Hierarchies of Abstract Machines

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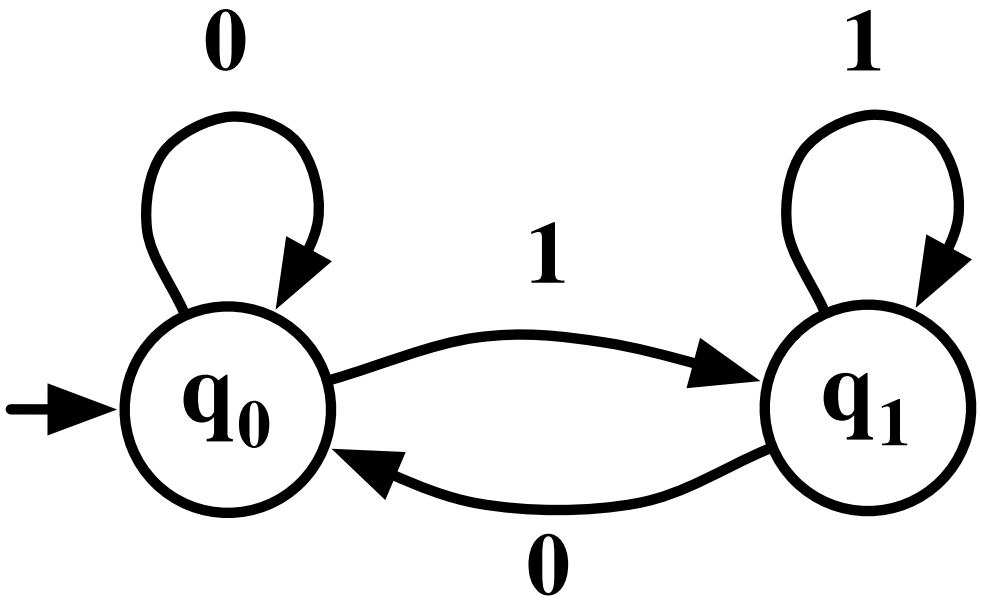

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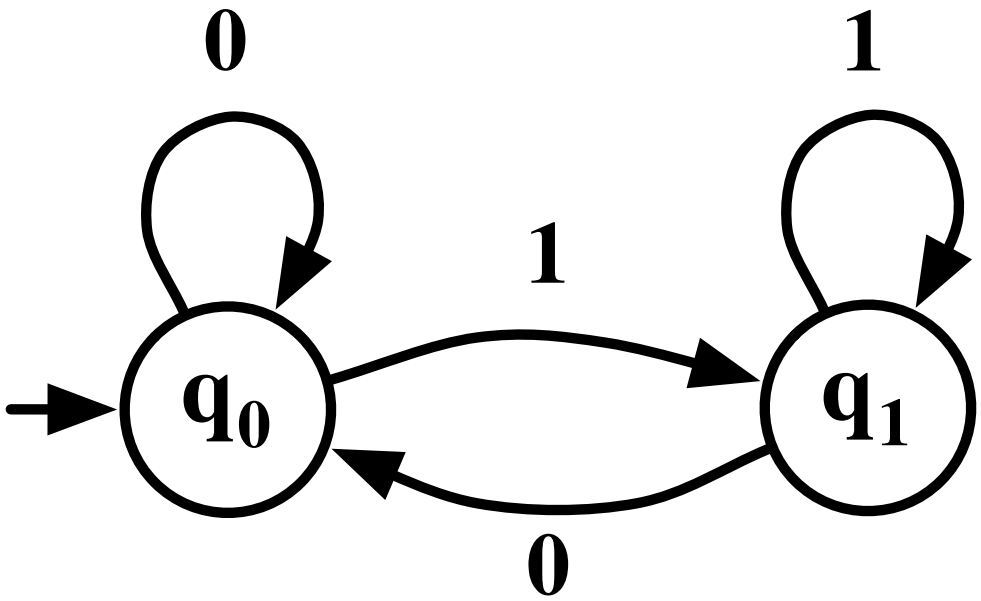

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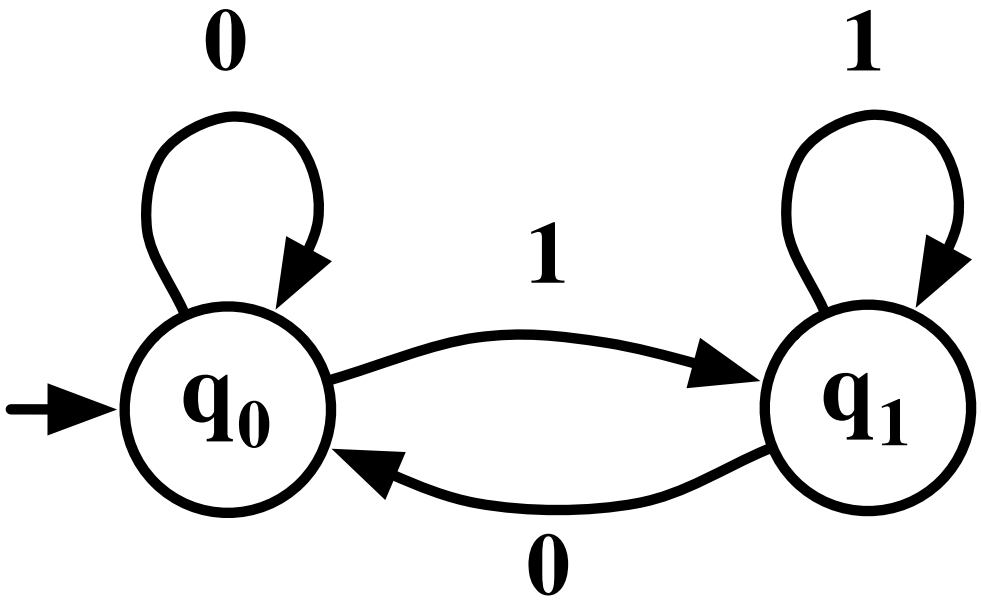

