

Recurrent Neural Networks and Super-Turing Computation

Jérémie Cabessa

Department of Information Systems
University of Lausanne
Switzerland

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Introduction

- ▶ We follow the so-called *mind-computer analogy* approach to cognitive science.
- ▶ We study the computational capabilities of basic models of recurrent neural networks.
- ▶ We show that recurrent neural networks provide a natural model of computation beyond the Turing limits.

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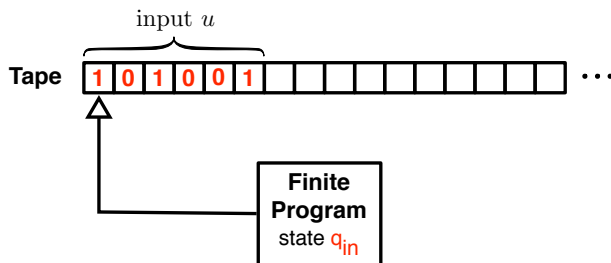
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Turing machine

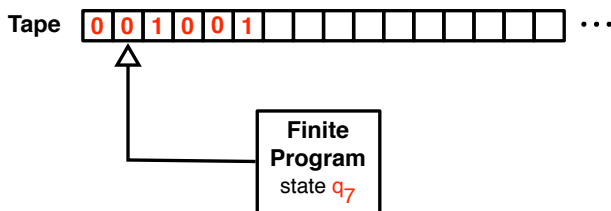
A Turing machine (TM) consists of an infinite tape, a read-write head, and a finite program.



- ▶ input u is *accepted* by \mathcal{M} if $\mathcal{M}(u)$ reaches the state q_{acc}
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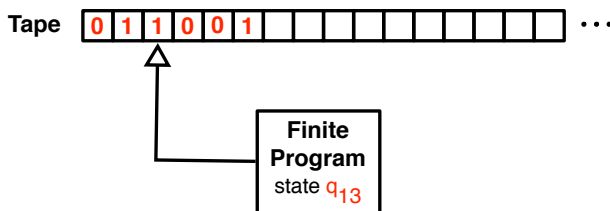
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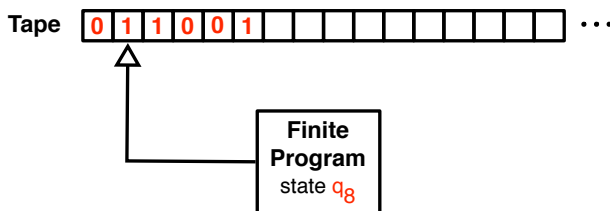
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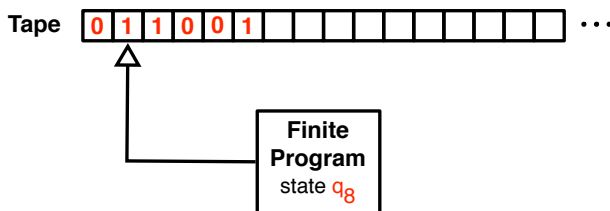
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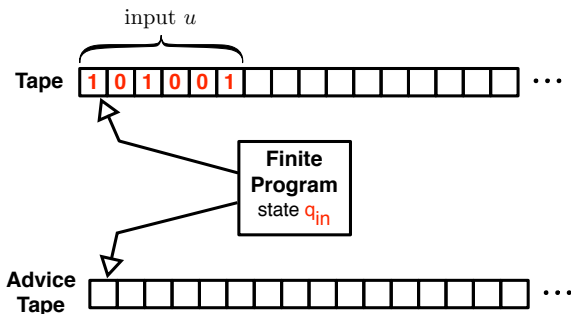
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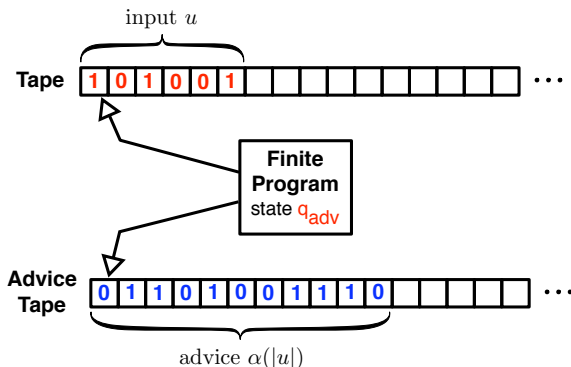
Turing machine with advice

