Finite State Machines and Bio-Inspired Neural Networks

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Introduction

- **Part 1:** We recall some results about the computational capabilities of recurrent neural networks.
- **Part 2:** We introduce a bio-inspired paradigm for neural computation.

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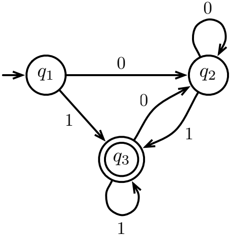
Graph composed of computational states (nodes) and transitions between those (edges).



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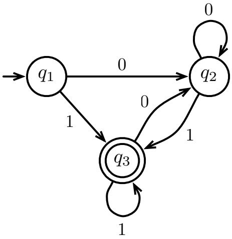

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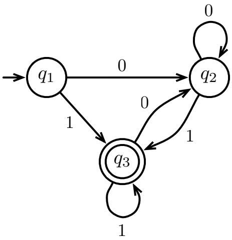

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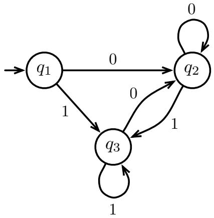
Muller Automaton (infinite words)

FSA with Muller acceptance condition: collection of sets of states.



$$\mathcal{T} = \{\{q_2\}, \{q_2, q_3\}\}$$

- Infinite word u *accepted (rejected)* by $\mathcal{A}$ if the infinite run $\mathcal{A}(u)$ satisfies $\inf(\rho_u) \in \mathcal{T}$ $(\inf(\rho_u) \notin \mathcal{T})$.
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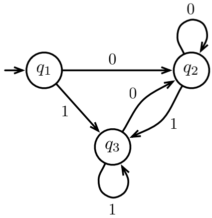

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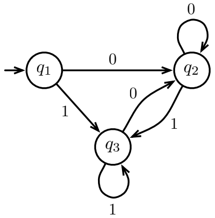

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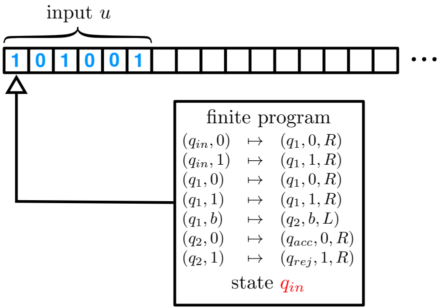

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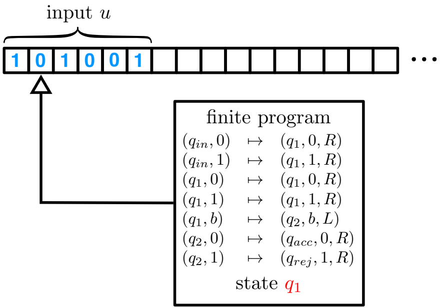

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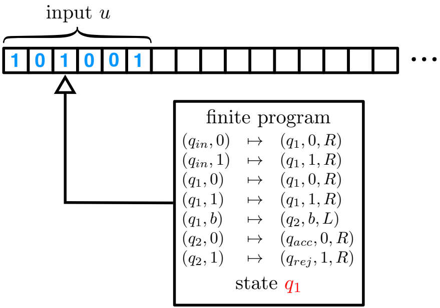

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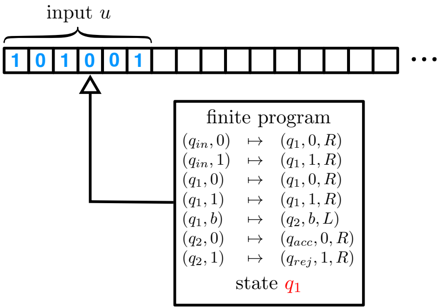

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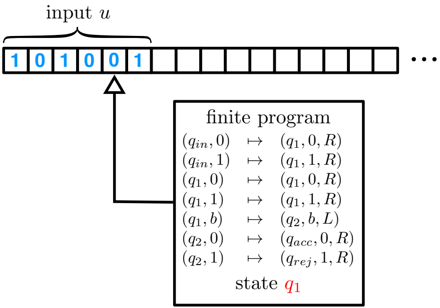

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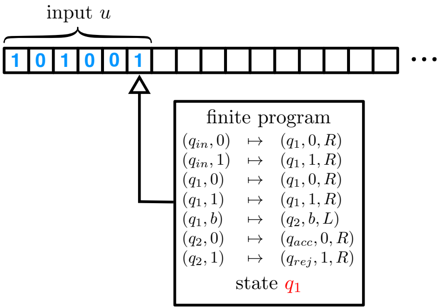

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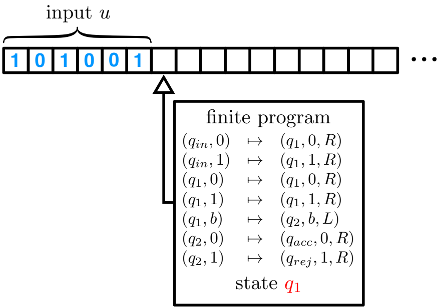

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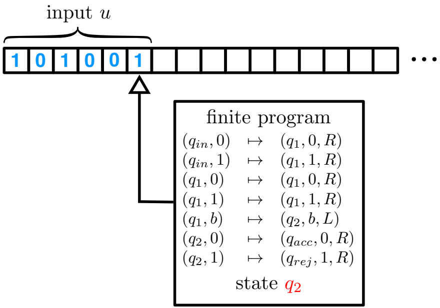

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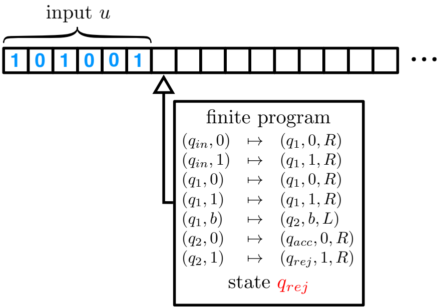

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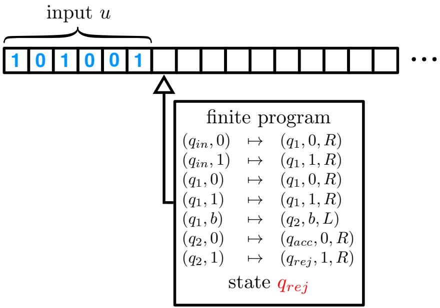

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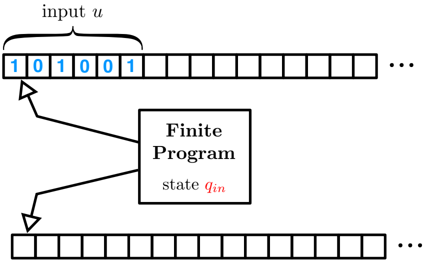
TM with additional tape and advice function $\alpha : \mathbb{N} \longrightarrow \{0,1\}^*$.



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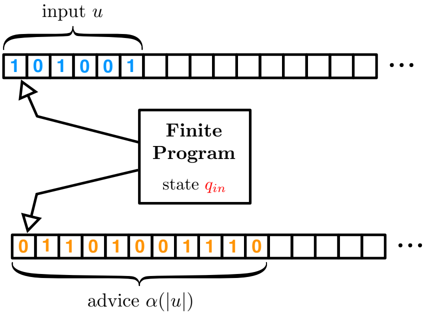

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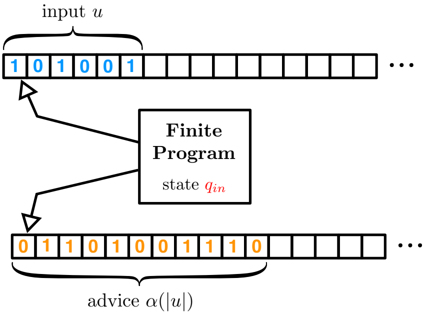

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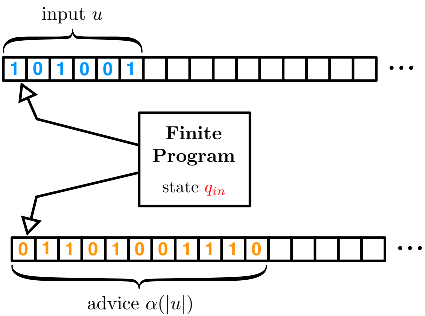

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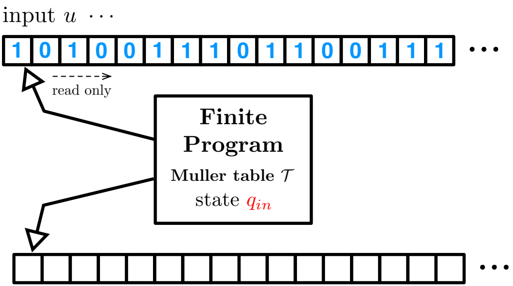

