Interactive Evolving Neural Networks are Super-Turing Universal

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Joint work with Alessandro E.P. Villa

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France

16 September 2014

Introduction

- We follow the so-called *mind-computer analogy* approach to cognitive science.
- We study the computational capabilities of basic models of recurrent neural networks.
- We show that recurrent neural networks provide a natural model of computation beyond the Turing limits.

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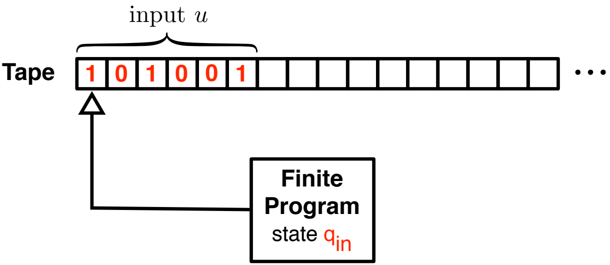
Turing machine

A Turing machine (TM) consists of an infinite tape, a read-write head, and a finite program.



- input u is *accepted* by $\mathcal{M}$ if $\mathcal{M}(u)$ reaches the state q_{acc}
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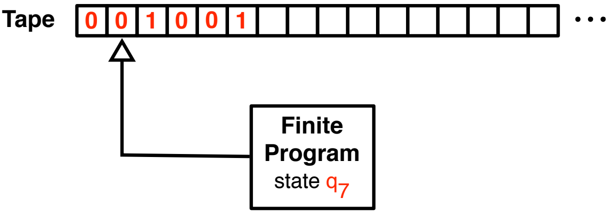

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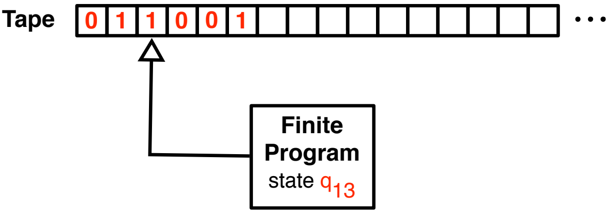

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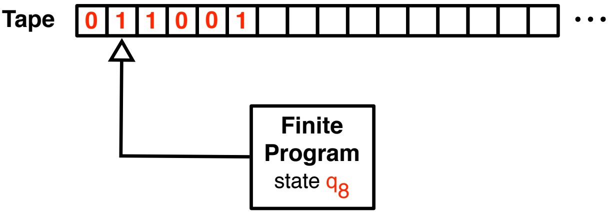

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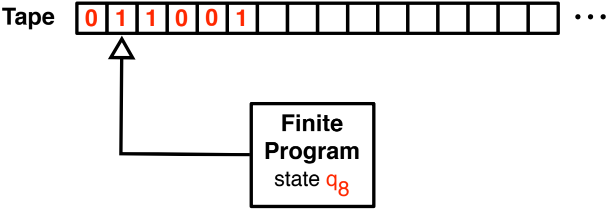

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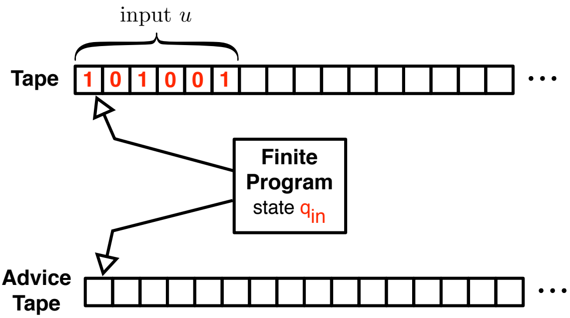