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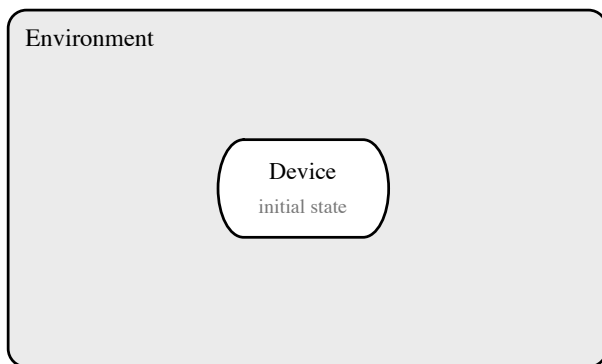


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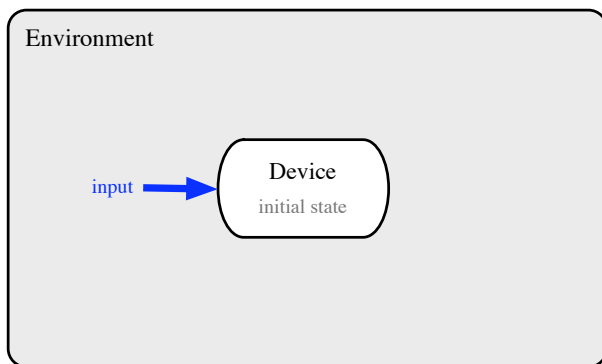


Classical Computation



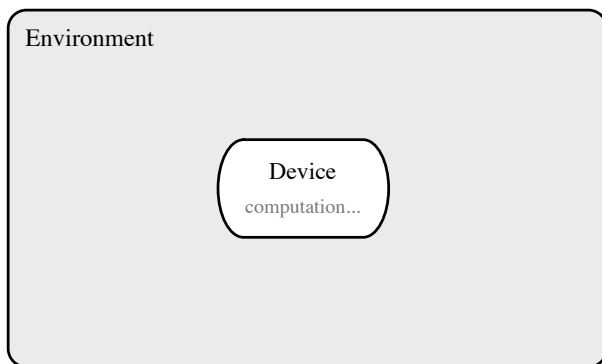
Closed-box and amnesic...