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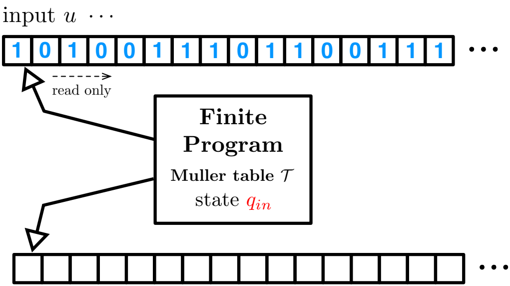

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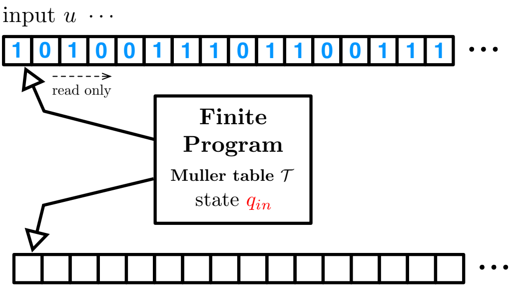

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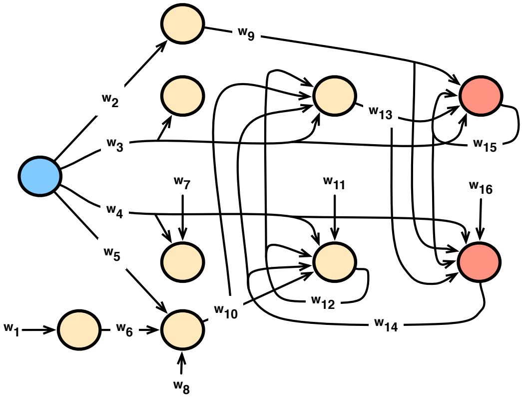
Recurrent Neural Network (RNN)



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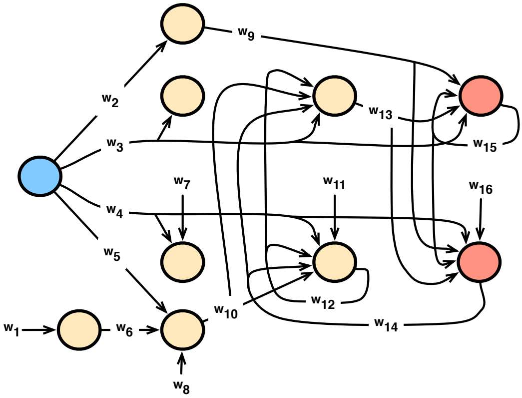

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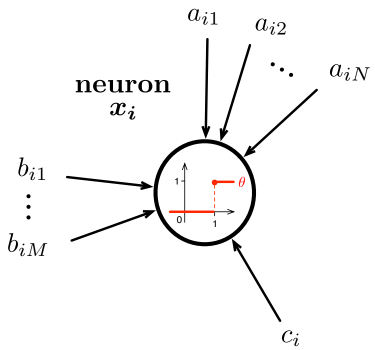
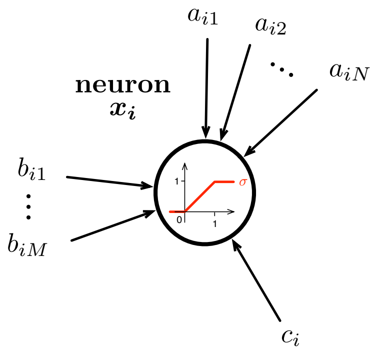
Boolean Recurrent Neural Network



$$x_i(t+1) = \color{red}{\theta}\color{black}{\left(\sum_{j=1}^{N} a_{ij} \cdot x_j(t) + \sum_{j=1}^{M} b_{ij} \cdot u_j(t) + c_i \right)}$$

Sigmoidal Recurrent Neural Network



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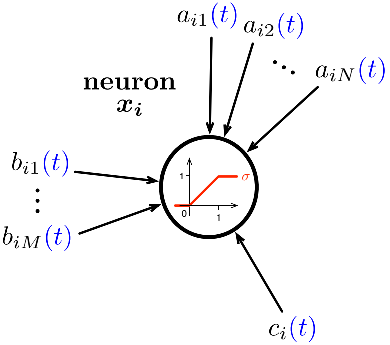
Evolving Recurrent Neural Network



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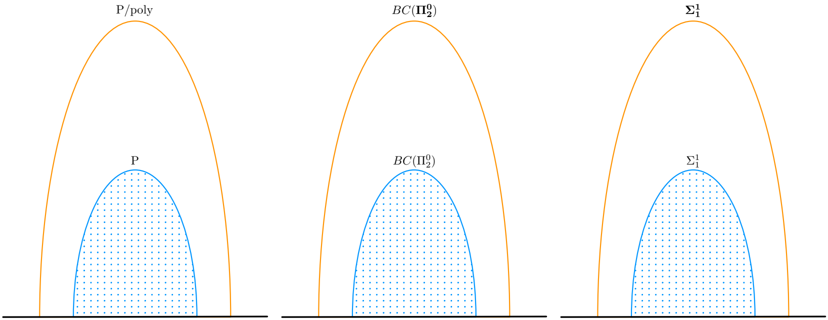
Hierarchy Theorems

- Stratifying the “super-Turing world”.
- Finite and infinite computation: we can define infinitely many complexity classes between the Turing and super-Turing levels.



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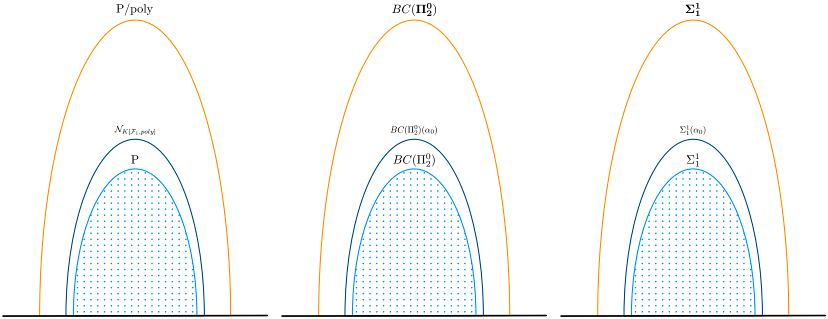

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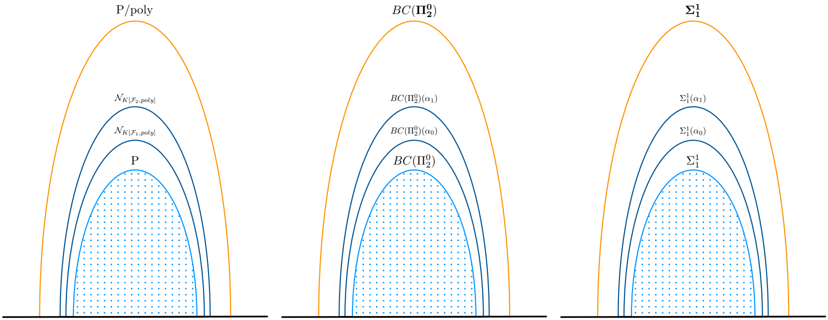

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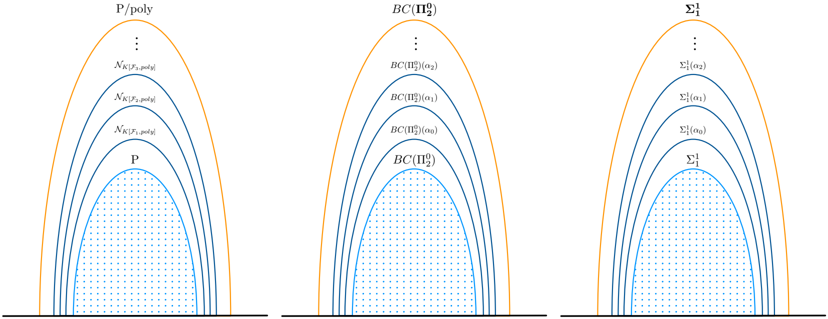

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  - ⋆ Computational states should rather be represented by *sustained activities of neural assemblies*, e.g., by *cyclic attractors*.
- Network is not robust to cell death, synaptic plasticity, architectural plasticity in general.
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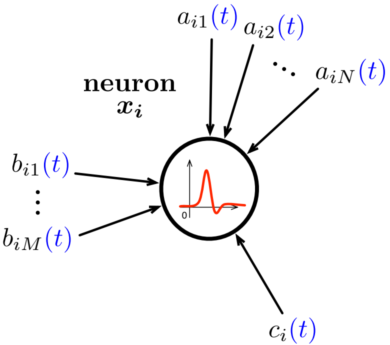
Bio-Inspired Recurrent Neural Network



- Dynamics governed by the Hodgkin-Huxley equations.

