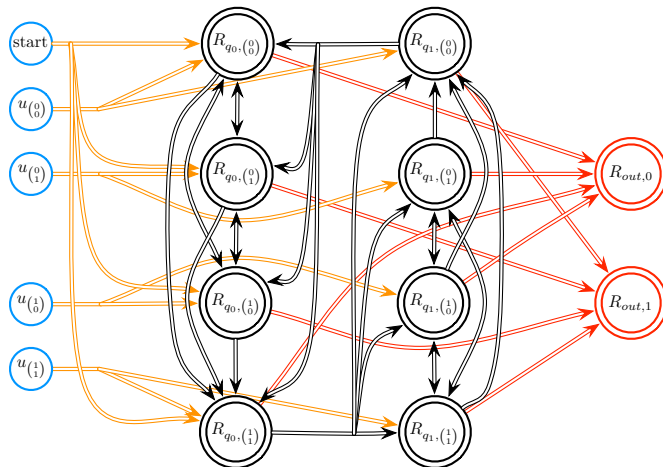
SIMULATION



GENERALIZATION TO SYNFIRE RING RNNs

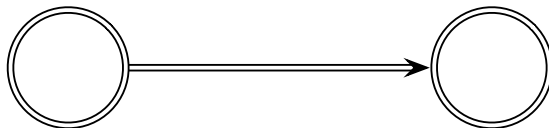


GENERALIZATION TO SYNFIRED RING RNNs



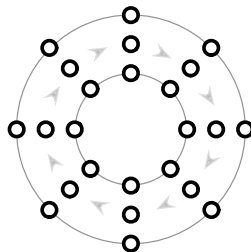
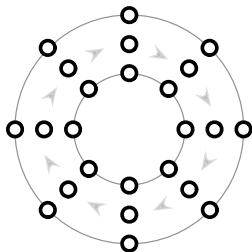
TRANSITION OF RING ACTIVITIES: FIBRES OF EXCITATORY & INHIBITORY CONNECTIONS

- ▶ The *refractory period* of the cells allows for a natural inhibitory system.