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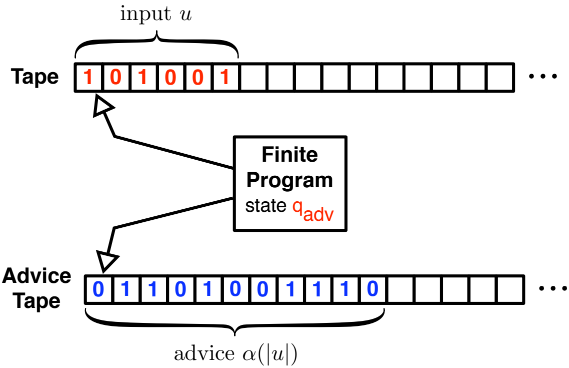
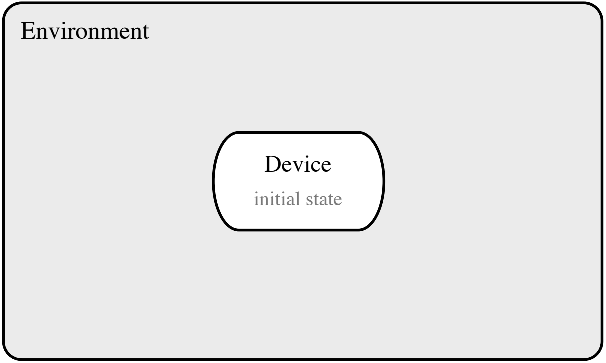


Classical Computation



Closed-box and amnesic...

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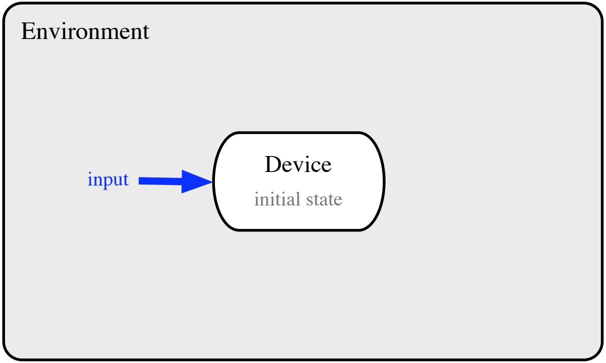

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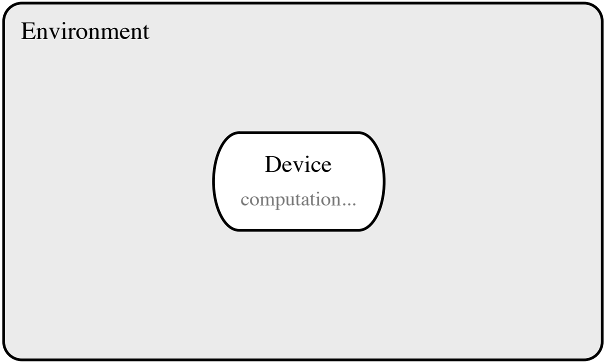

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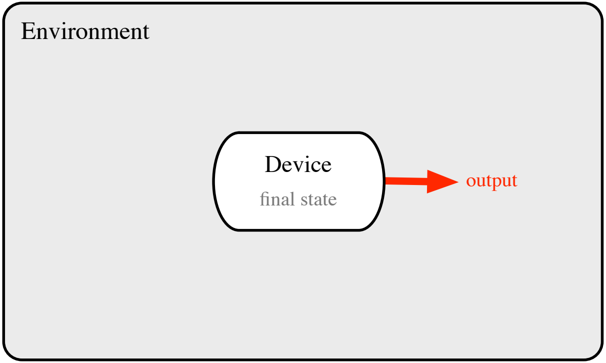

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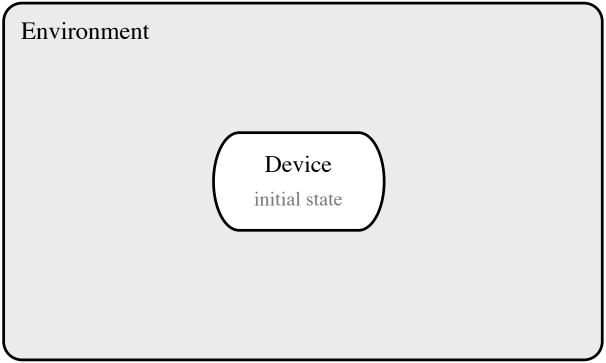

