A Hierarchical Classification of First-Order Recurrent Neural Networks

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Joint work with Alessandro Villa

University Joseph Fourier – Grenoble 1

24 May 2010

Introduction

The fields of artificial neural networks and theoretical computer science have been linked since their inception (McCulloch and Pitts 1943, Kleene 1956, Minsky 1967).

Synaptic weights	Activation function	Computational power
rational	hard-threshold	finite state automaton
rational	(linear) sigmoid	Turing machine
real	(linear) sigmoid	beyond Turing limits

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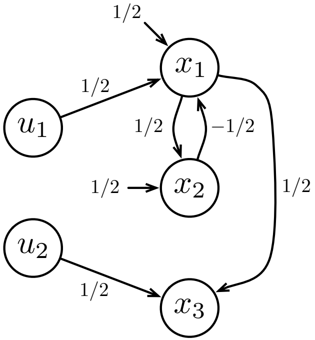

Every stimulation s induced an evolution e_s. For instance:

time	$t =$	0	1	2	3	$\cdots\cdots$
stimulation	$s =$	$\begin{pmatrix}1\\0\end{pmatrix}$	$\begin{pmatrix}0\\1\end{pmatrix}$	$\begin{pmatrix}1\\1\end{pmatrix}$	$\begin{pmatrix}1\\1\end{pmatrix}$	$\cdots\cdots$
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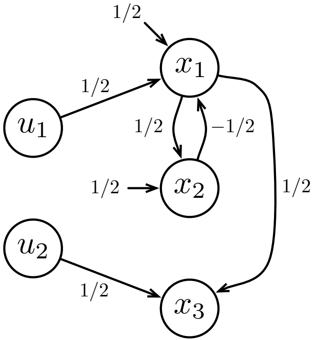

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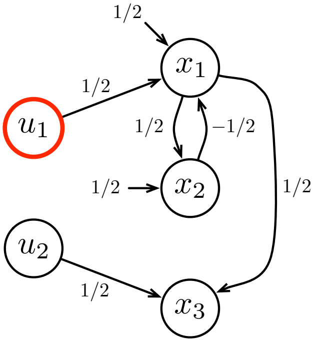

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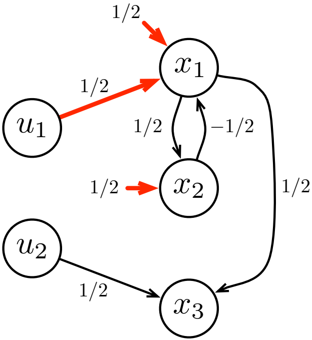

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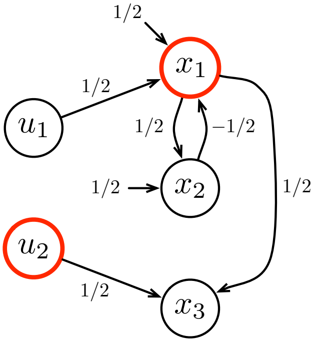

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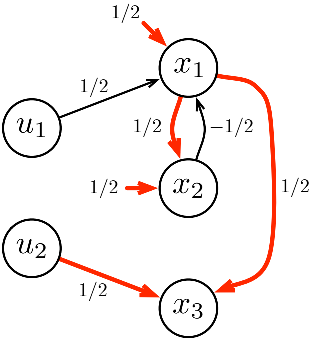

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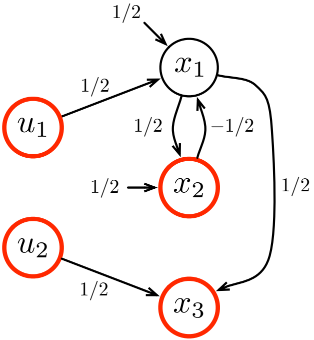

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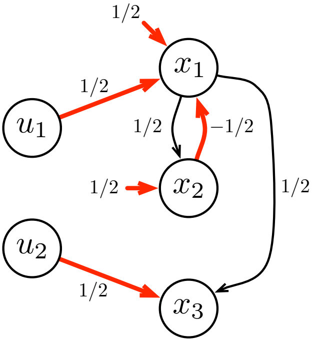