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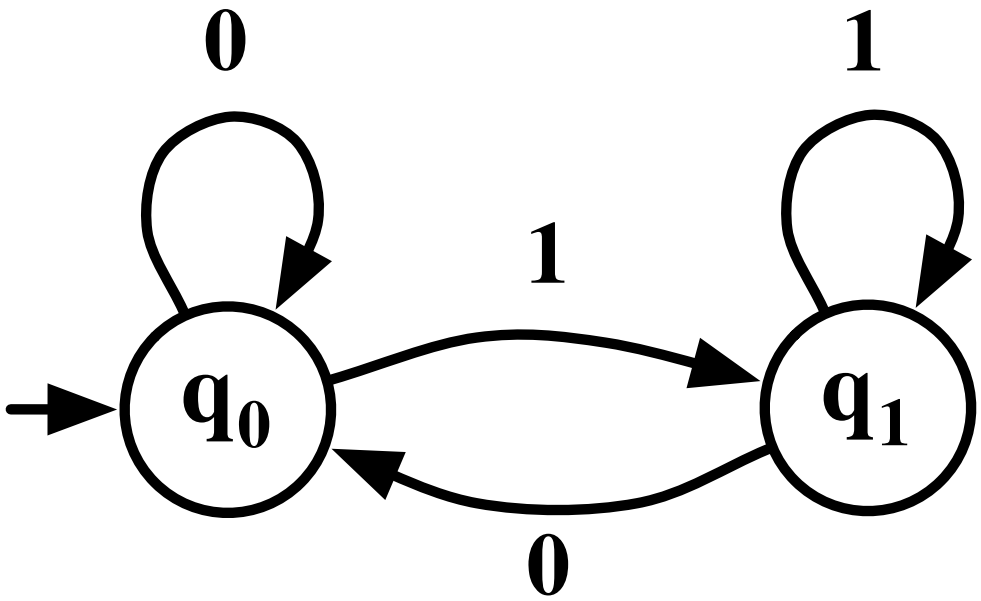

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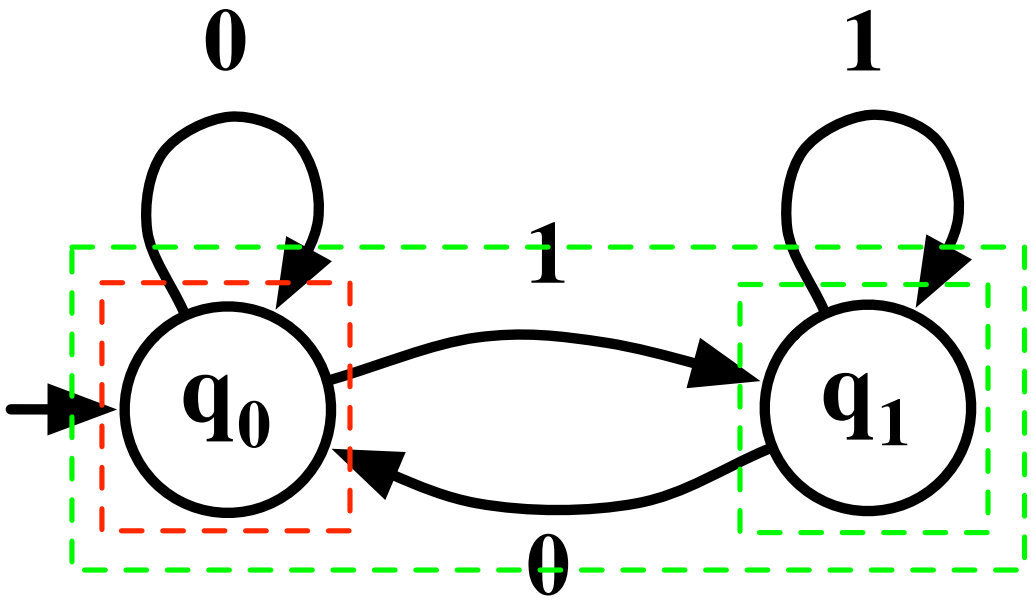