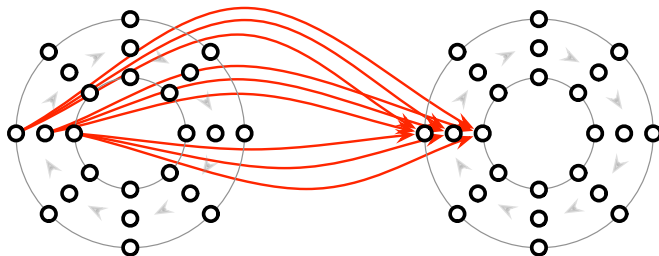
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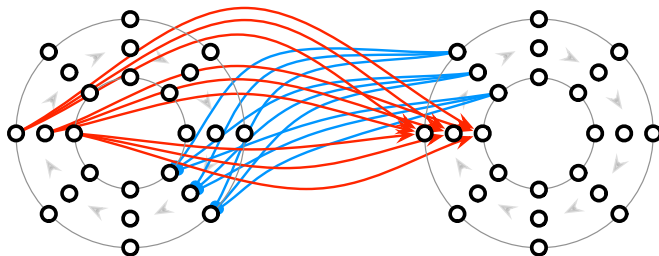
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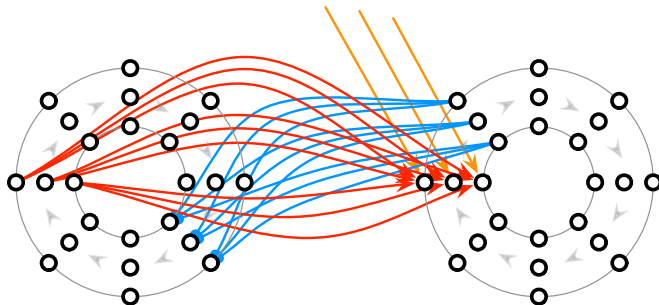
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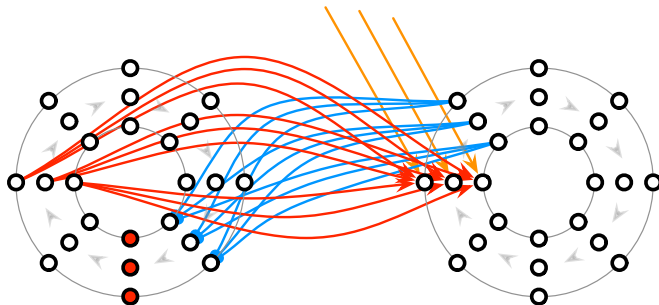
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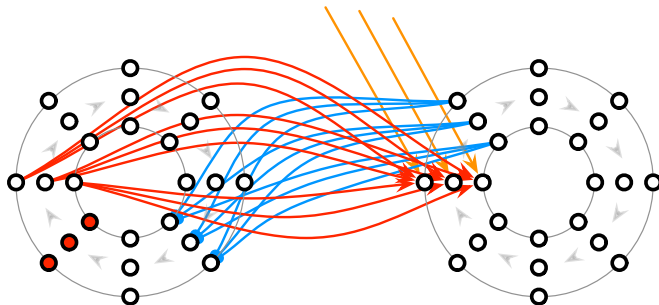
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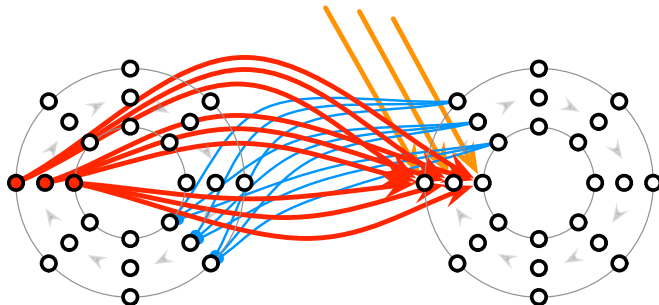
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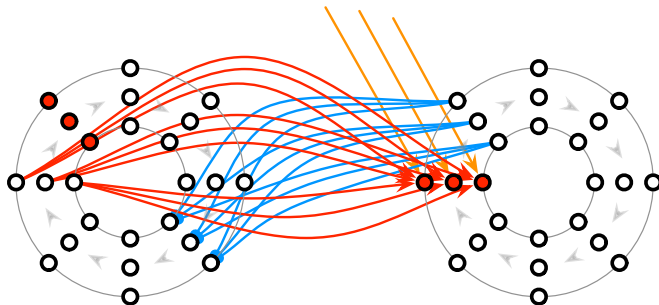
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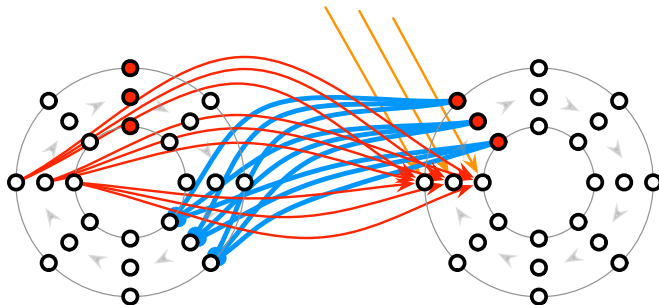
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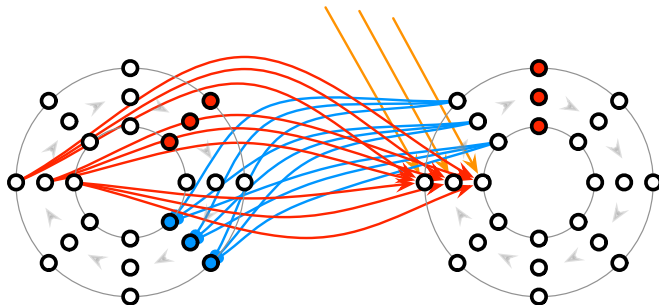
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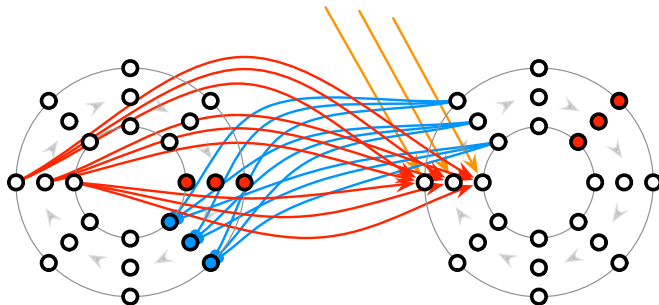
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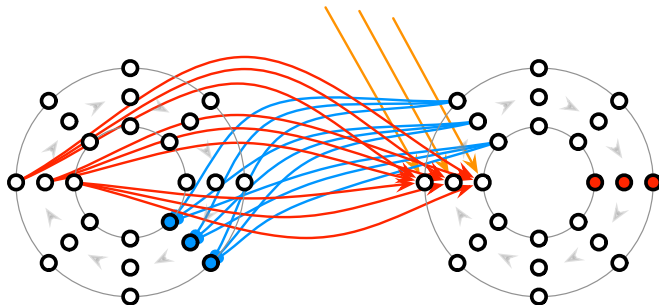
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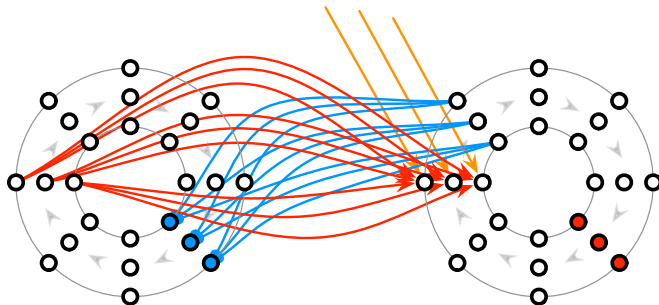
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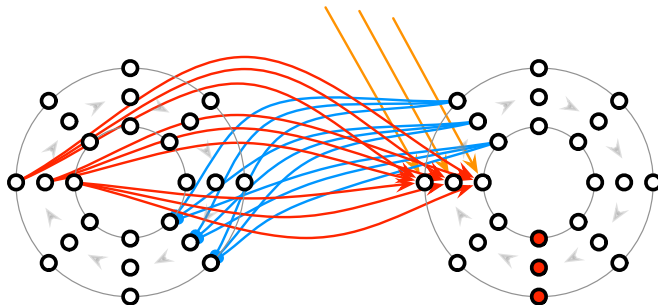
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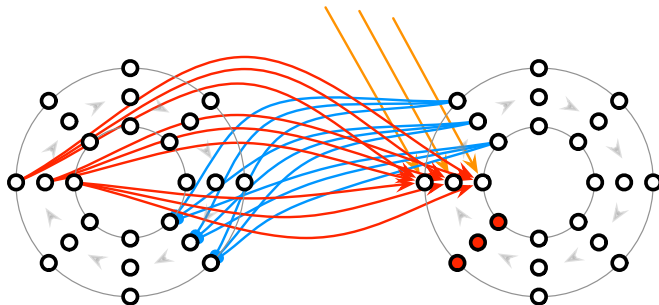
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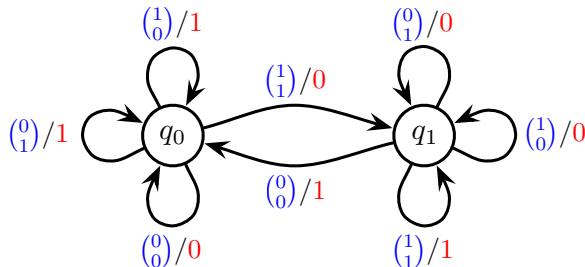