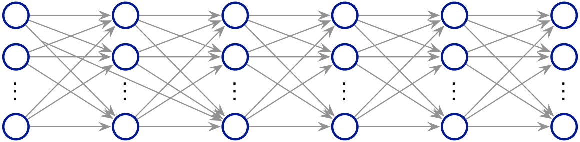
Hodgkin-Huxley Model

$$
\begin{aligned}
C \cdot \frac{dV}{dt} &= -I_L - I_{Na} - I_K - I_C - I_{input} \quad \text{where} \\
I_L &= g_L \cdot (V - V_L) \\
I_{Na} &= g_{Na} \cdot m \cdot h \cdot (V - V_{Na}) \\
& \quad \frac{dm}{dt} = \frac{m_\infty - m}{\tau_m} \qquad m_\infty = \frac{1}{1 + \mathrm{e}^{-s_m \cdot (V - V_{hm})}} \\
& \quad \frac{dh}{dt} = \frac{h_\infty - h}{\tau_h} \qquad h_\infty = 1 - \frac{1}{1 + \mathrm{e}^{-s_h \cdot (V - V_{hh})}} \\
I_K &= g_K \cdot n \cdot (V - V_K) \\
& \quad \frac{dn}{dt} = \frac{n_\infty - n}{\tau_n} \qquad n_\infty = \frac{1}{1 + \mathrm{e}^{-s_n \cdot (V - V_{hn})}} \\
I_C &= w_{intra}^{exc} + w_{inter}^{exc} + w_{inter}^{inh} + w_{output}^{exc} + w_{output}^{inh} \\
I_{input} &= a_{input}^{exc} \cdot \chi t_{input}(t)
\end{aligned}
$$

V: membrane potential; C membrane capacitance; I_L: leakage current; I_{Na}, I_K: sodium and the potassium fast currents; I_C: synaptic currents coming from the neighboring neurons; I_{input}: pulse-like input current. **(show simulator)**

Synfire Chains

- *Synfire chains* have been theorized as fundamental neuronal structures (Abeles 82).
- They allow for robust and highly precise transmission of information in neural networks.



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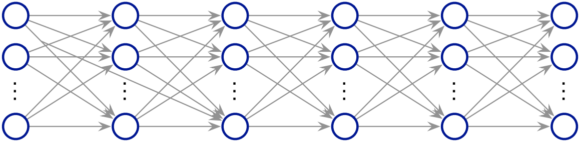

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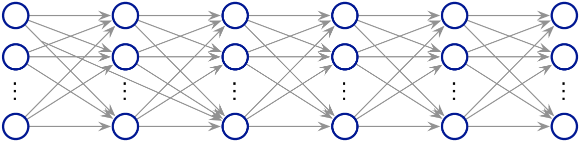

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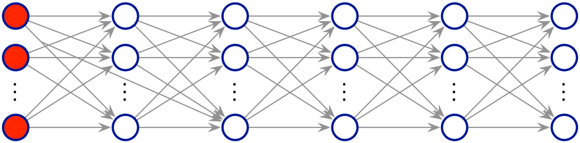

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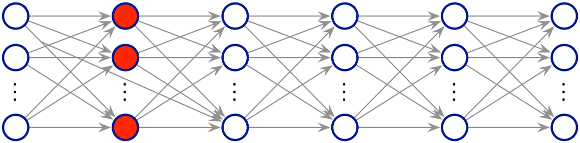

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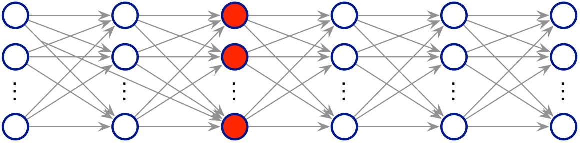

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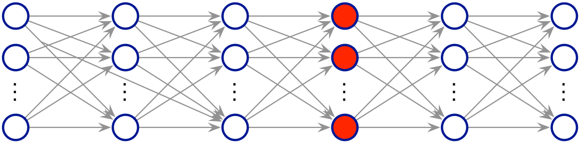

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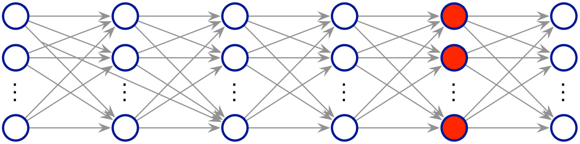

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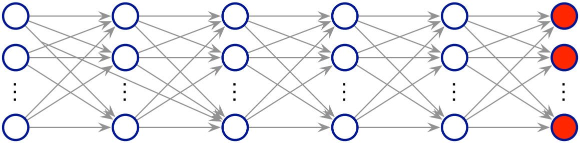

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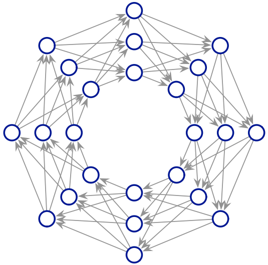

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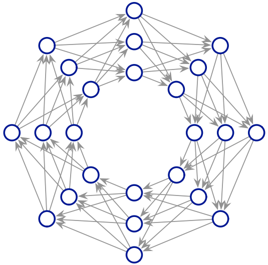

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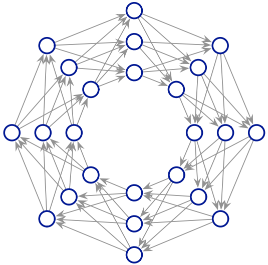

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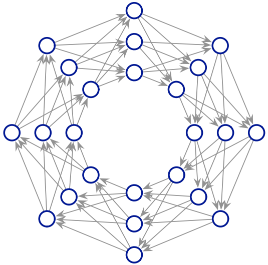

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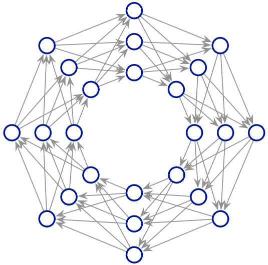

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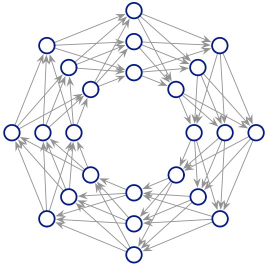

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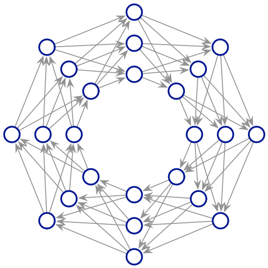

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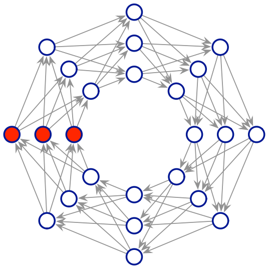

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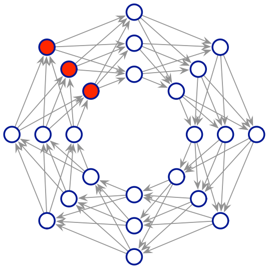

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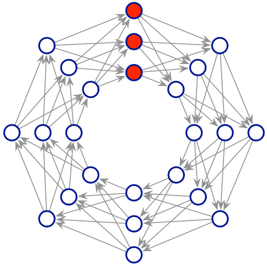

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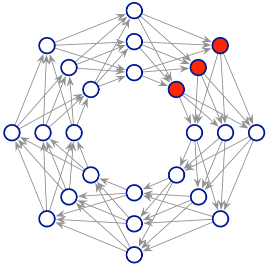

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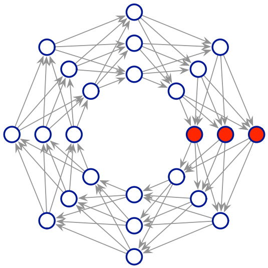

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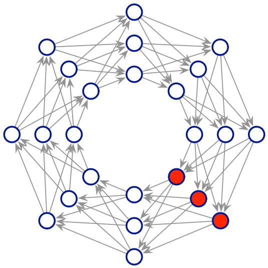

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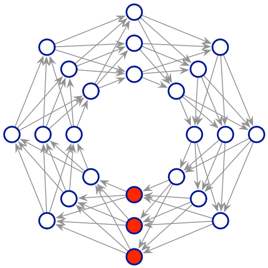

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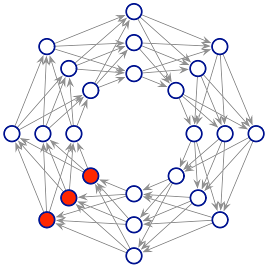