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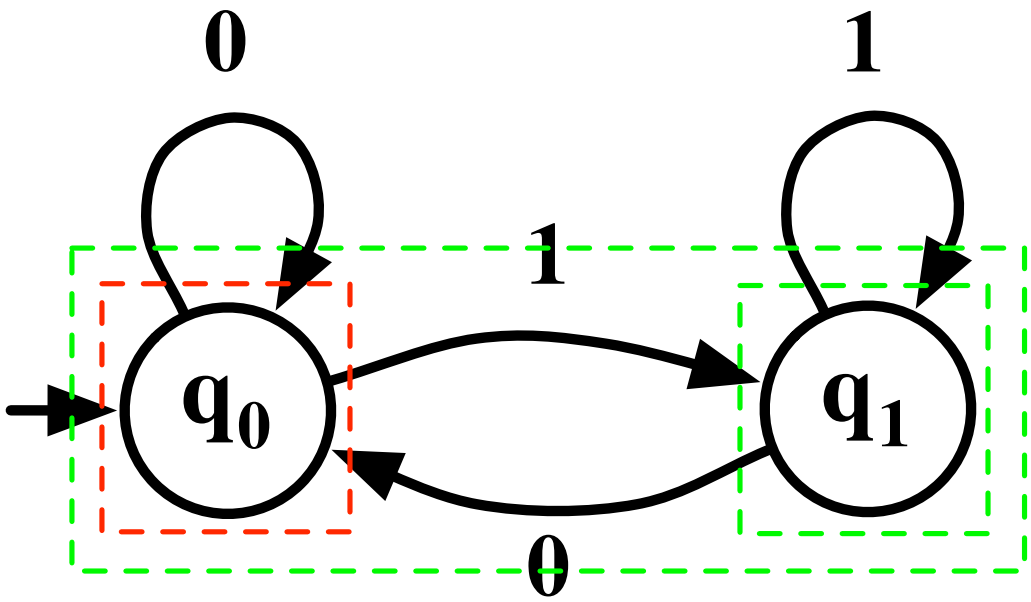

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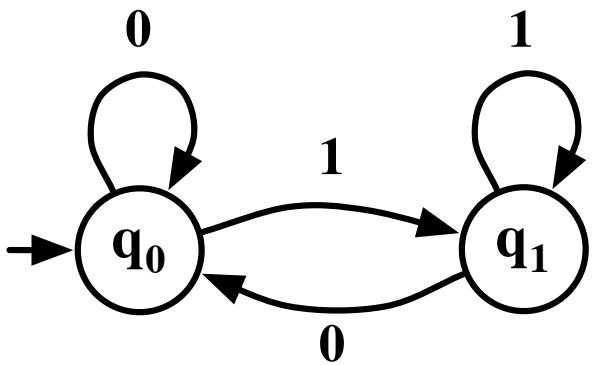
The accepted words are exactly those containing infinitely many 1's. This is the *language recognized by this automaton.*

Definition

A finite *Muller automaton* is a tuple $\mathcal{A} = (Q, A, \delta, i, \mathcal{T})$, where

- Q is a finite set of states,
- A is an alphabet,
- $\delta : Q \times A \longrightarrow Q$ is the transition function,
- $i \in Q$ is the initial state,
- $\mathcal{T} \subseteq \mathcal{P}(Q)$ is the table of $\mathcal{A}$.



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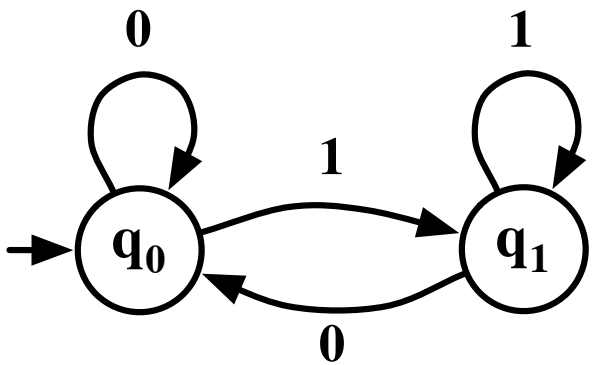

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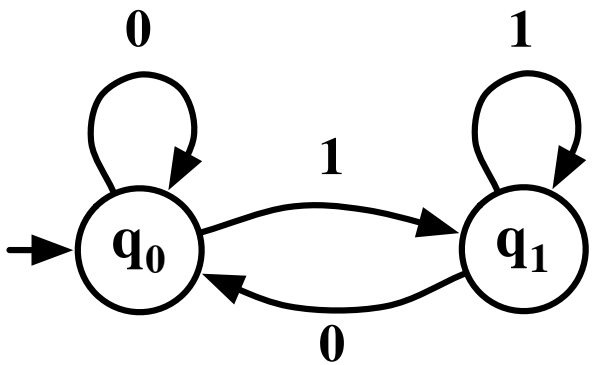

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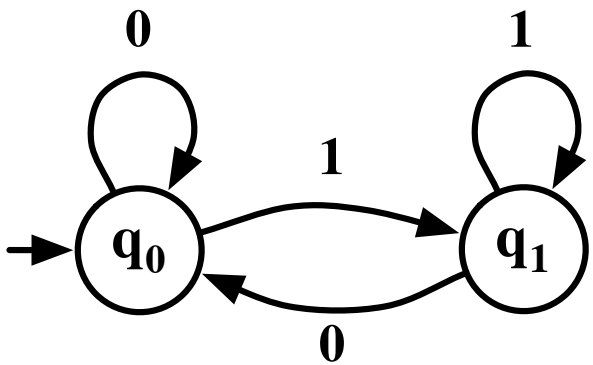