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SIMULATION



Play movie

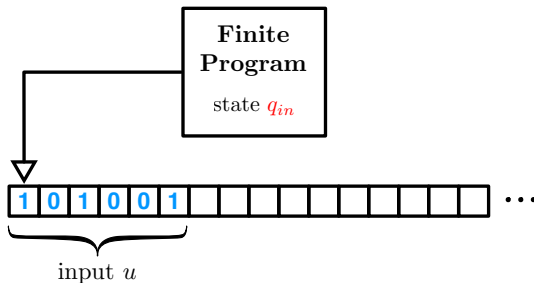
AUTOMATA & HODGKIN-HUXLEY RNNs WITH SYNFIRES RINGS

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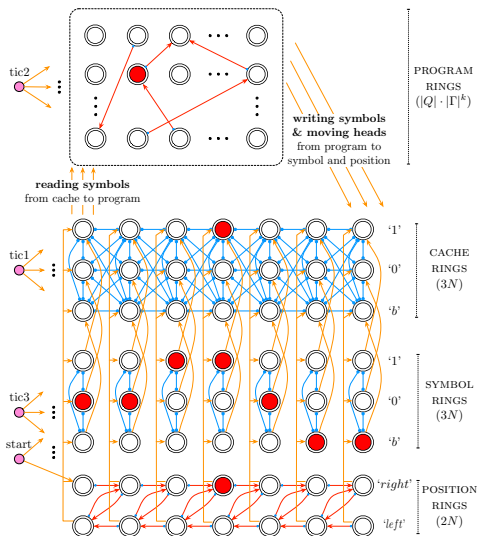
THEOREM

Any finite state automaton can be simulated by some Hodgkin-Huxley based neural network composed of synfire rings.