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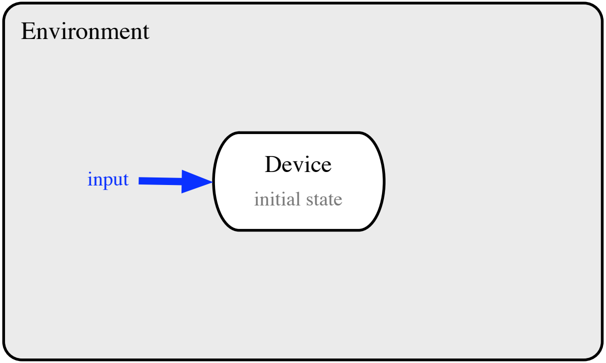

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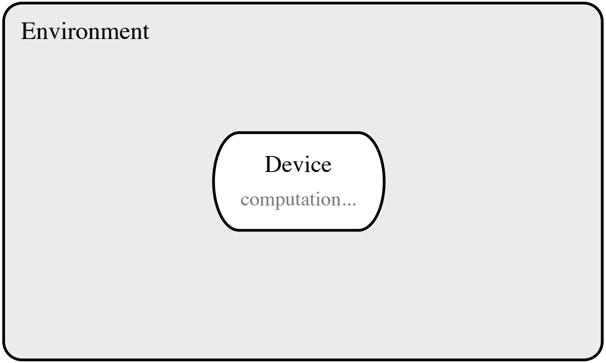

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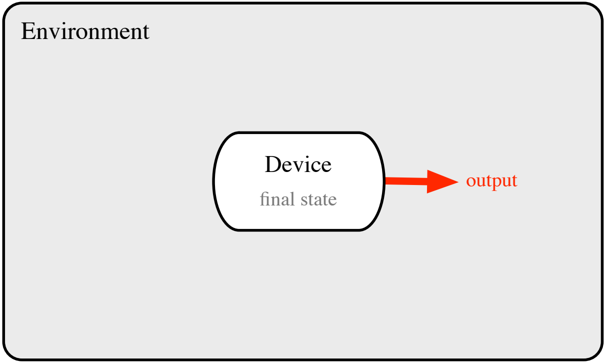

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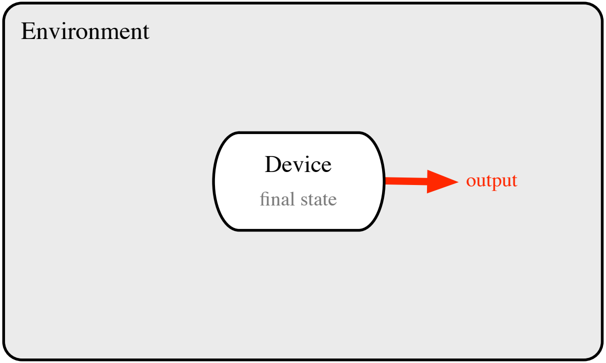

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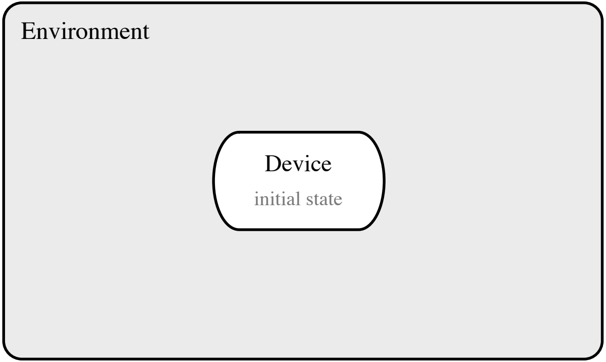

Sequentially interactive and memory active...