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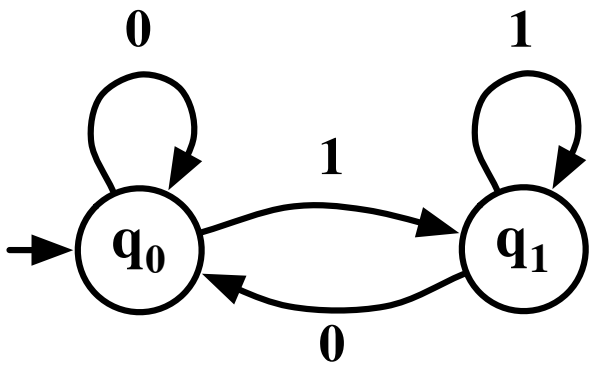

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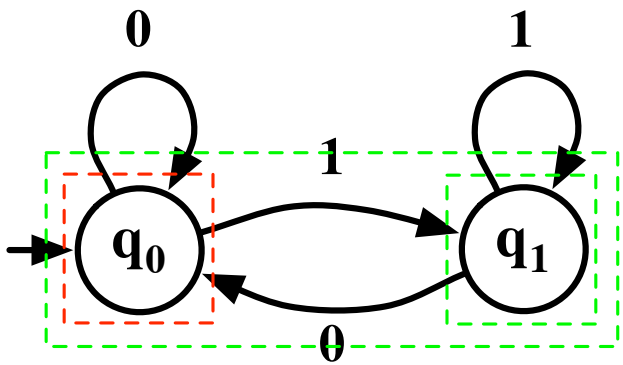

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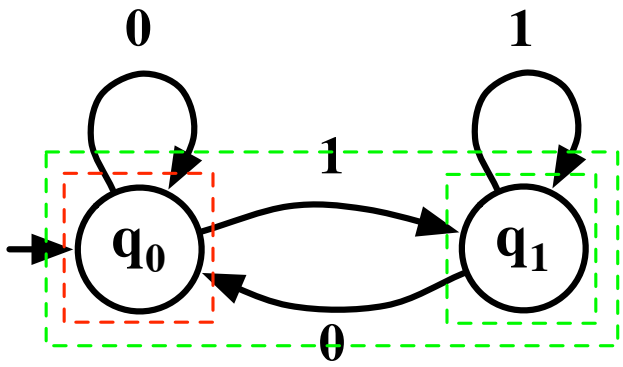

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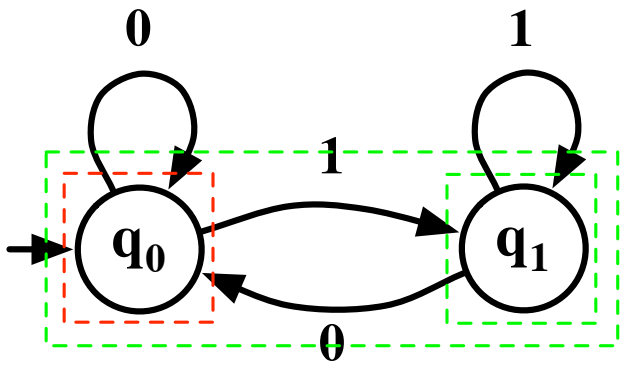

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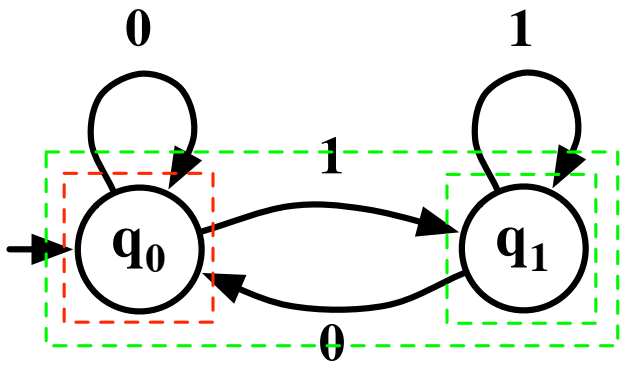

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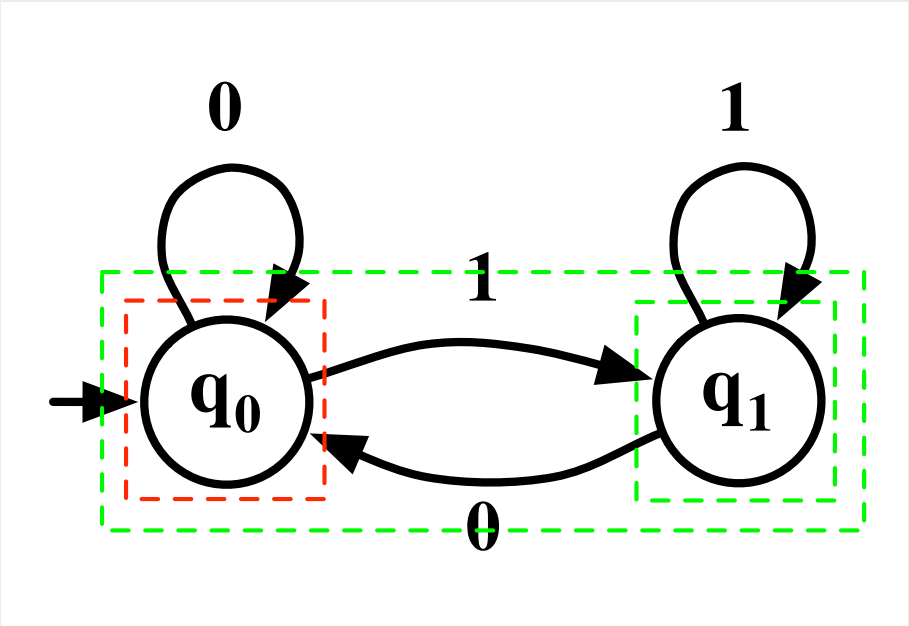
Example