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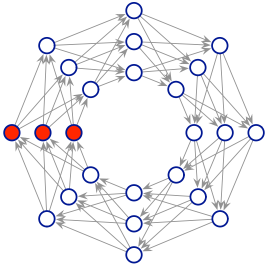

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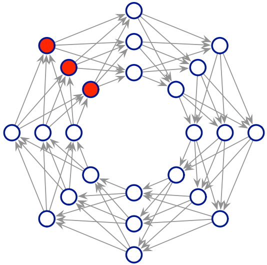

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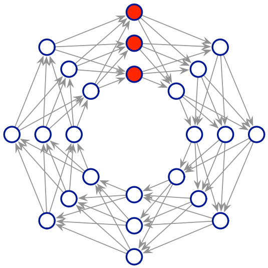
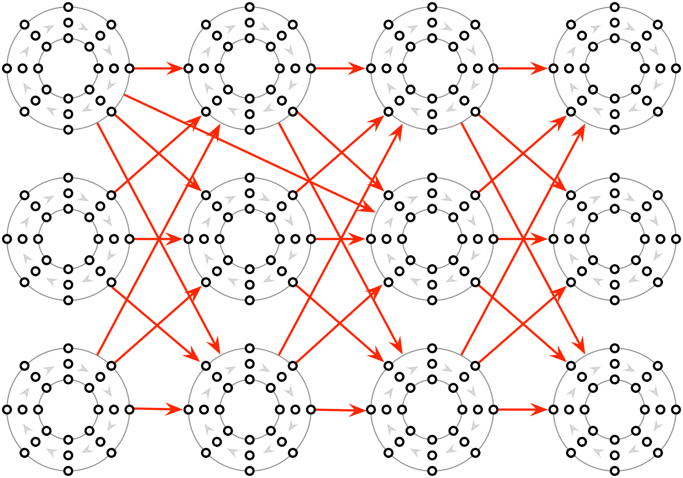
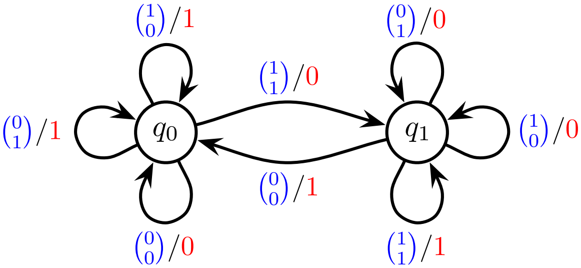


Synfire Ring Architecture



Binary Adder Automaton



$$\begin{array}{cccccccc} & {}^{1} & 1^{1} & 0^{1} & 1 & 1 & 0^{1} & 1 \\ + & & 1 & 1 & 1 & 0 & 0 & 1 \\ \hline & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{array}$$

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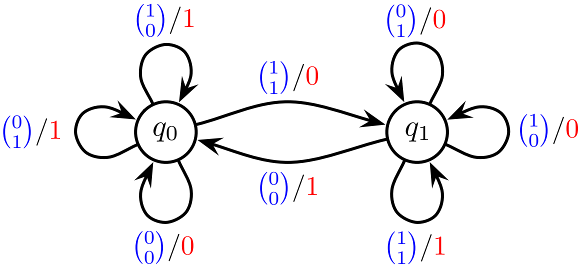

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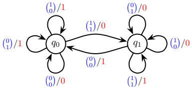

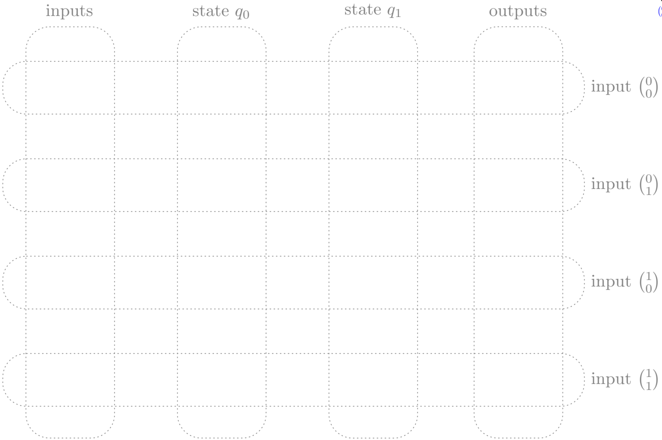

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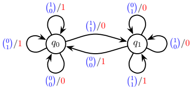

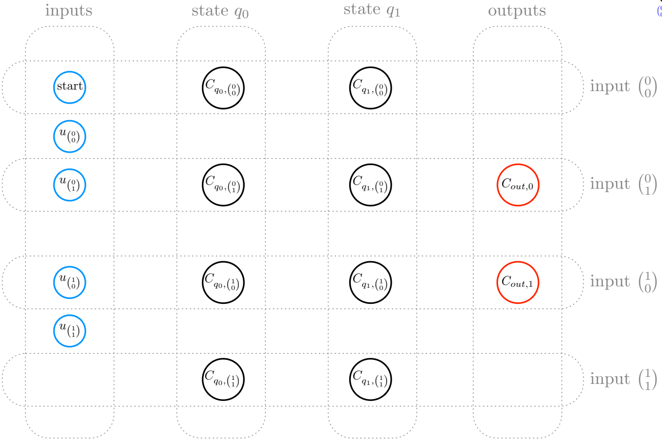

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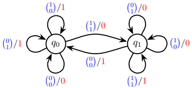

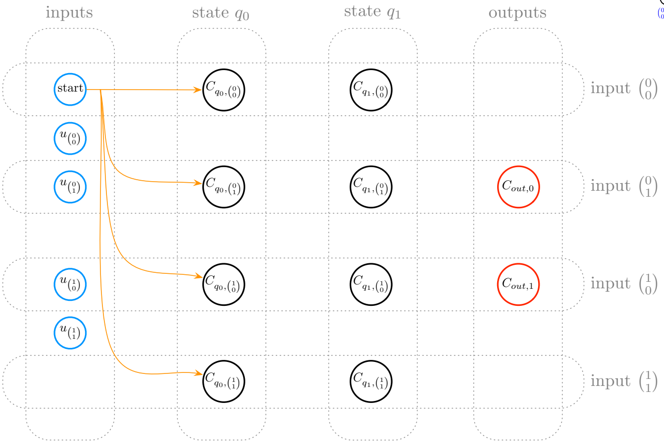

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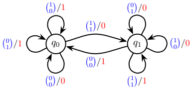

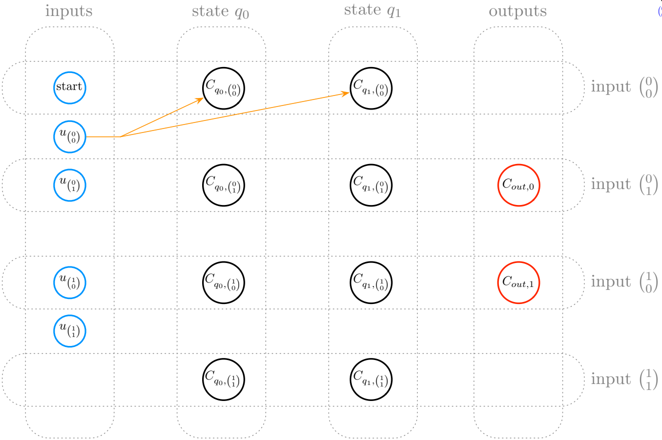

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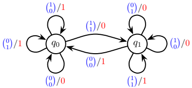

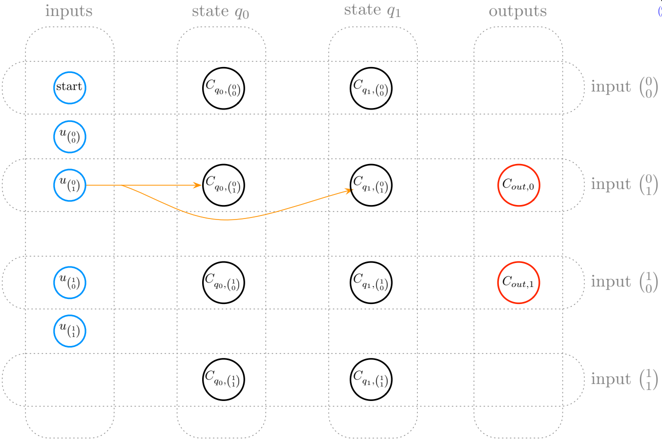

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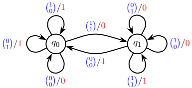