TM AND RNN

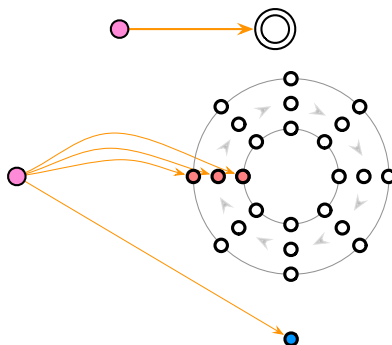


TM AND RNN



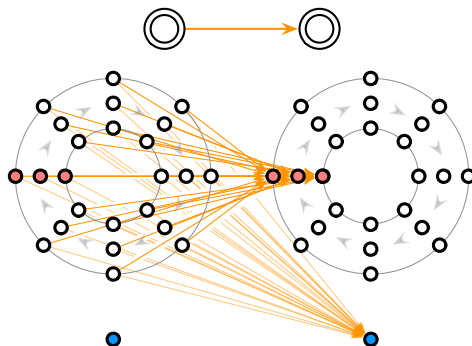
FIBRES OF CONNECTIONS

► Cell-to-ring one-shot excitation



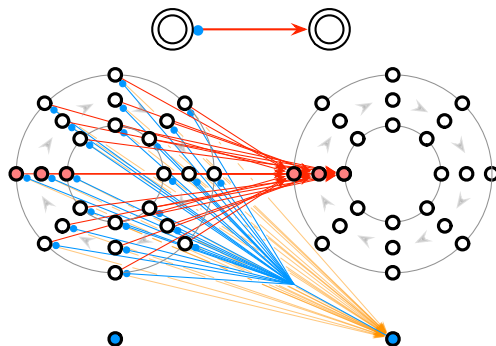
FIBRES OF CONNECTIONS

► Ring-to-ring constant excitation



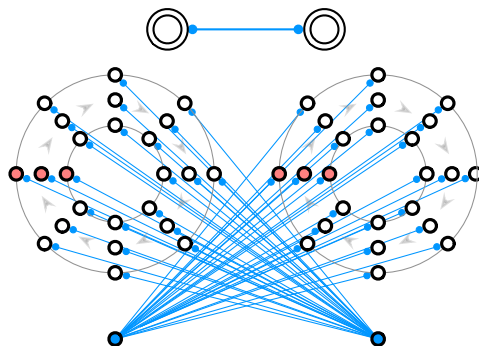
FIBRES OF CONNECTIONS

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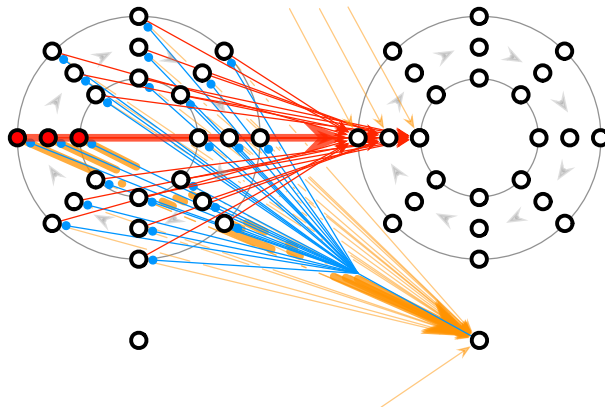


FIBRES OF CONNECTIONS

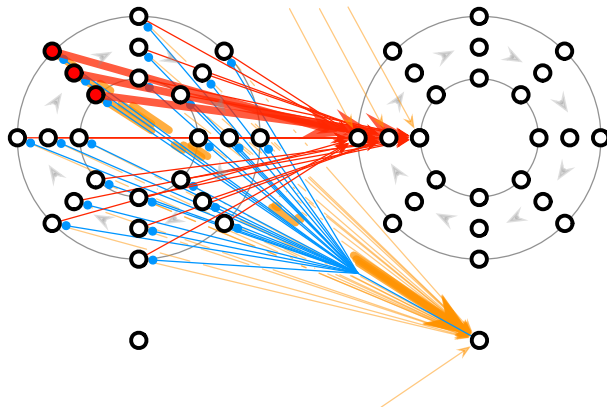
- ▶ Ring-to-ring bidirectional one-shot inhibition