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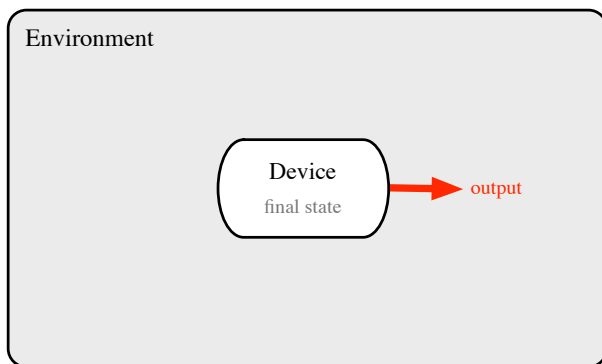
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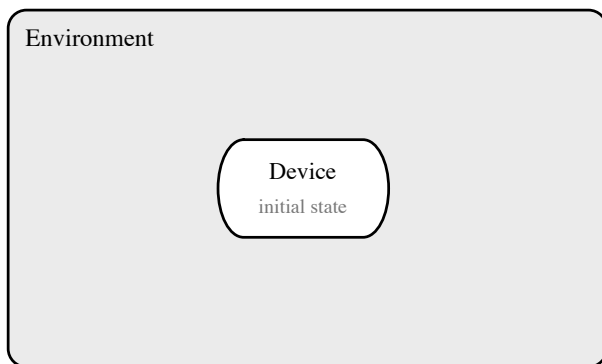
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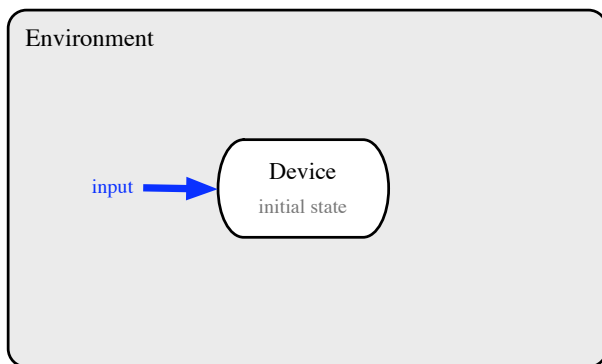
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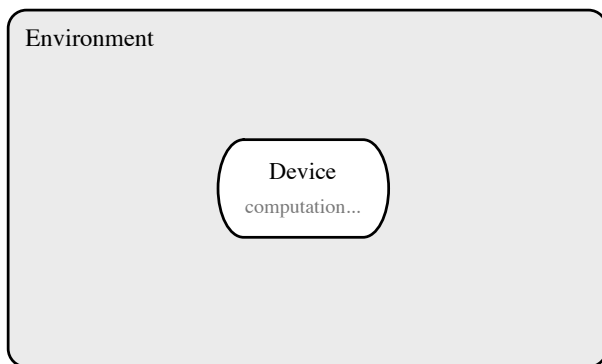
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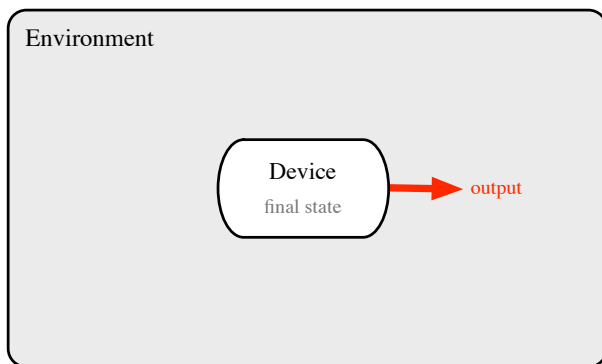
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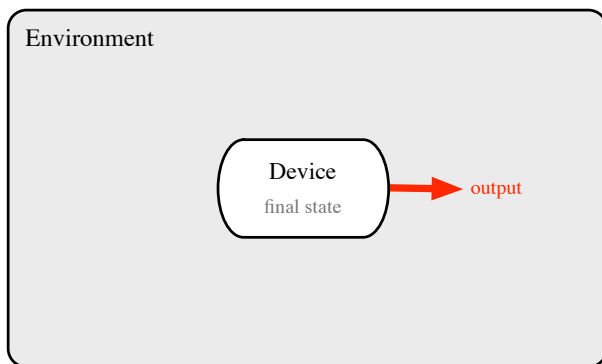
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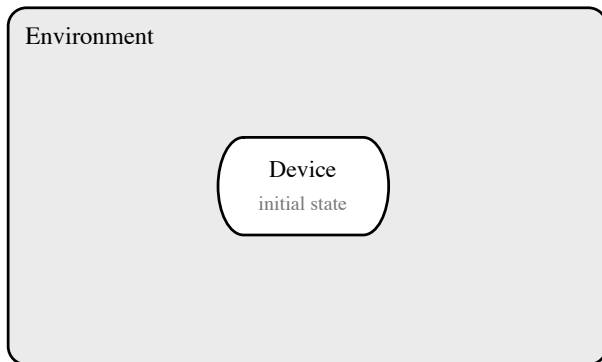
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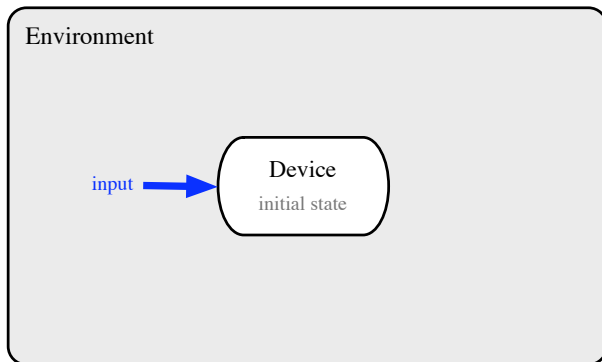
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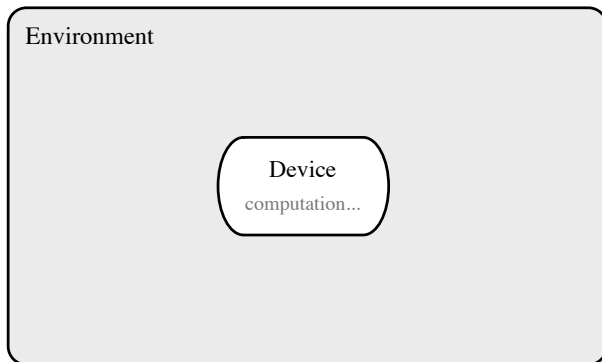
Sequentially interactive and memory active...