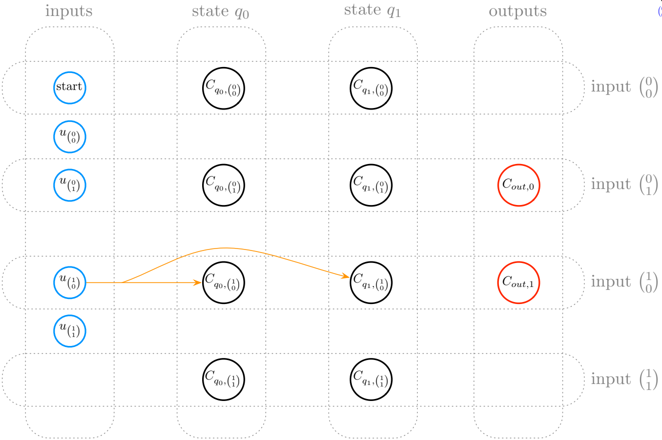

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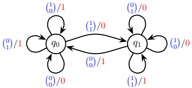

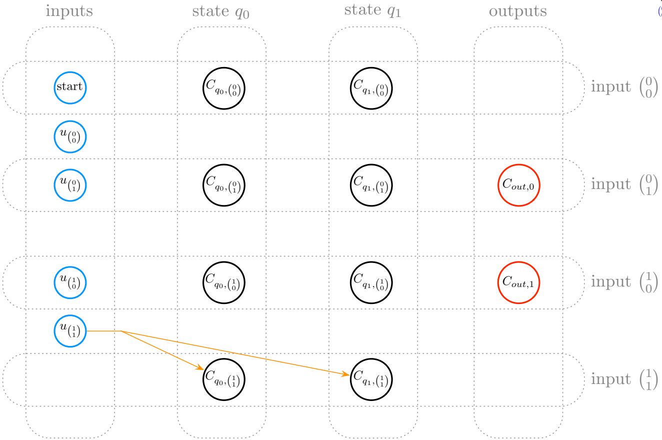

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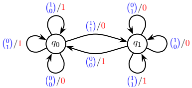

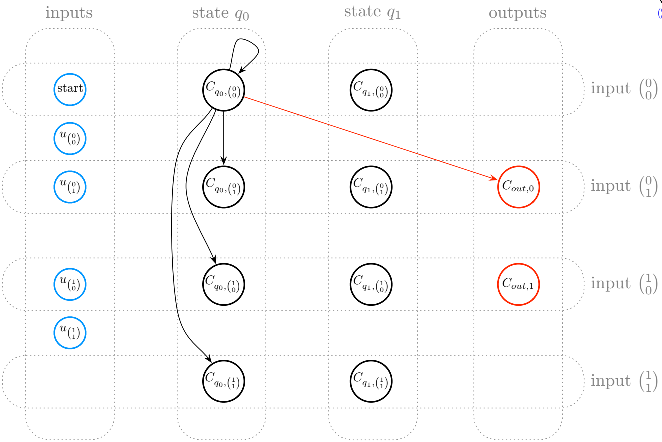

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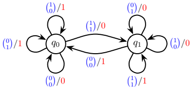

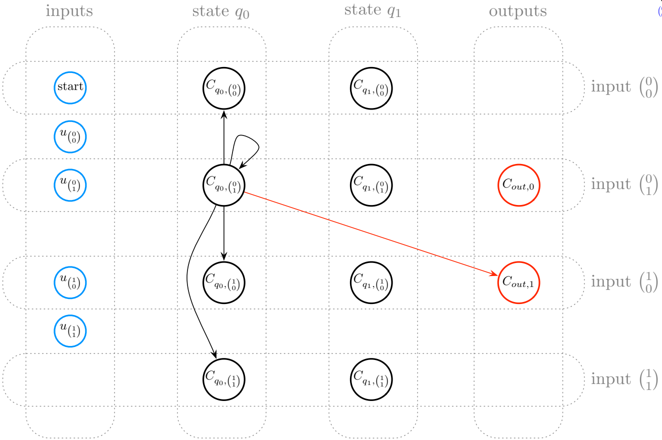

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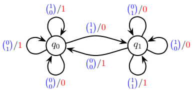

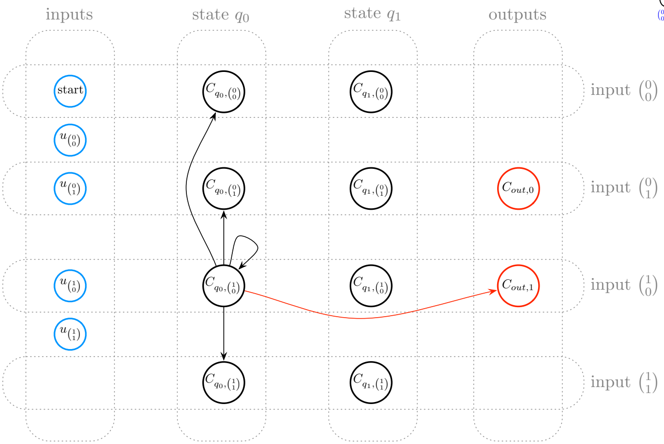

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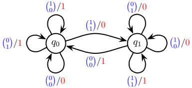

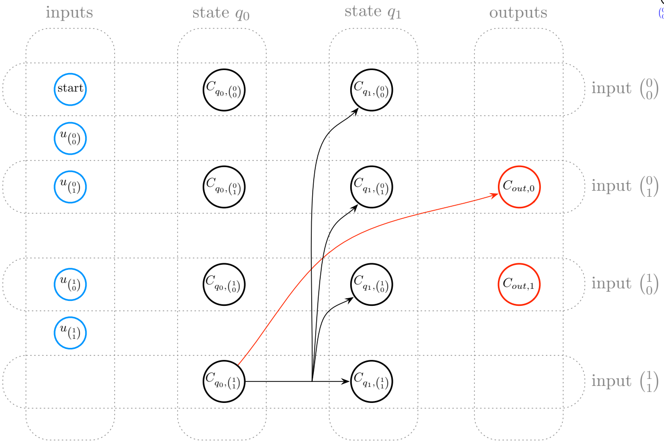

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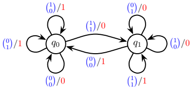

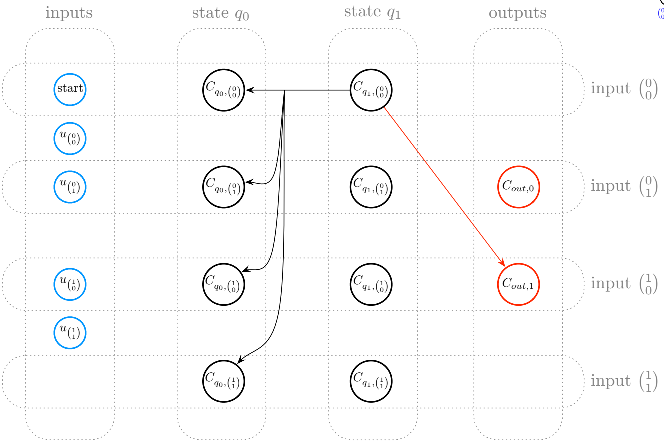

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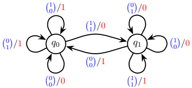

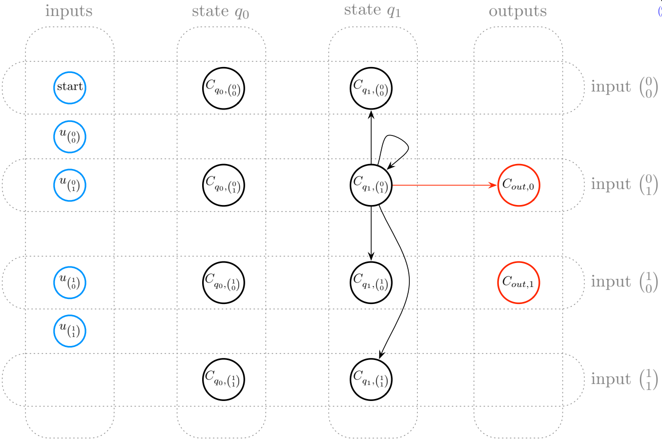

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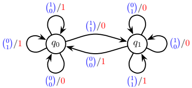