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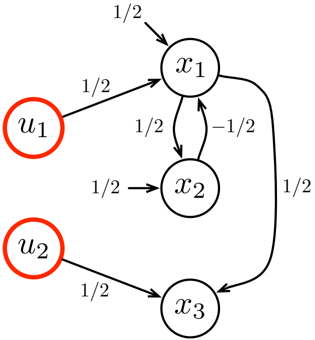

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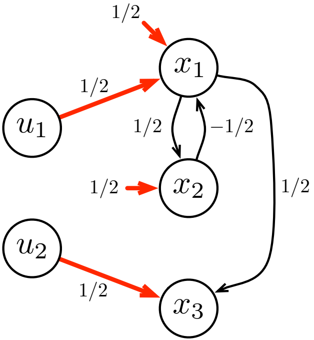

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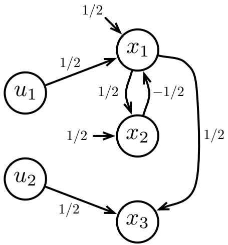

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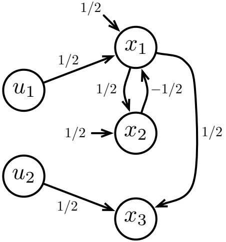

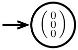

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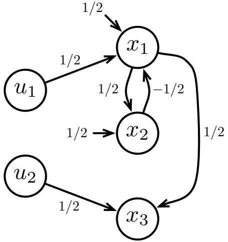

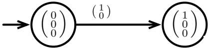

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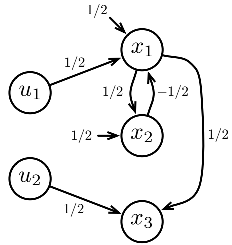

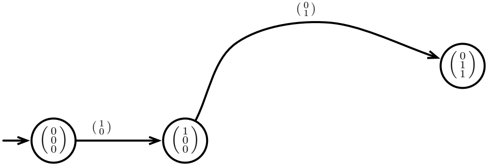

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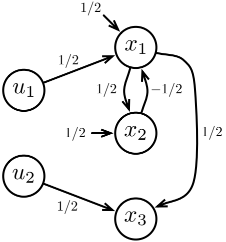

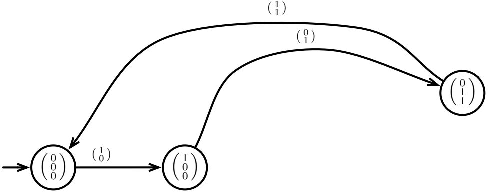

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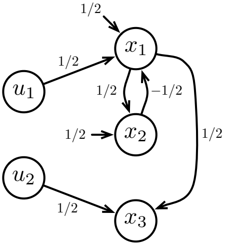

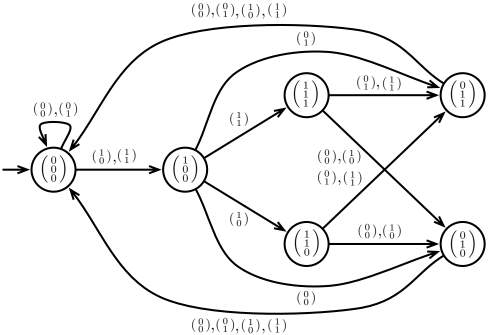

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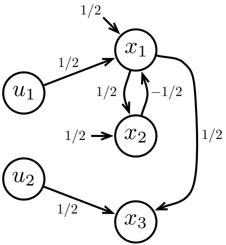
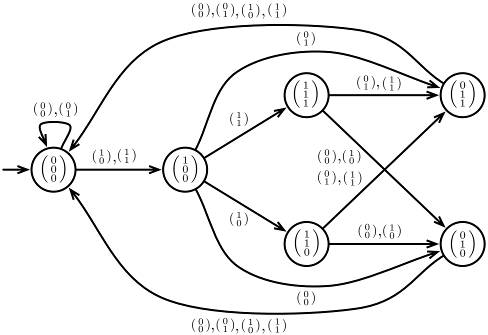
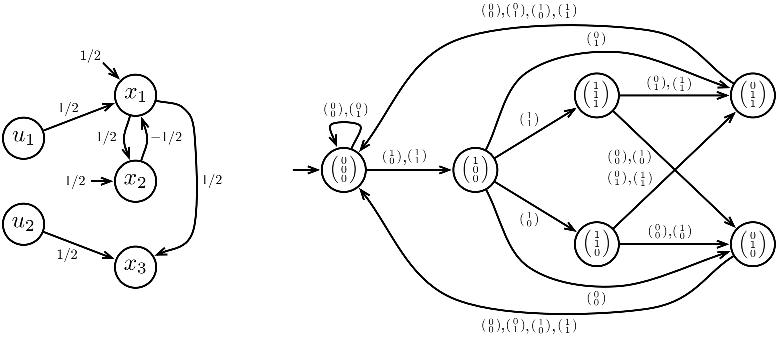