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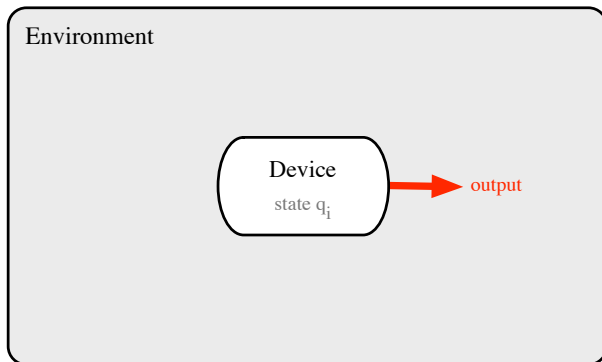
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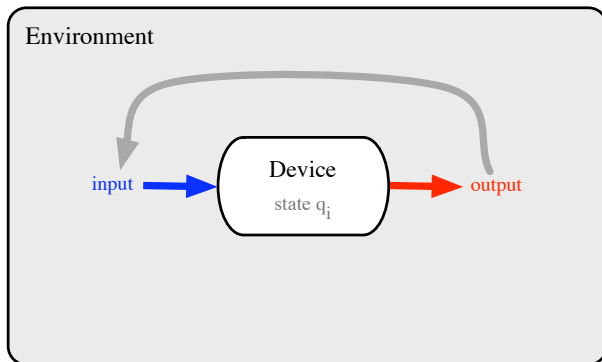
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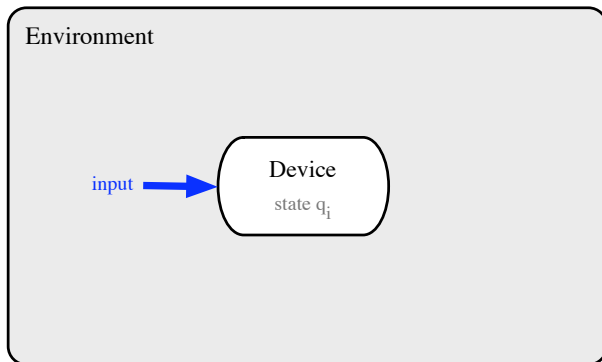
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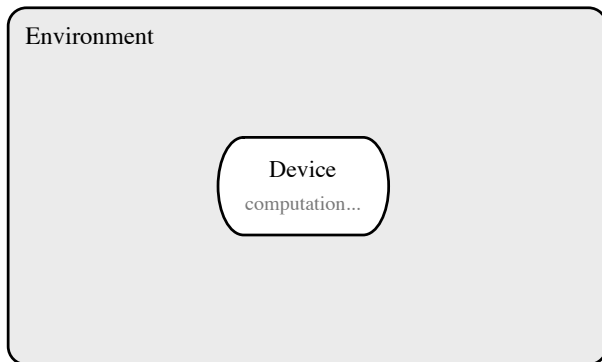
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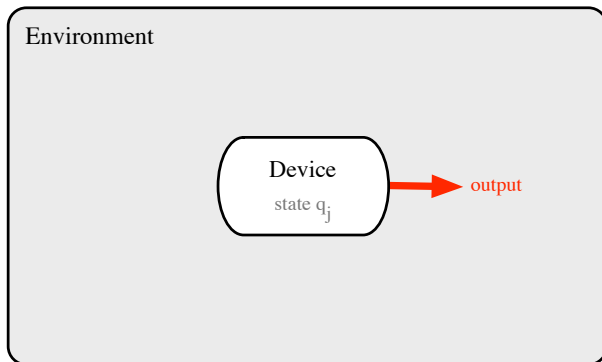
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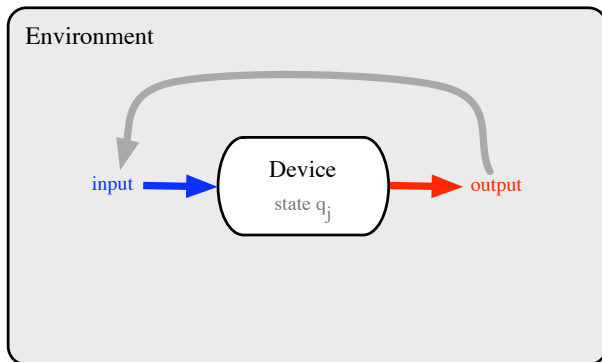
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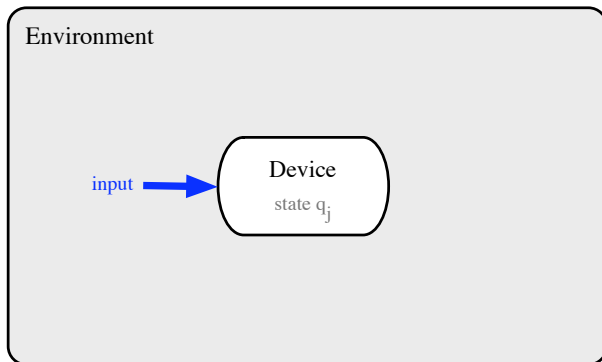