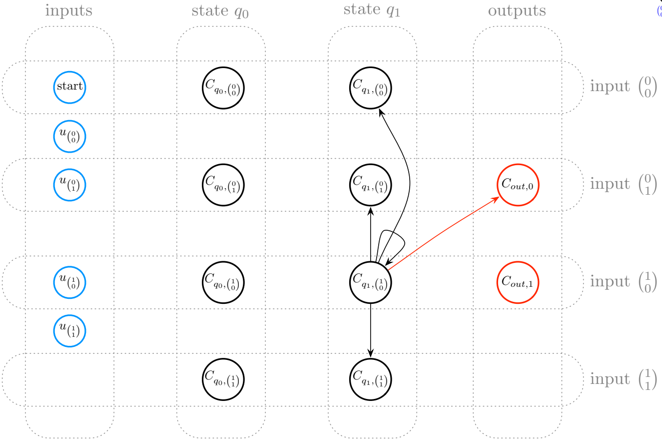

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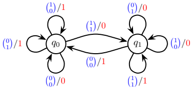
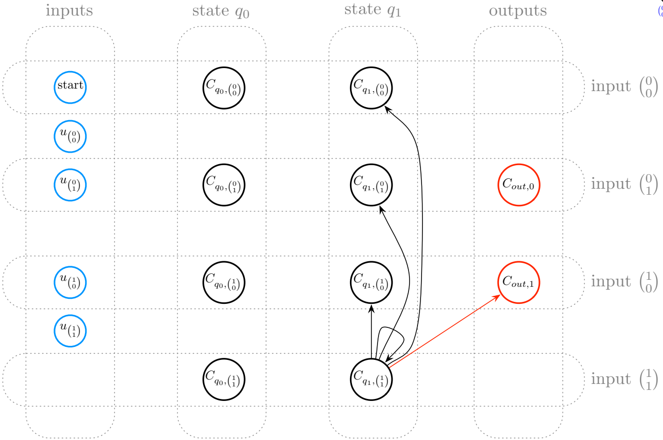
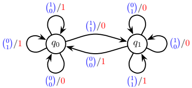
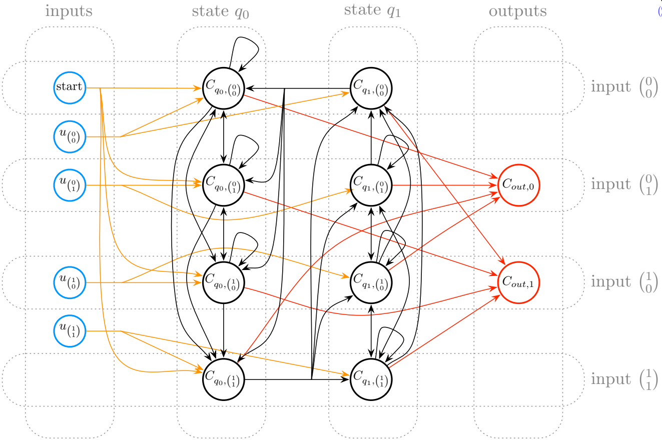


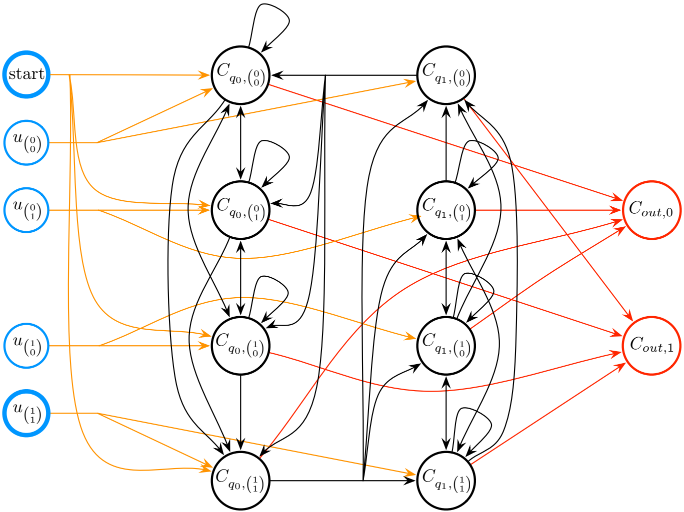


Binary Adder Boolean Neural Network





Simulation



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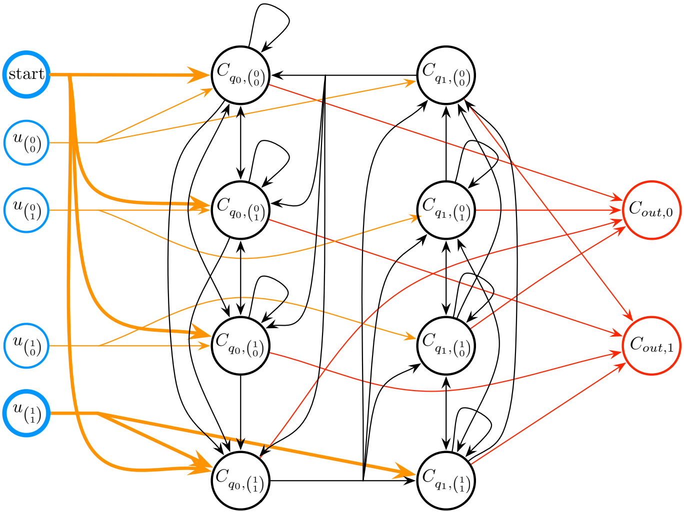
Simulation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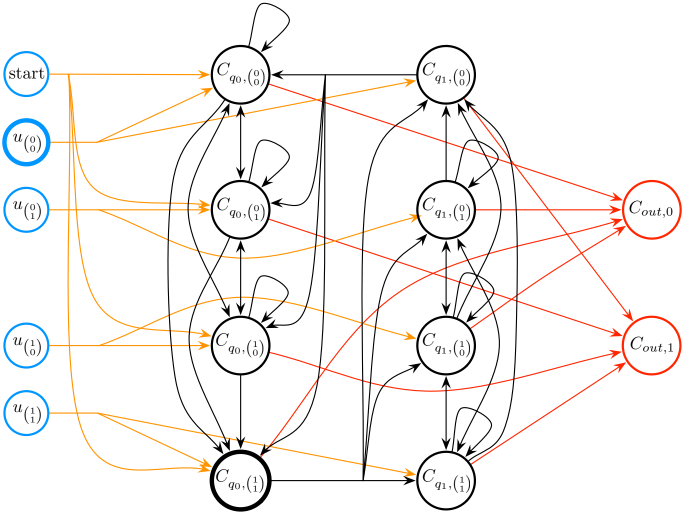
Simulation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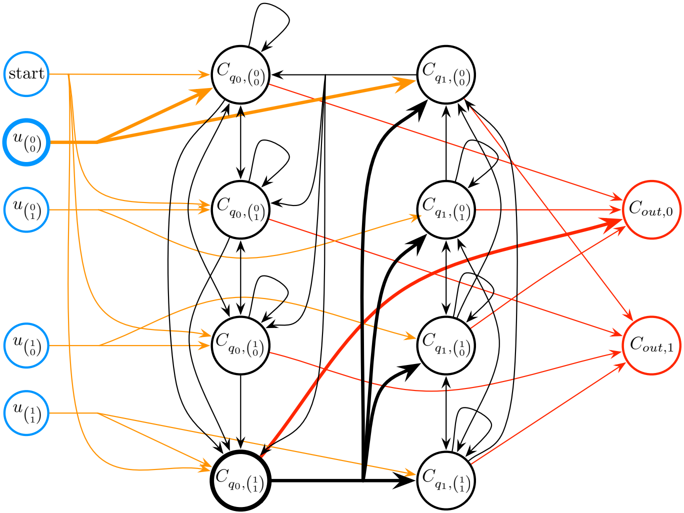
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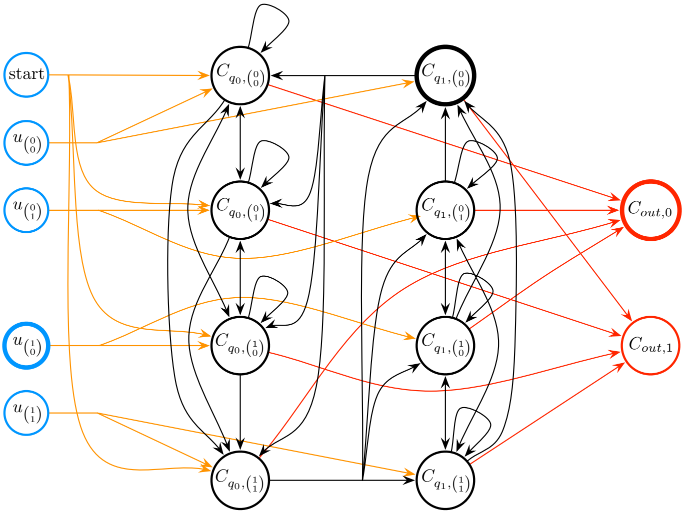
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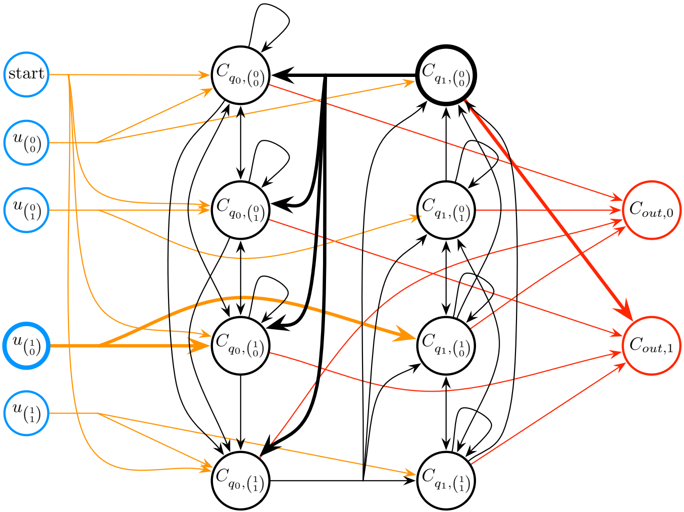
Simulation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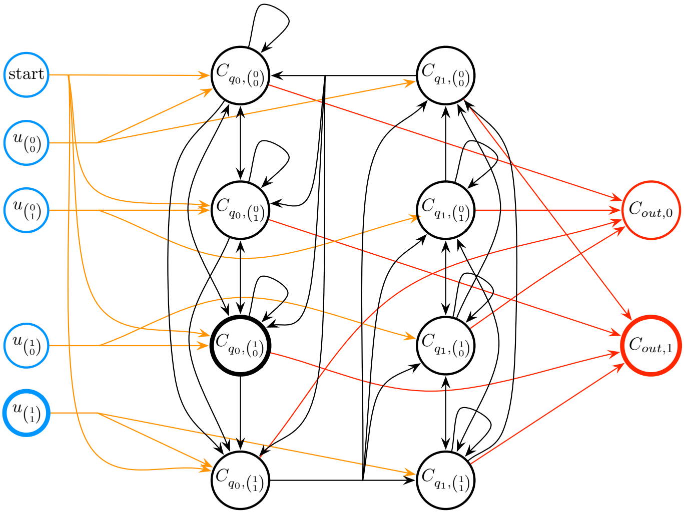