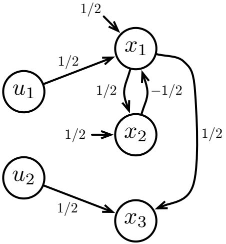


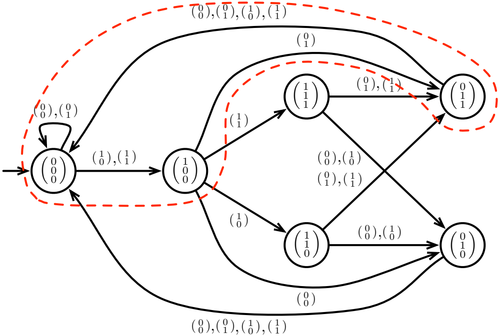
An *attractor* is a set of states visited infinitely often by the network along some infinite evolution.



Evolution of the net $\leftrightarrow$ *Path* in the transition state diagram

Attractor of the net $\leftrightarrow$ *Cycle* in the transition state diagram





Evolution of the net $\leftrightarrow$ *Path* in the transition state diagram

Attractor of the net $\leftrightarrow$ *Cycle* in the transition state diagram

- We assume that attractors can be of two kinds: either *meaningful* or *spurious*.
- An infinite stimulation $s \in [\mathbb{B}^k]^\omega$ is *accepted* by $\mathcal{N}$ if its corresponding evolution eventually gets trapped by a meaningful attractor.
- The set $L(\mathcal{N}) \subseteq [\mathbb{B}^k]^\omega$ of all infinite stimulations accepted by $\mathcal{N}$ is called the *neural language* recognized by $\mathcal{N}$.

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Proposition

Let $L \subseteq [\mathbb{B}^k]^\omega$. Then the following are equivalent:

- *L is recognizable by some network.*
- *L is recognizable by some Muller automaton.*

Proof.

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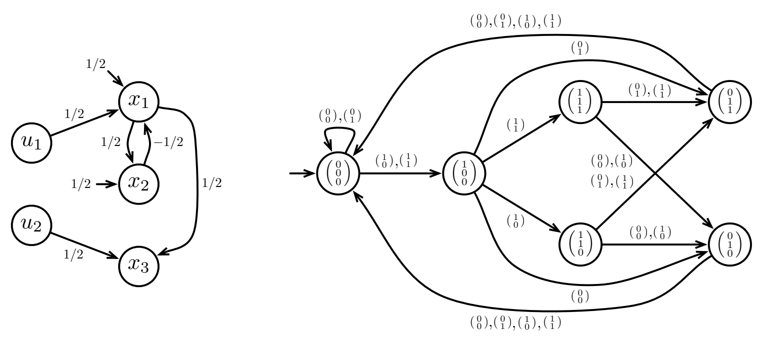

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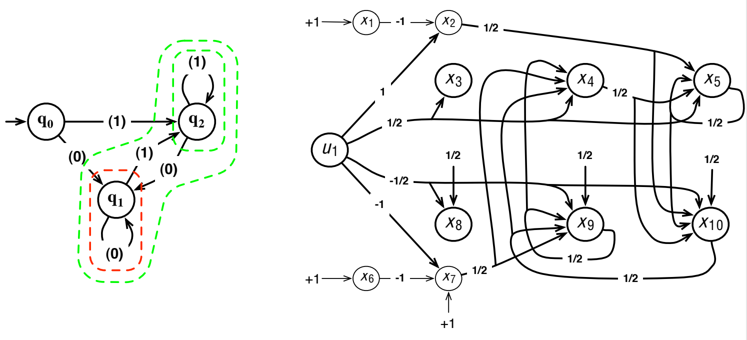

We translate the Wadge classification theory from the automata to the neural network context ...

$$\mathcal{N} \leq_W \mathcal{N}' \quad \text{iff} \quad \text{there exists } f : [\mathbb{B}^k]^\omega \to [\mathbb{B}^l]^\omega \text{ continuous s.t. } s \in L(\mathcal{N}) \Leftrightarrow f(s) \in L(\mathcal{N}')$$

$$\mathcal{N} <_W \mathcal{N}' \quad \text{iff} \quad \mathcal{N} \leq_W \mathcal{N}' \text{ and } \mathcal{N}' \not\leq_W \mathcal{N}$$

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Definition

The collection of all nets ordered by $\leq_W$ is called *the RNN hierarchy*.