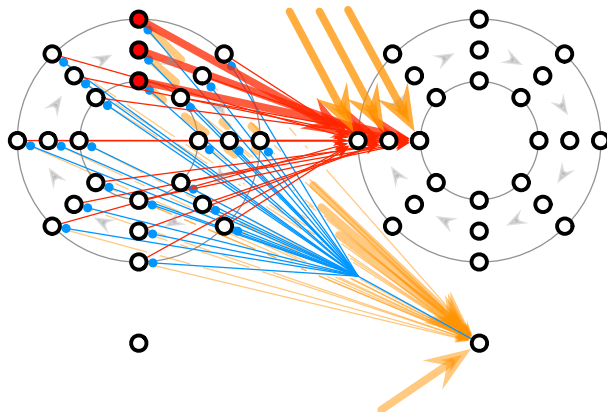
TRANSITION OF RING ACTIVITIES



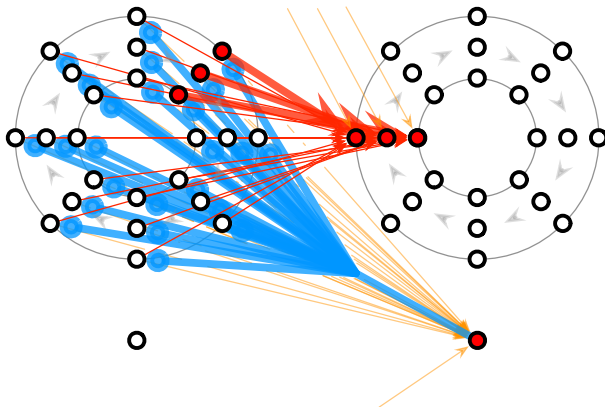
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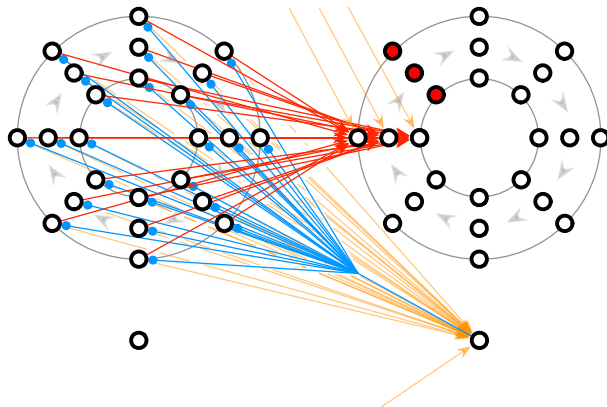
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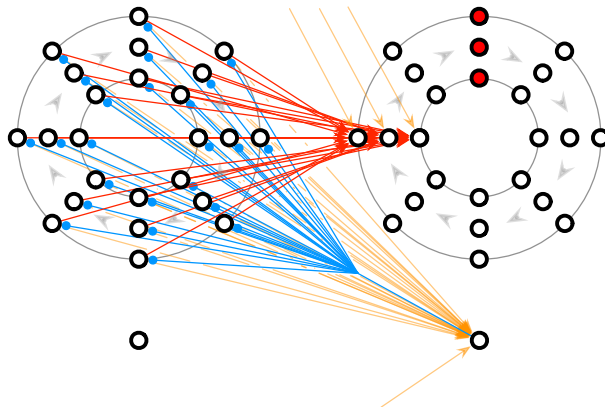
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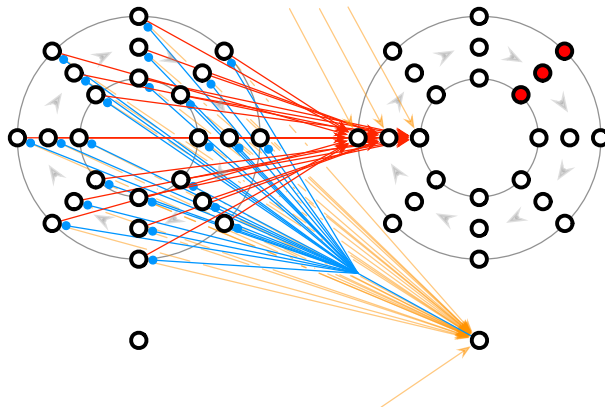
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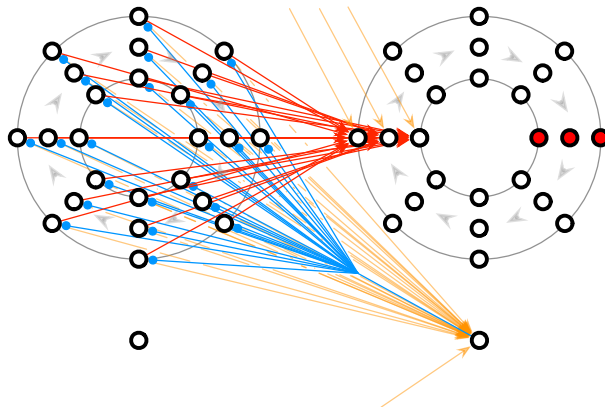
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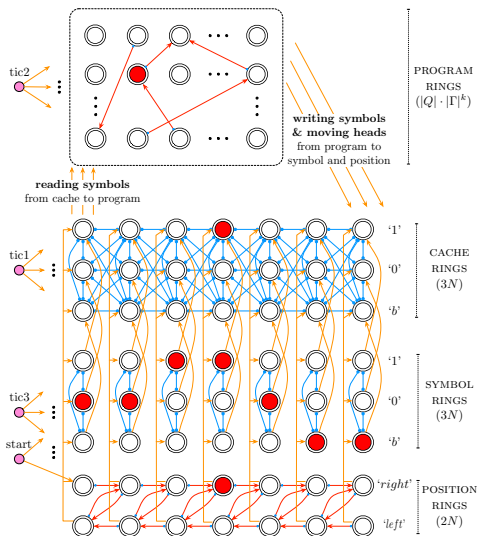
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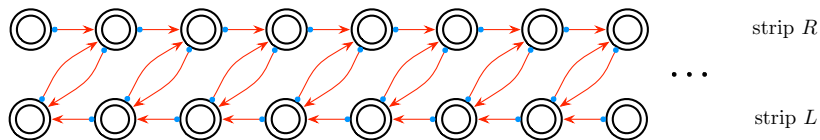
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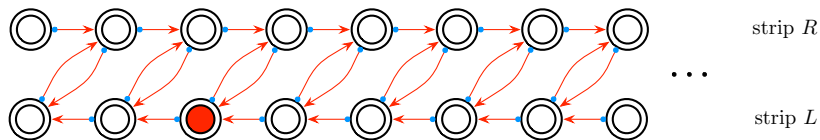
POSITION RINGS