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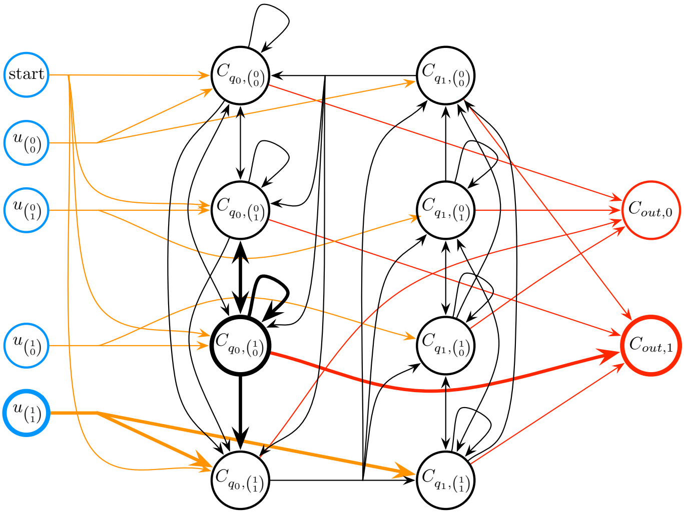
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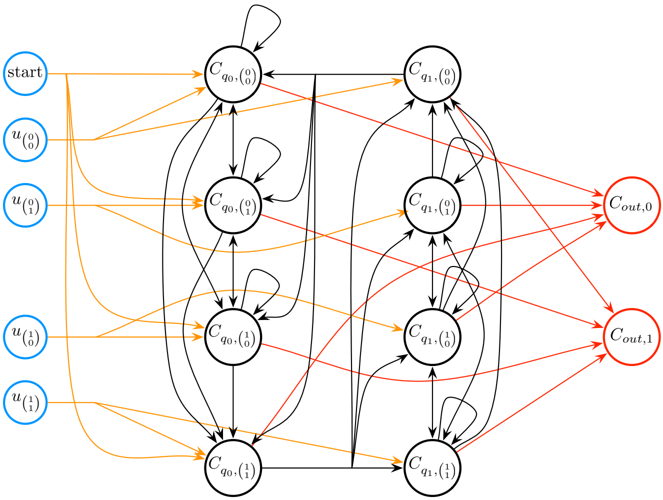
Generalization to Synfire Ring RNNs



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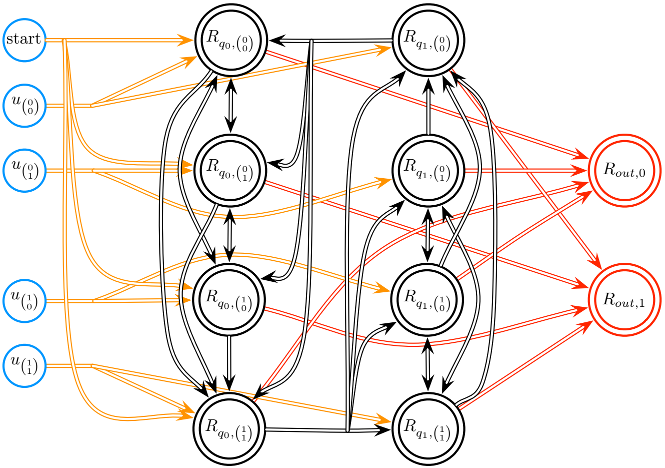
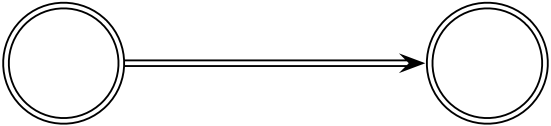


Transition of ring activities: fibres of excitatory & Inhibitory connections

- The *refractory period* of the cells allows for a natural inhibitory system.



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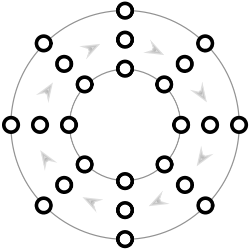

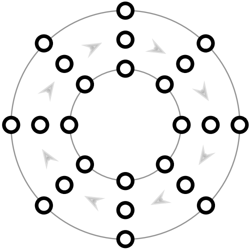

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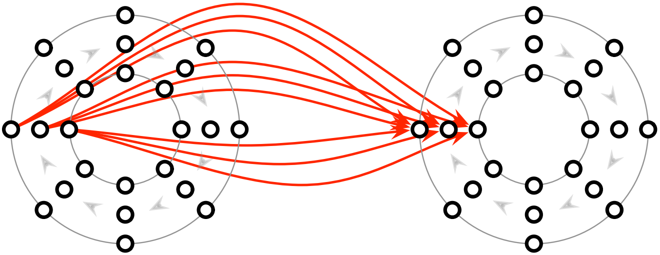

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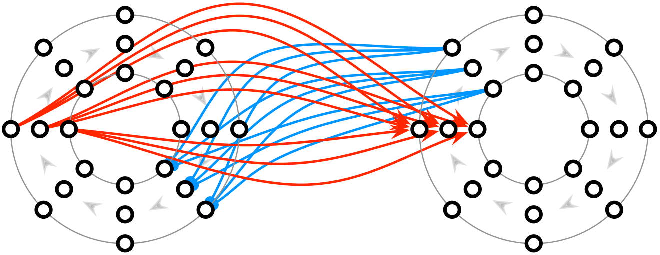

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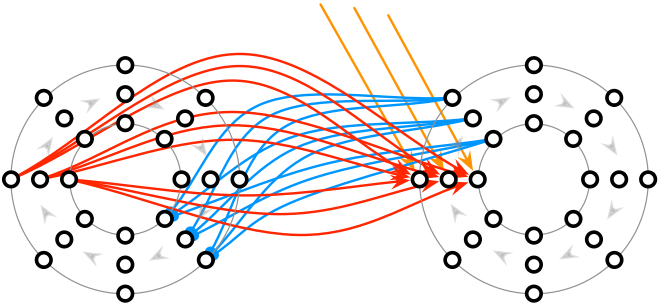

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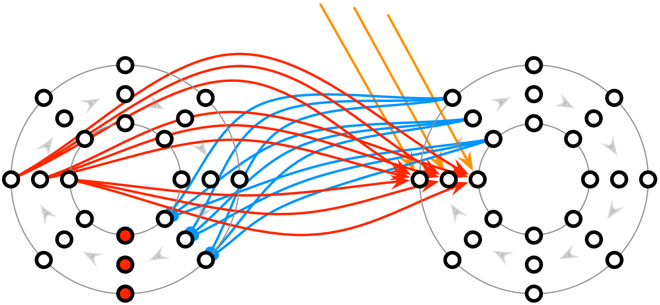