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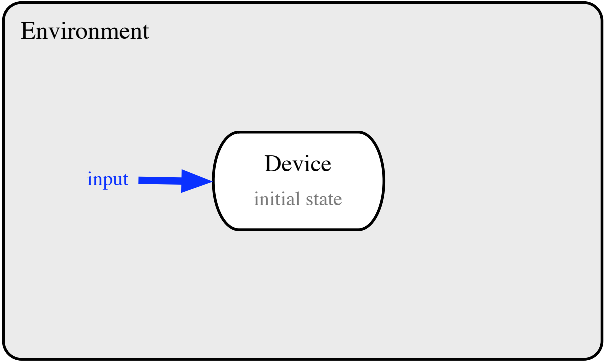

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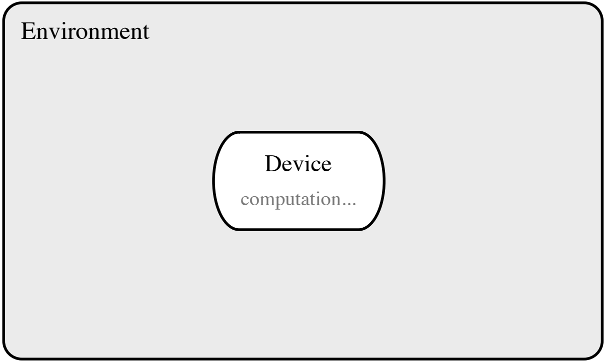

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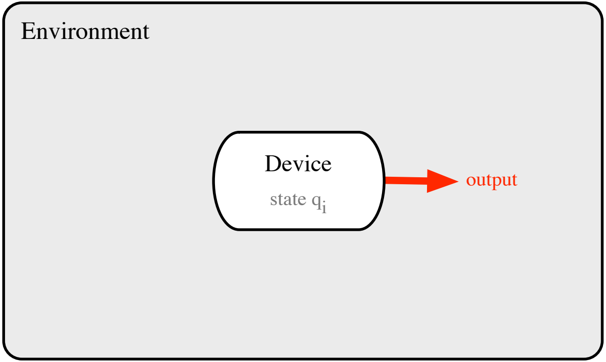

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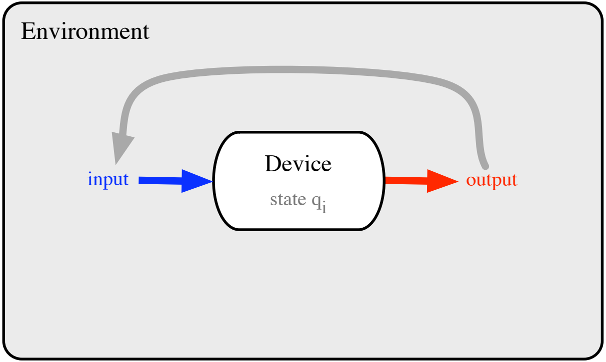

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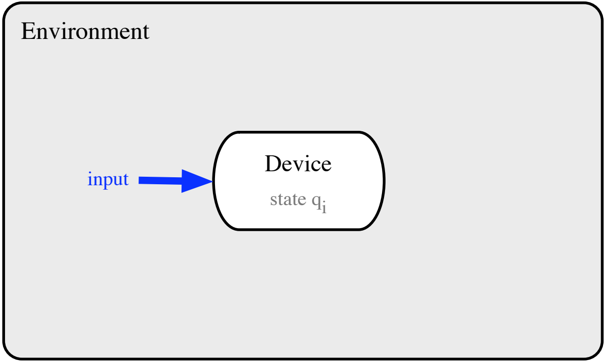