Consider the following automaton $\mathcal{A}$. Then $L(\mathcal{A})$ is the set of words containing infinitely many 1's.



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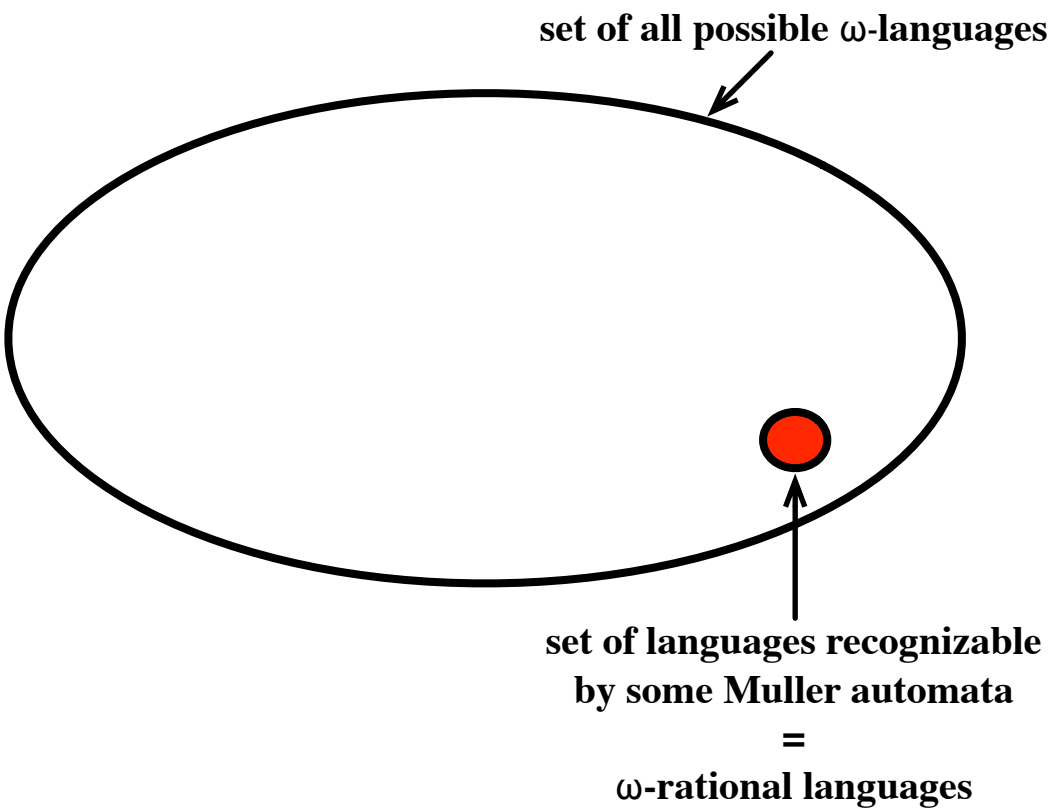

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Definition (alt. def. of the same reduction relation)

We say that $\mathcal{A} \leq_W \mathcal{B}$ iff there exists a continuous function f such that $L(\mathcal{A}) = f^{-1}(L(\mathcal{B}))$.

- We set $\mathcal{A} <_W \mathcal{B}$ if both $\mathcal{A} \leq_W \mathcal{B}$ and $\mathcal{B} \not\leq_W \mathcal{A}$.
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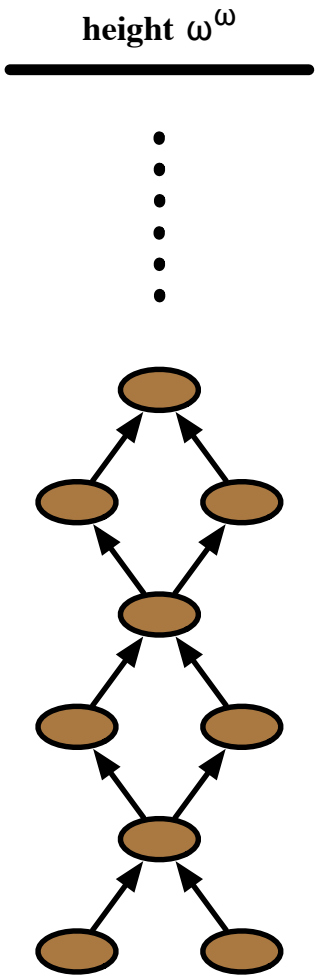
Definition

The collection of all Muller automata (or ω-rational languages) ordered by the relation $\leq_W$ is called the *Wagner hierarchy*.