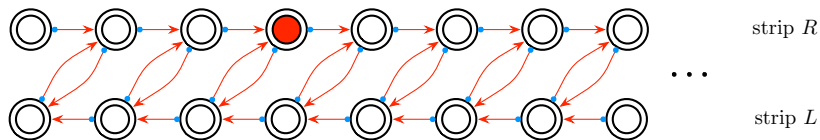
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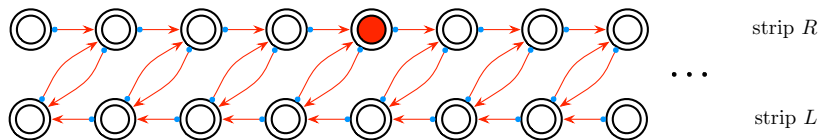
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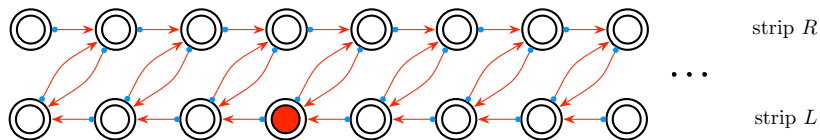
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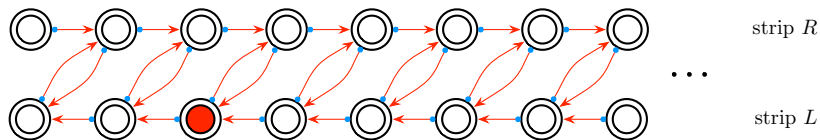
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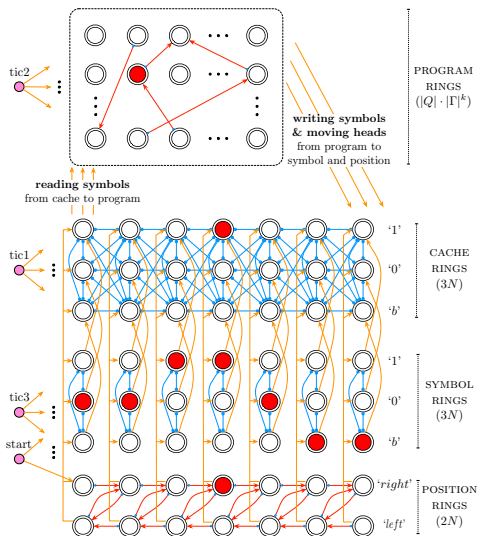
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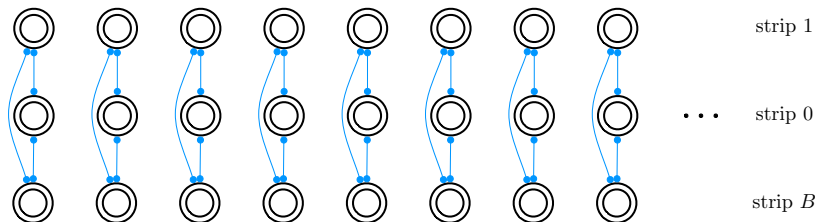
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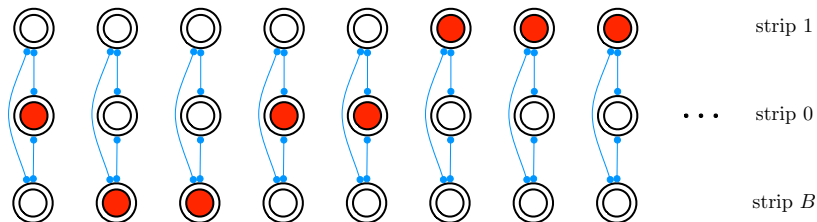
SYMBOL RINGS



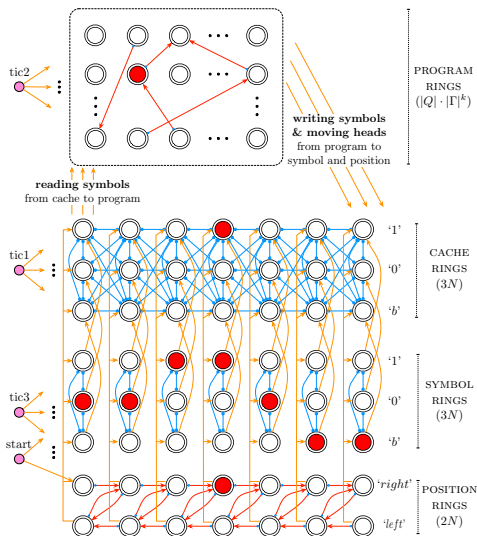
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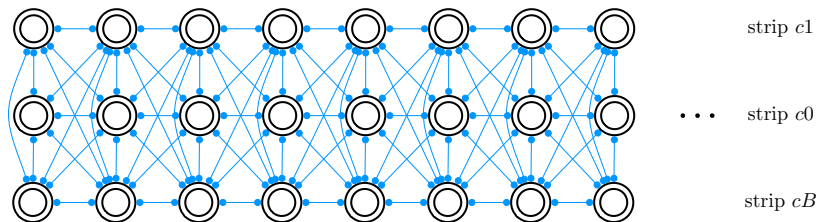
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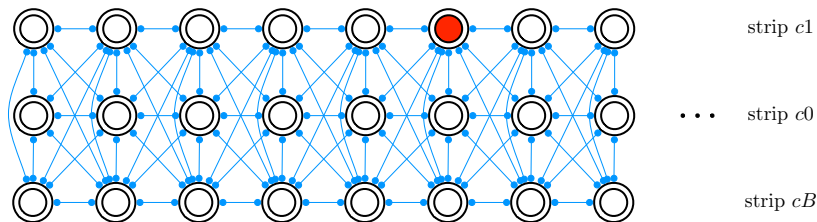
CACHE RINGS