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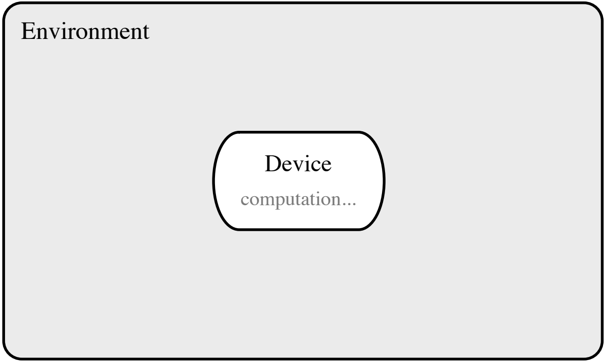

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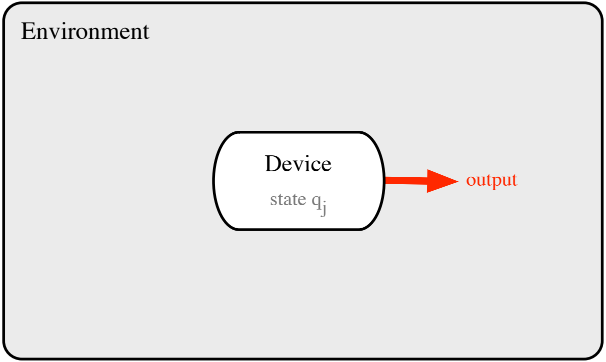

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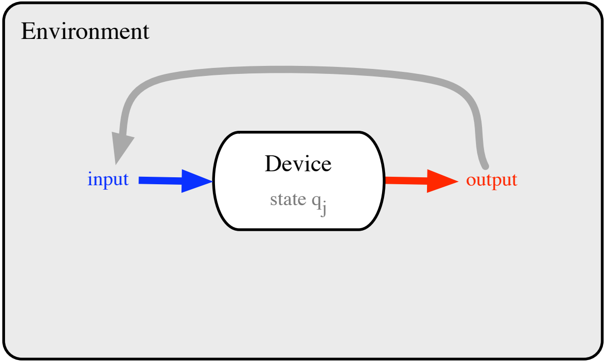

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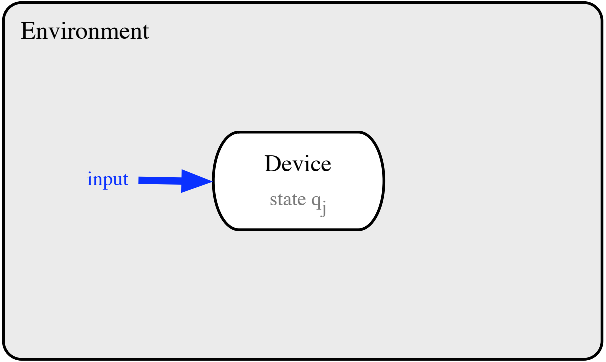

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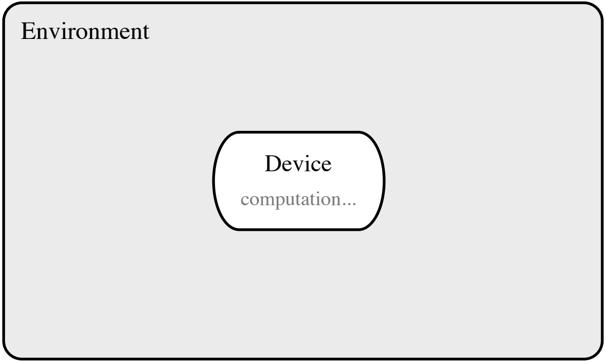

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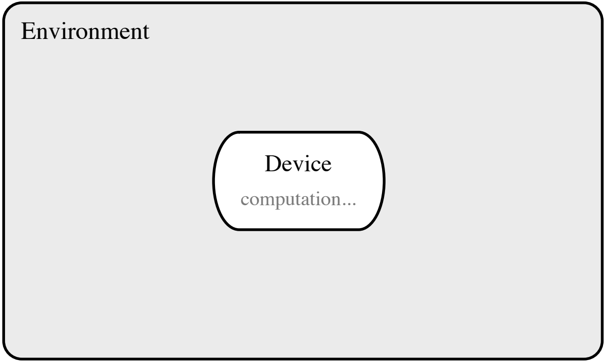