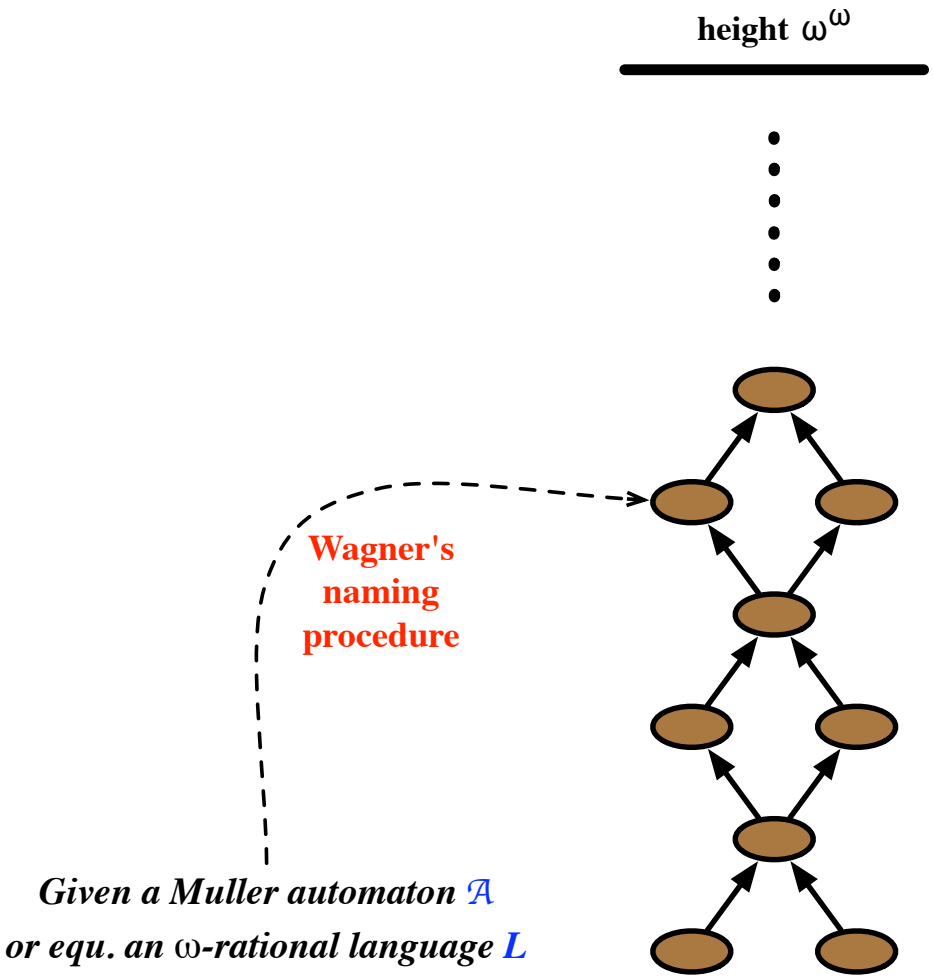
Theorem

The Wagner hierarchy has width 2 and height ω^ω.



Theorem

The Wagner hierarchy is decidable.



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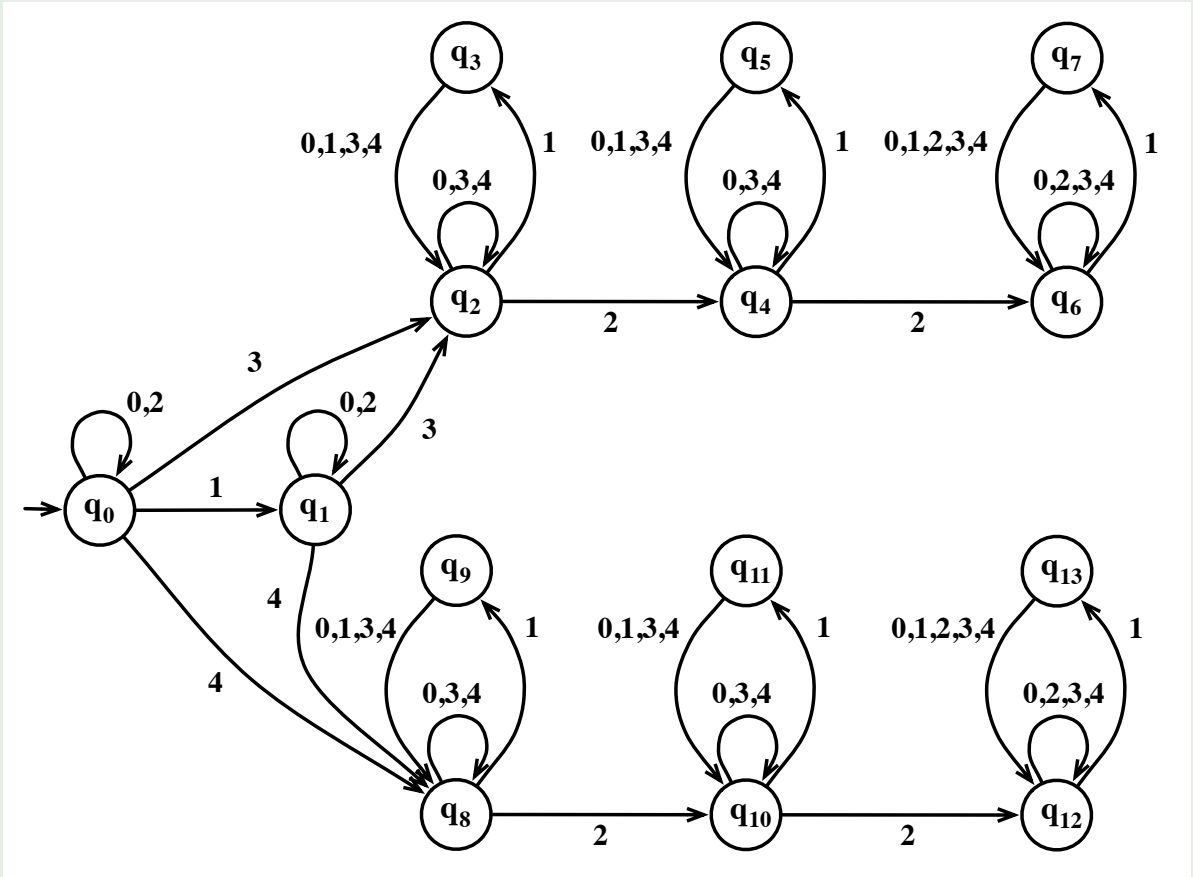