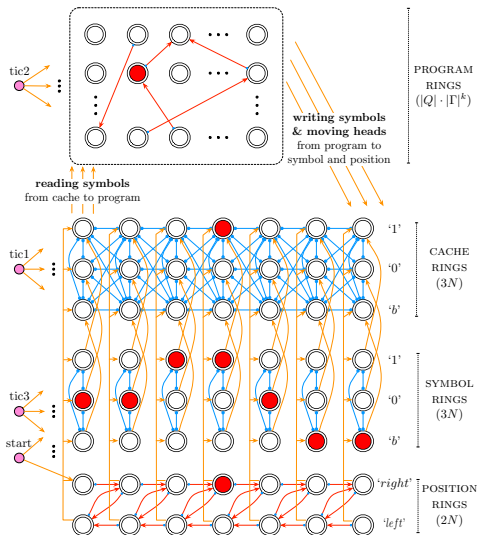
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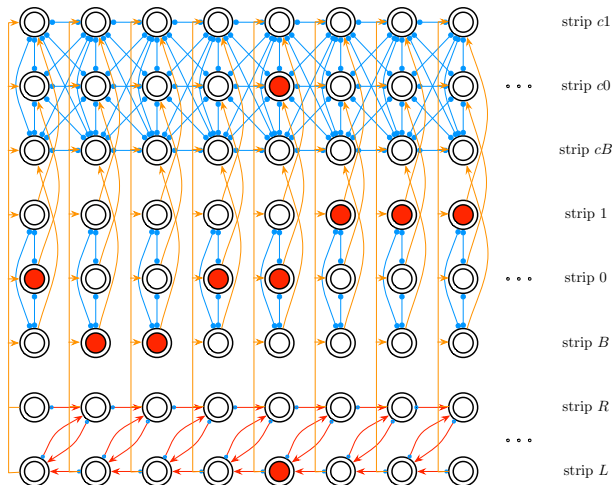
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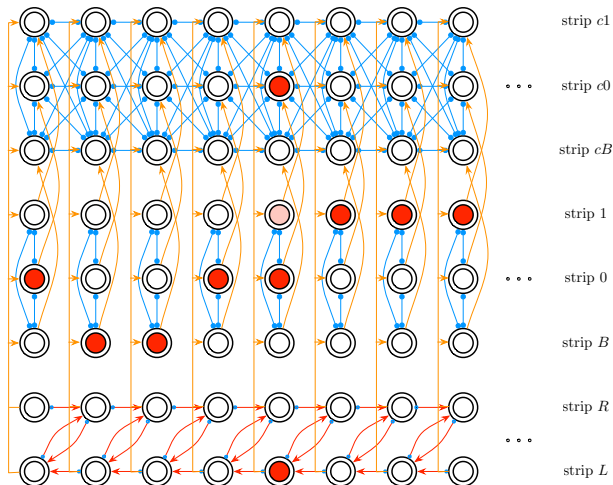
POSITION-SYMBOL-CACHE CONNECTIONS