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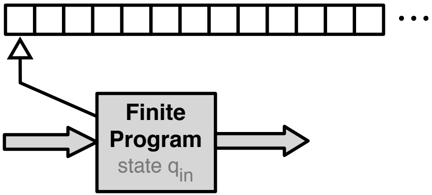
Interactive Turing machine



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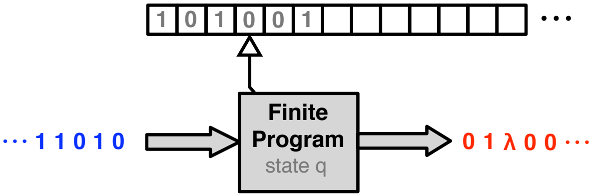
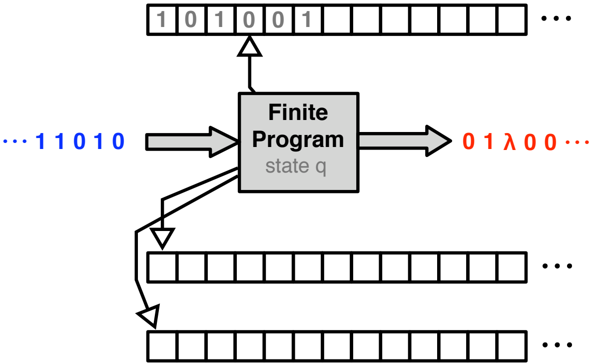


Interactive Turing machine with advice

An Int-TM provided with additional advice input and output tapes and advice function $\alpha : \mathbb{N} \longrightarrow \{0,1\}^*$



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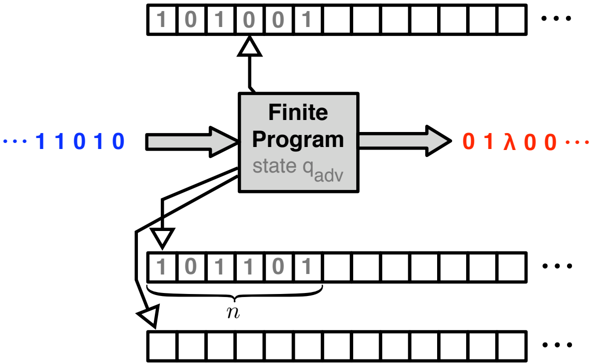

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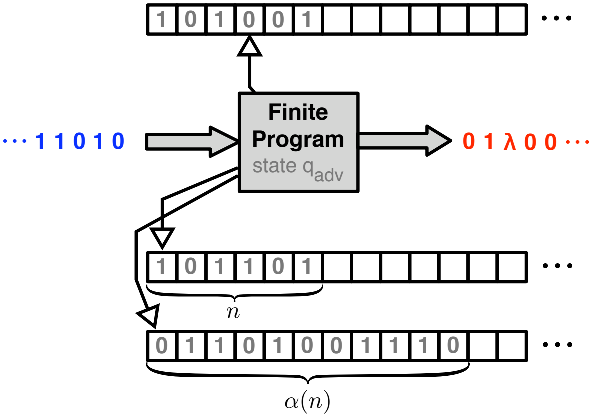

Lemma

Turing machines with advice are strictly more powerful than Turing machines.

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Interactive Turing machines with advice are strictly more powerful than interactive Turing machines.