We translate the Wadge classification theory from the automata to the neural network context . . .

$\mathcal{N} \leq_W \mathcal{N}'$ iff there exists $f : [\mathbb{B}^k]^\omega \longrightarrow [\mathbb{B}^l]^\omega$ continuous s.t. $s \in L(\mathcal{N}) \Leftrightarrow f(s) \in L(\mathcal{N}')$

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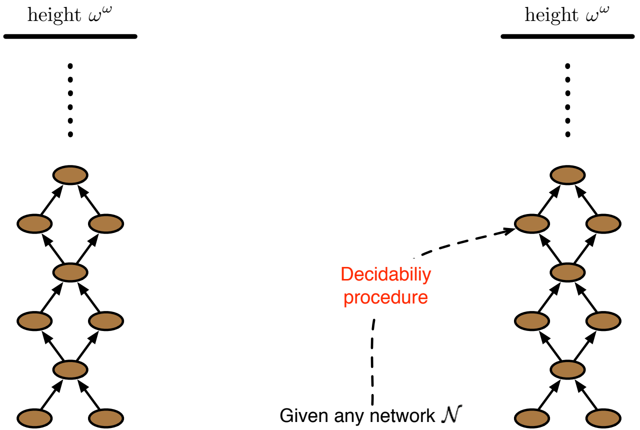
Theorem

- *The RNN hierarchy is well-founded, has width 2 and height ω^ω.*
- *The RNN hierarchy is decidable.*



Alternating and co-alternating tree of length $\alpha < \omega^\omega$ in a graph:

$$\alpha = \omega^{n_p} \cdot m_p + \omega^{n_{p-1}} \cdot m_{p-1} + \ldots + \omega^{n_1} \cdot m_1 + \omega^{n_0} \cdot m_0$$

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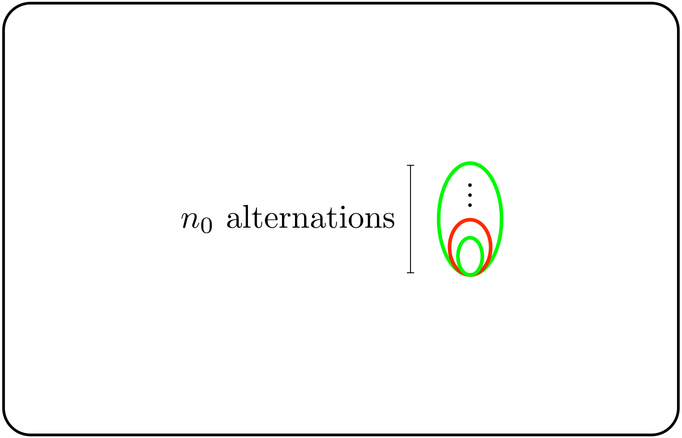

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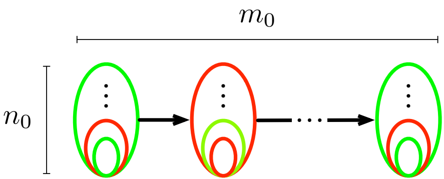

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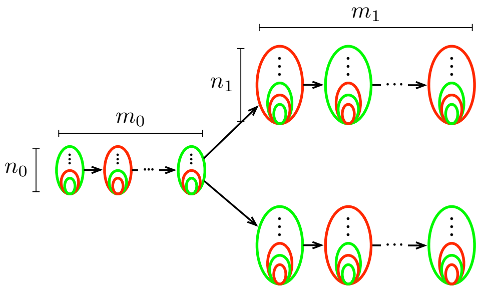