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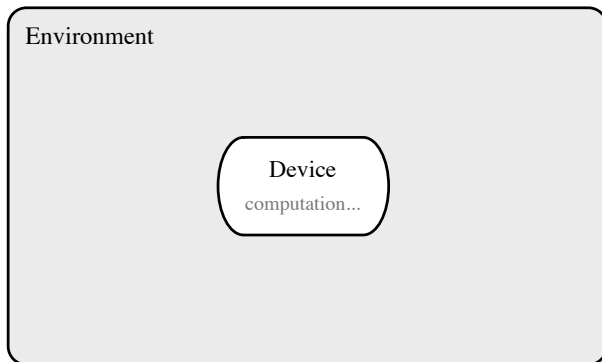
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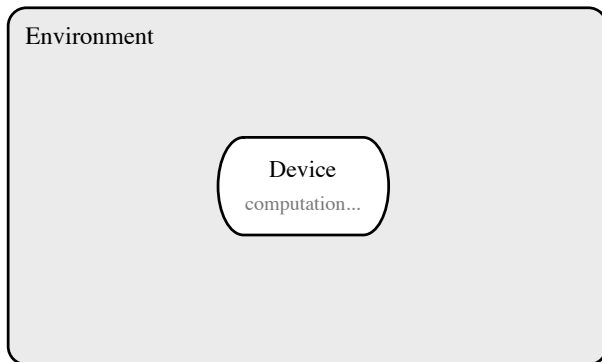
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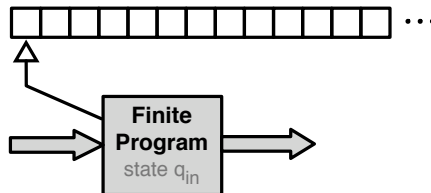
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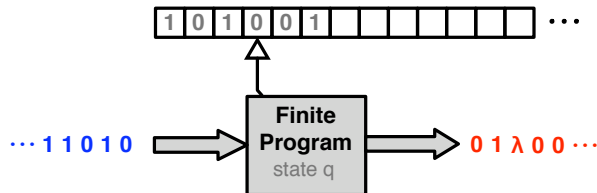
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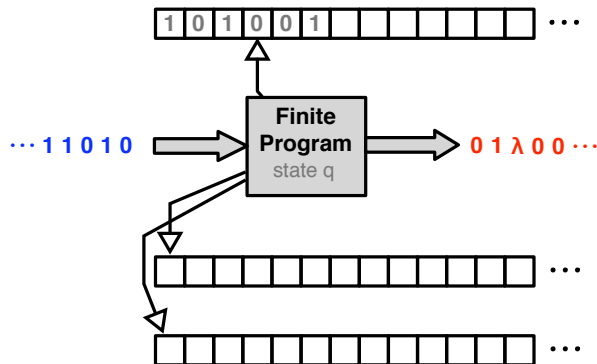


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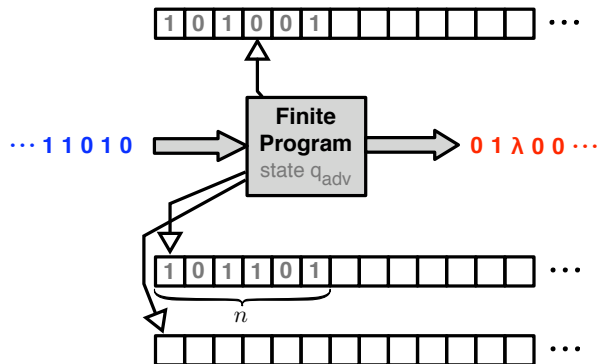
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An Int-TM provided with additional advice input and output tapes and advice function $\alpha : \mathbb{N} \rightarrow \{0, 1\}^*$