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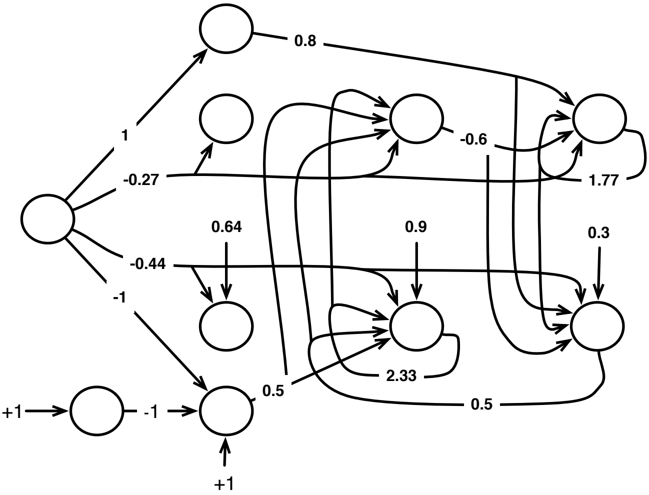
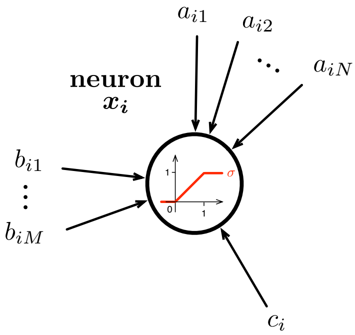
Recurrent Neural Networks



Dynamics: static synaptic weights

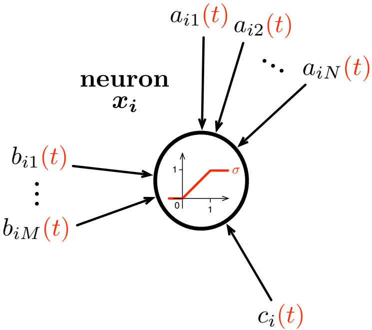


$$x_i(t+1) = \sigma\left(\sum_{j=1}^{N} a_{ij} \cdot x_j(t) + \sum_{j=1}^{M} b_{ij} \cdot u_j(t) + c_i\right)$$

Results

	Static Architecture
$\mathbb{Q}$	Turing Siegelmann & Sontag 95 (classical comp.) Cabessa & Siegelmann 12 (interactive comp.)
$\mathbb{R}$	Super-Turing Siegelmann & Sontag 94 (classical comp.) Cabessa & Siegelmann 12 (interactive comp.) Cabessa & Villa 12 (interactive comp.)

Dynamics: evolving synaptic weights



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	Evolving Architecture
$\mathbb{Q}$	Super-Turing Cabessa & Siegelmann 11 & 14 (classical comp.) Cabessa & Villa 13 (interactive comp.)
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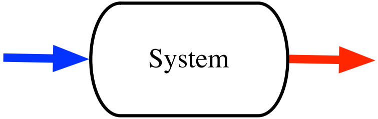
Interactive Scenario

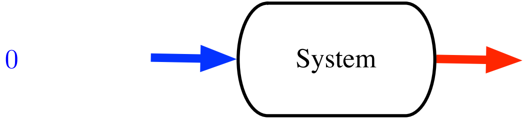


In this way, any deterministic interactive system $\mathcal{S}$ realises an ω-*translation* $\varphi_{\mathcal{S}} : \{0,1\}^{\omega} \to \{0,1\}^{\leq\omega}$.

Definition

An ω-translation ψ is said to be *interactively computable* if there exists a deterministic interactive system $\mathcal{S}$ (of any kind) such that $\varphi_{\mathcal{S}} = \psi$.