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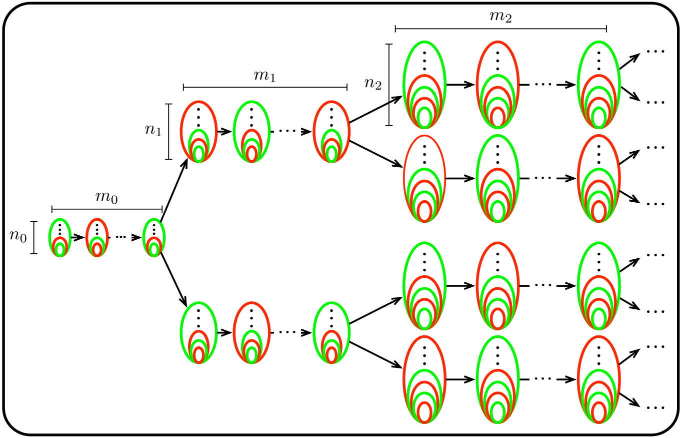

Decidability procedure:

1. Compute the state transition diagram of $\mathcal{N}$.
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$A = \left\{(0,0,0)^T, (1,0,0)^T, (0,1,1)^T\right\}$ is the only meaningful attractor for $\mathcal{N}$

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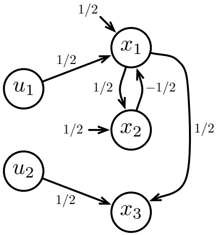

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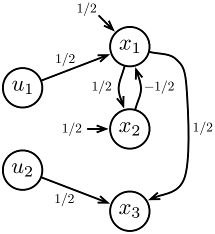

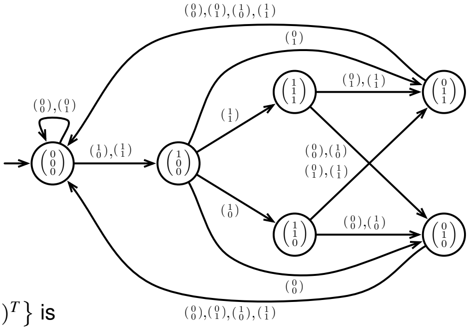

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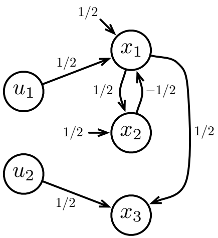

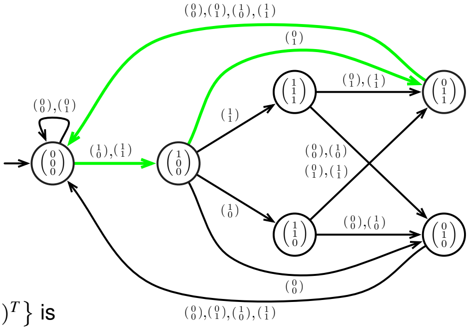

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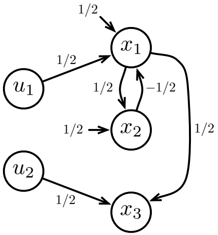

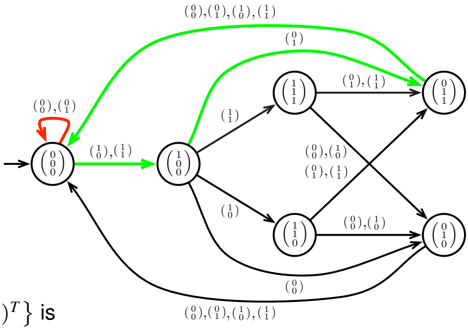

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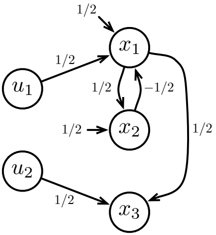

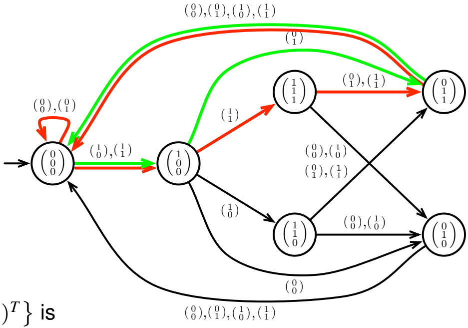

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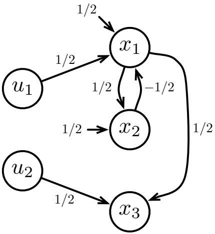

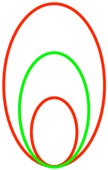

$$d(\mathcal{N}) = \omega^2 \cdot 1 = \omega^2$$

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