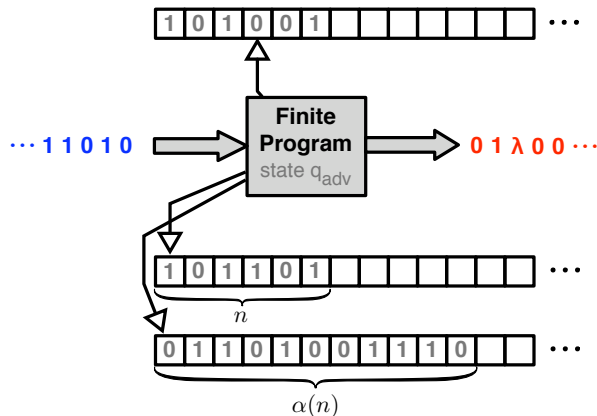
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Turing machines with advice are strictly more powerful than Turing machines.

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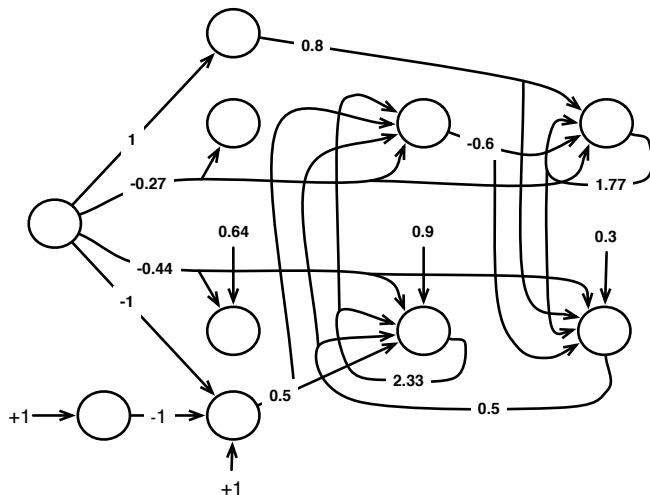
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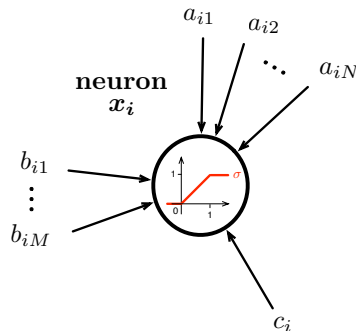
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Recurrent Neural Networks



Dynamics: static synaptic weights

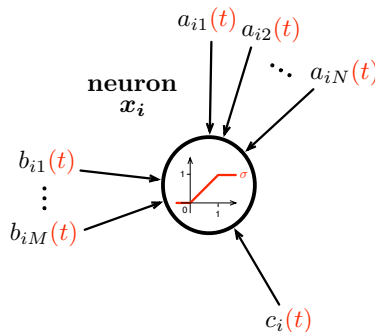


$$x_i(t+1) = \sigma \left(\sum_{j=1}^N a_{ij} \cdot x_j(t) + \sum_{j=1}^M b_{ij} \cdot u_j(t) + c_i \right)$$

Results

	Static Architecture
\mathbb{Q}	<p>Turing</p> <p>Siegelmann & Sontag 95 (classical comp.) Cabessa & Siegelmann 12 (interactive comp.)</p>
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Dynamics: evolving synaptic weights



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Summary

	Static	Evolving
\mathbb{Q}	Turing	Super-Turing
\mathbb{R}	Super-Turing	Super-Turing

Conclusions

- ▶ Evolving-RNNs provide a natural abstract computational model beyond the Turing limits.
- ▶ *Architectural Evolution* is an alternative way to the *power of the continuum* to achieve super-Turing capabilities.
- ▶ The results support the idea that *architectural evolution* might play a crucial role in the computational capabilities of biological neural networks.
- ▶ Future work: study the computational power of more biologically oriented neural models involved in more bio-inspired computational frameworks.
- ▶ The results do not prove that the brain is super-Turing...

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