Interactive Scenario



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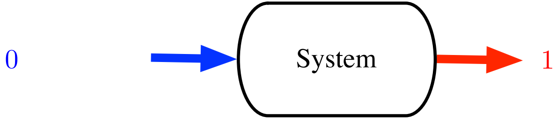

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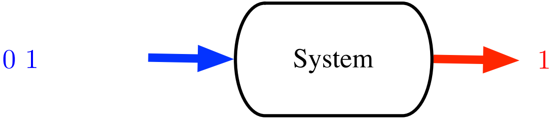

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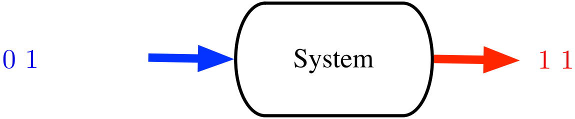

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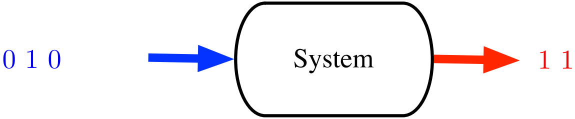

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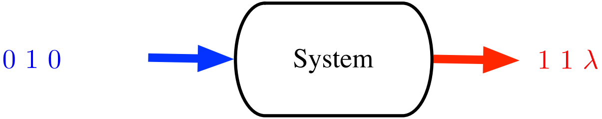

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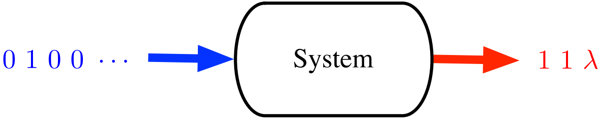

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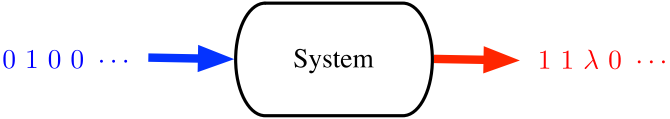

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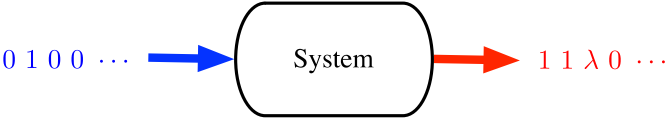

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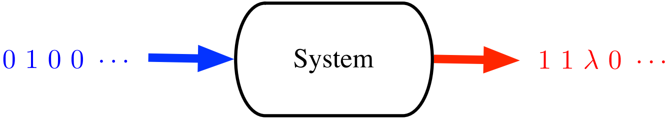

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Results

Theorem

Interactive Evolving RNNs are Super-Turing universal.

More precisely, let ψ be some ω-translation. The following conditions are equivalent:

1. *ψ is interactively computable (by* any *det. int. system)*
2. *ψ is realisable by some Int-Ev-RNN[$\mathbb{B}$]*
3. *ψ is realisable by some Int-Ev-RNN[$\mathbb{Q}$]*
4. *ψ is realisable by some Int-St-RNN[$\mathbb{R}$]*
5. *ψ is realisable by some Int-TM/A*
6. *ψ is continuous (i.e. "limit" of monotone function)*

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3. ψ *is realisable by some Int-Ev-RNN[$\mathbb{Q}$]*
4. ψ *is realisable by some Int-St-RNN[$\mathbb{R}$]*
5. ψ *is realisable by some Int-TM/A*
6. ψ *is continuous (i.e. "limit" of monotone function)*

Results

Theorem

Interactive Evolving RNNs are Super-Turing universal.

More precisely, let ψ be some ω-translation. The following conditions are equivalent:

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Summary

	Static	Evolving
$\mathbb{Q}$	Turing	Super-Turing universal (interactive case)
$\mathbb{R}$	Super-Turing universal (interactive case)	Super-Turing universal (interactive case)

Conclusions

- The results are theoretical: any implementation of an evolving RNN will never be super-Turing.
- Evolving-RNNs provide a natural abstract computational model beyond the Turing limits.
- *Architectural Evolution* is an alternative way to the *power of the continuum* to achieve super-Turing capabilities.
- The results support the idea that *architectural evolution* might play a crucial role in the computational capabilities of biological neural networks.
- Future work: study the computational power of more biologically oriented neural models involved in more bio-inspired computational frameworks.
- The results do not prove that the brain is super-Turing...

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