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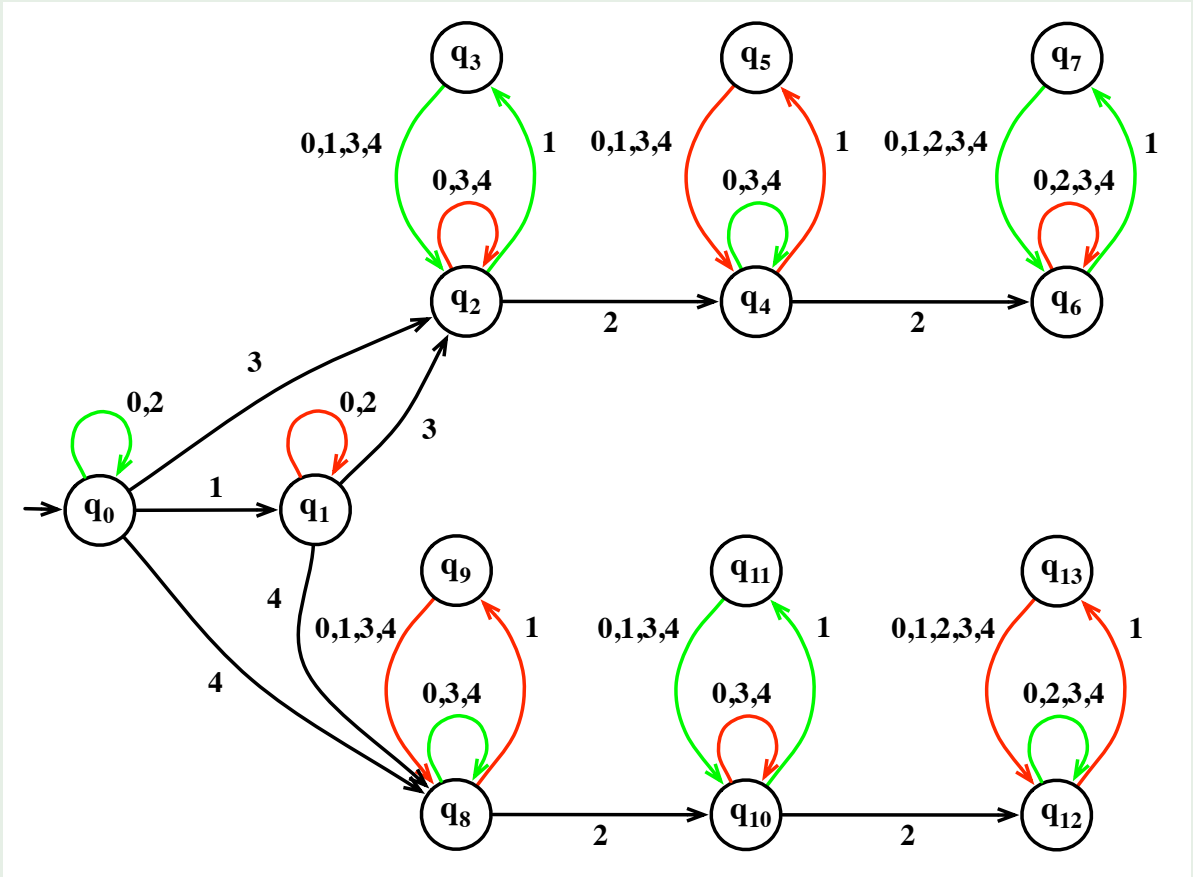

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Main kind abstract Machines reading infinite words:

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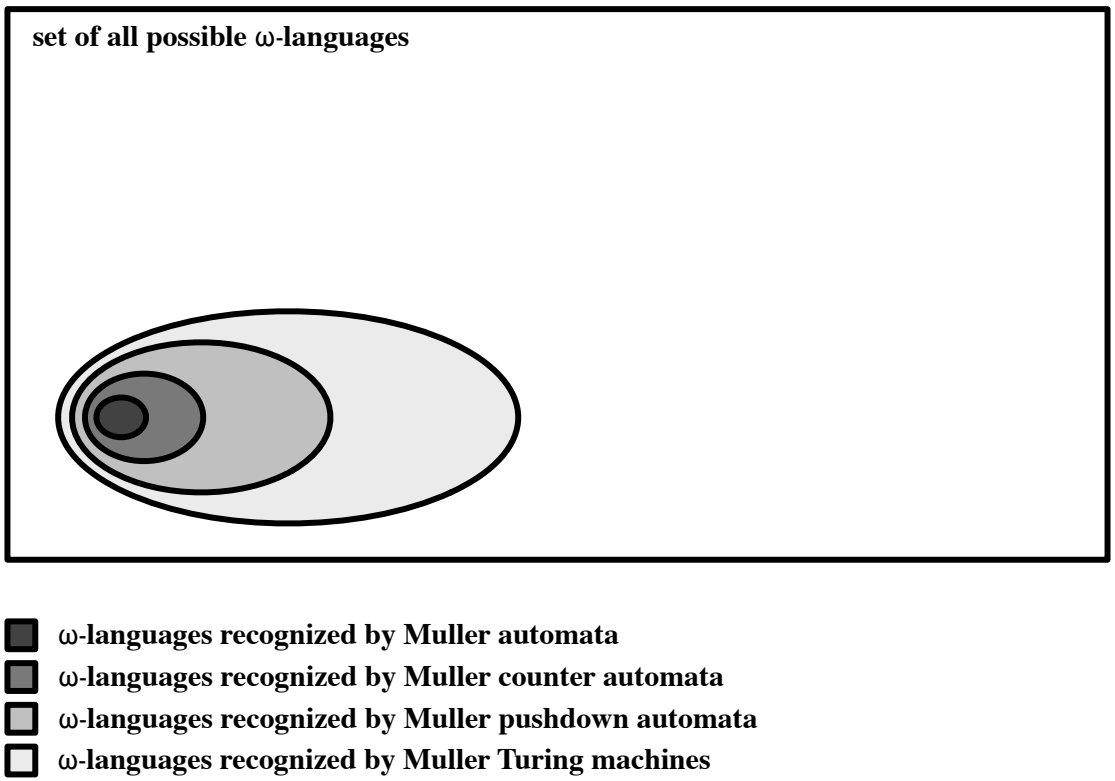
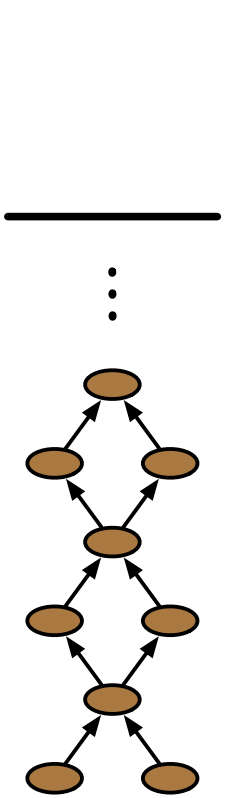
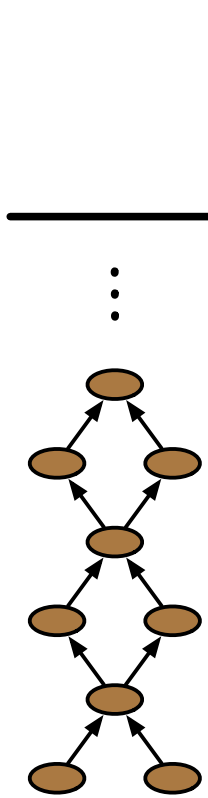
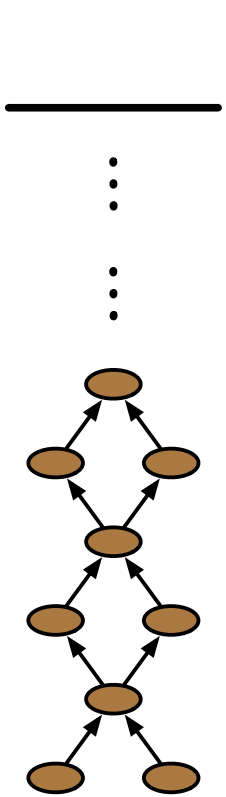
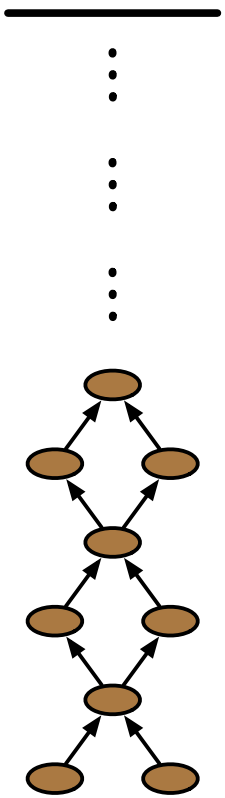


DetMA	DetMCA	DetMPDA	DetMTM
height ω^ω	height $(\omega^2)^\omega = \omega^\omega$	height $(\omega^\omega)^\omega$	height $(\omega_1^{CK})^\omega$
			
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