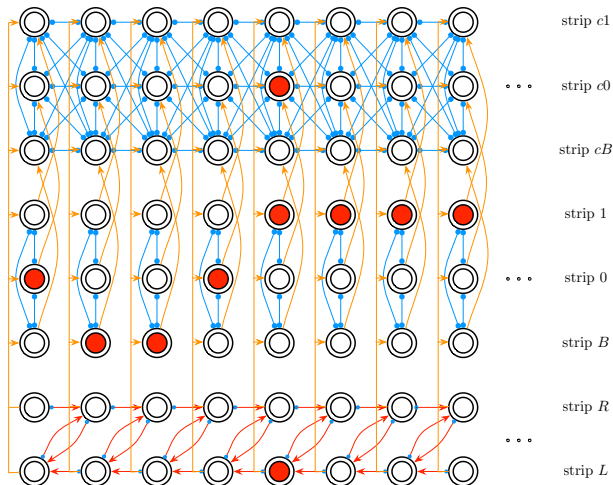
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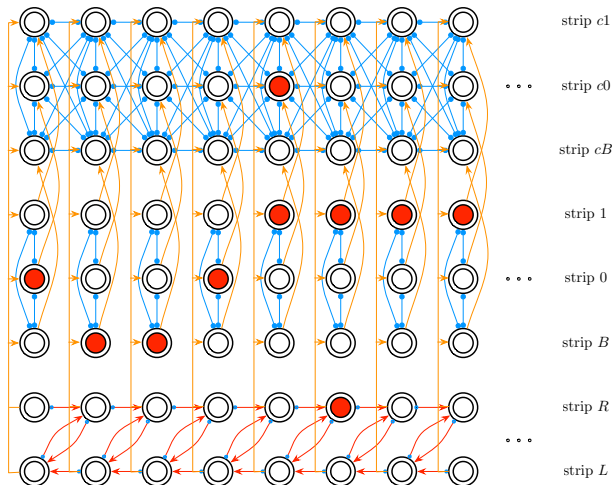
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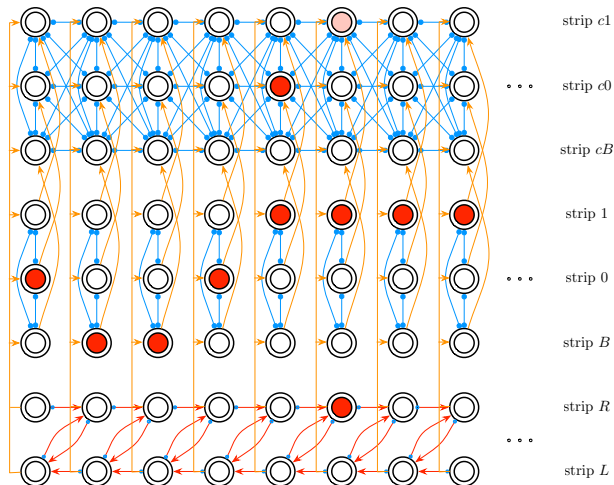
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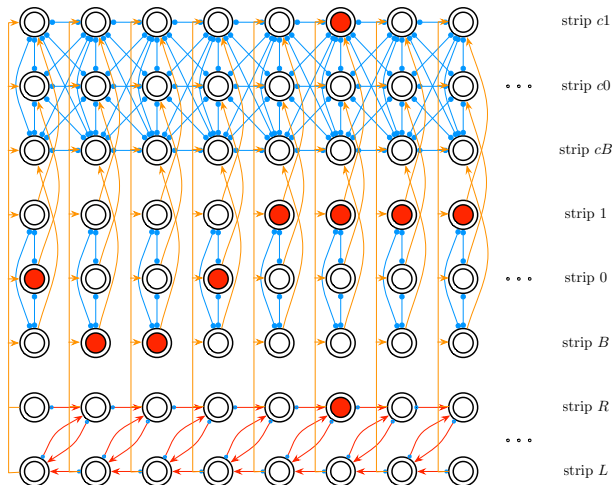
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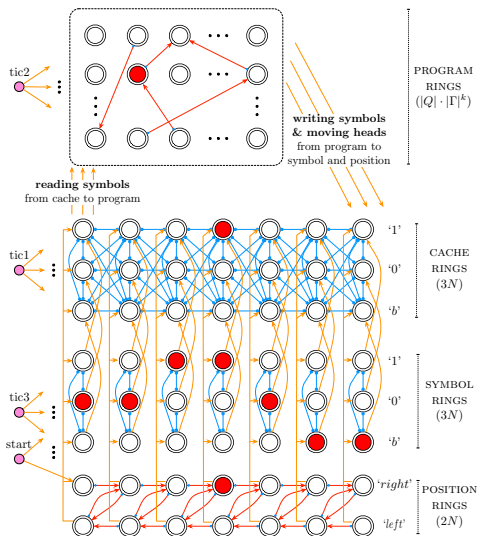
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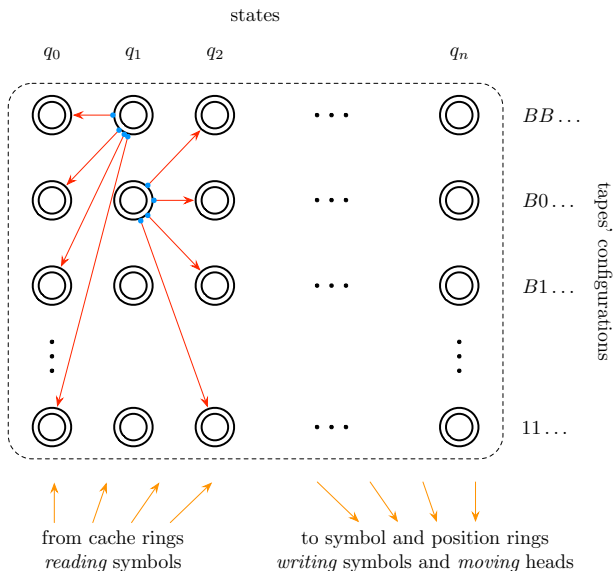
POSITION-SYMBOL-CACHE CONNECTIONS



PROGRAM RINGS



PROGRAM RINGS



TURING MACHINES & BOOLEAN RNNs WITH SYNfire RINGS

Since the construction is generic, the following results hold:

THEOREM

- ▶ *Let \mathcal{M} be a fixed-space k -tape TM whose every tape is of length N . Then, there exists some B-RNN composed of $\mathcal{O}(N)$ synfire rings and cells that simulates \mathcal{M} in linear time.*
- ▶ *Let \mathcal{M} be a k -tape TM. Then, there exists a B-RNN composed of infinitely many synfire rings that simulates \mathcal{M} in linear time.*

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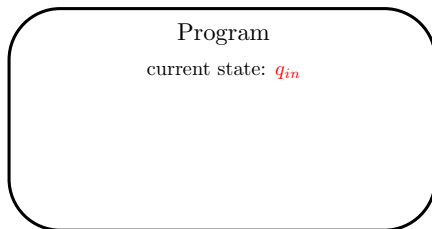
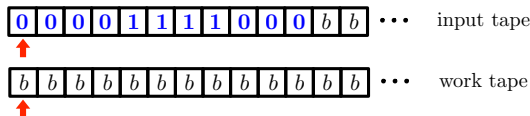
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SIMULATION

- ▶ TM recognizing the non-regular language $L = \{0^n 1^n 0^n : n \geq 0\}$.



Play movie

FUTURE WORKS

1. RESERVOIR COMPUTING: ECHO STATE NETWORKS (ESN)
/ LIQUID STATE MACHINES (LSM)
 - ▶ Introduce *learning* – via synaptic plasticity, intrinsic plasticity, etc. – within this bio-inspired neural architecture.
2. NEUROMORPHIC COMPUTING
 - ▶ Implement bio-inspired into neuromorphic hardwares.
3. BIOLOGY
 - ▶ Implement this architecture into cultured neural networks (*in vitro*): towards neuronal computers...

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