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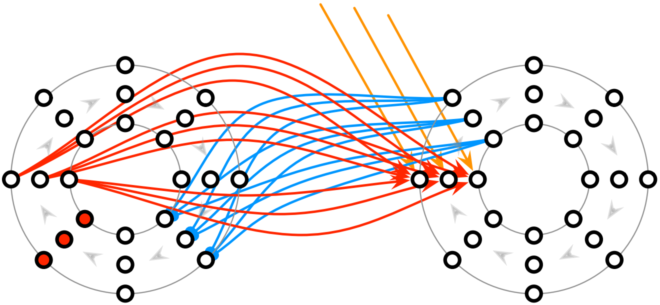

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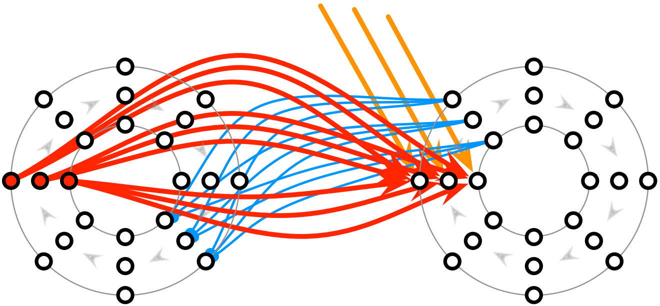

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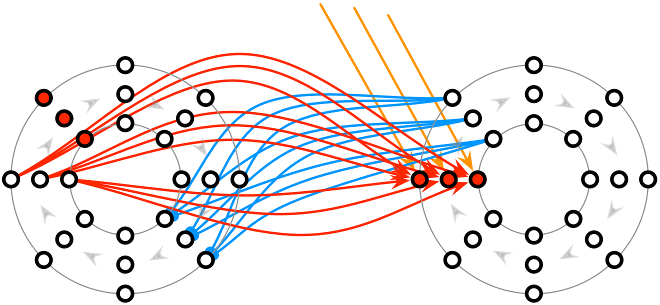

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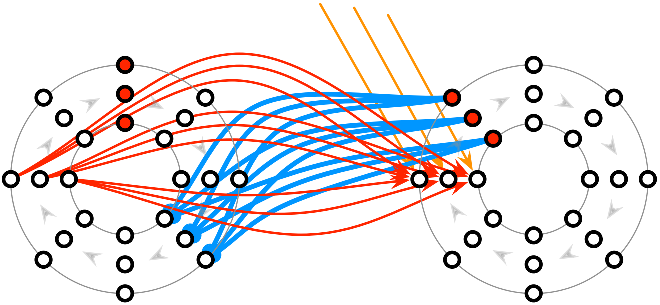

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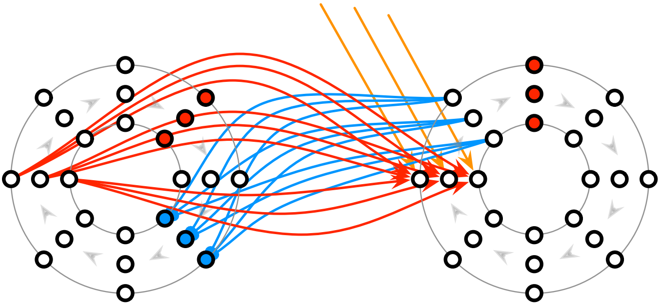

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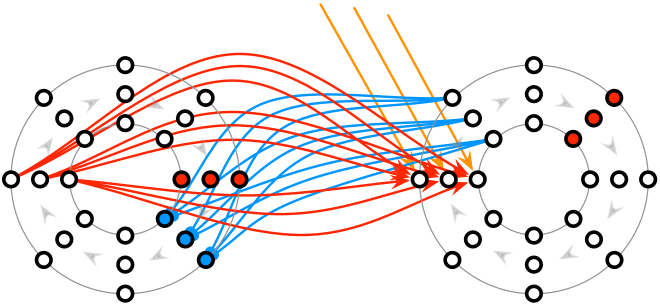

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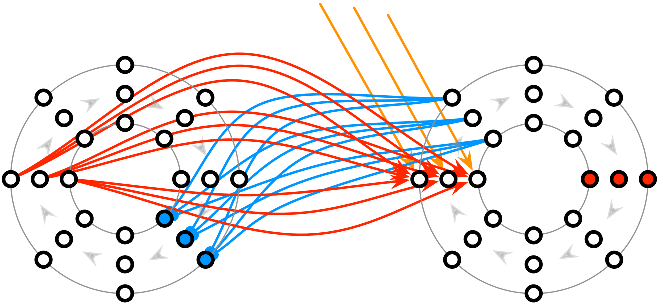

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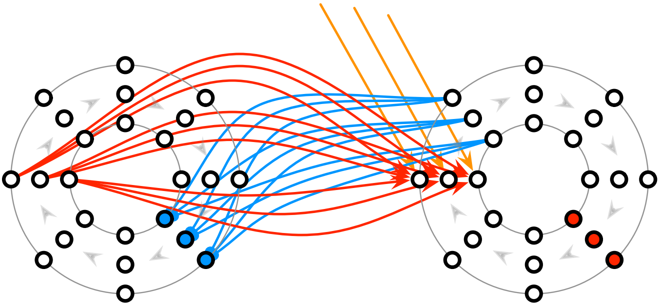

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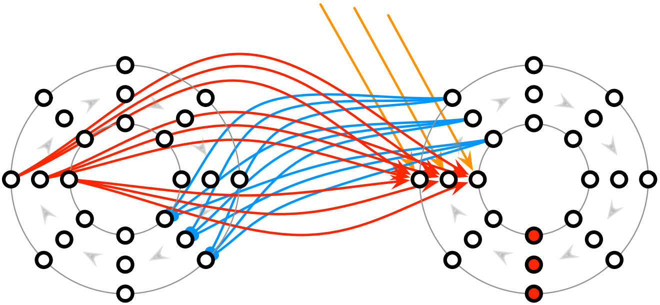

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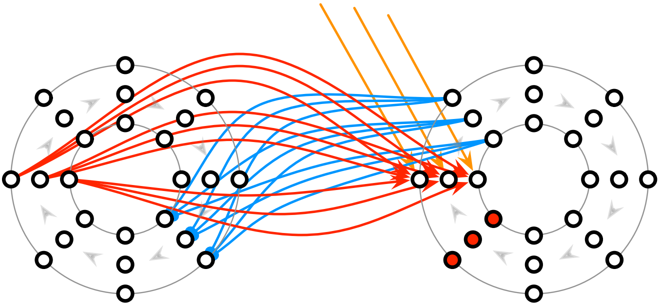
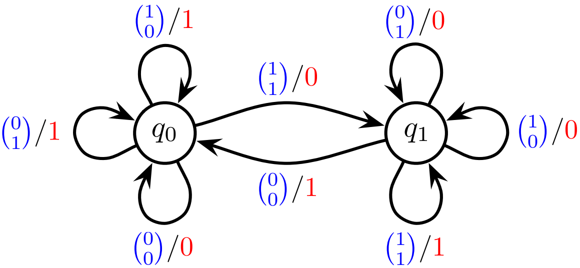


Simulation



Play movie

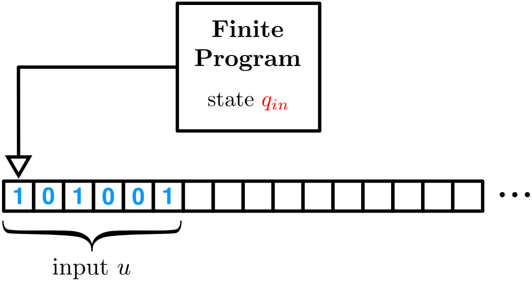
Automata & Hodgkin-Huxley RNNs with Synfire Rings

Since the construction is generic, the following result ensues:

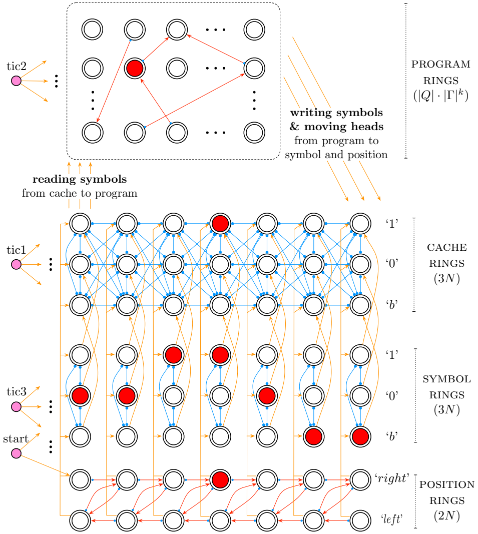
Theorem

Any finite state automaton can be simulated by some Hodgkin-Huxley based neural network composed of synfire rings.

TM and RNN

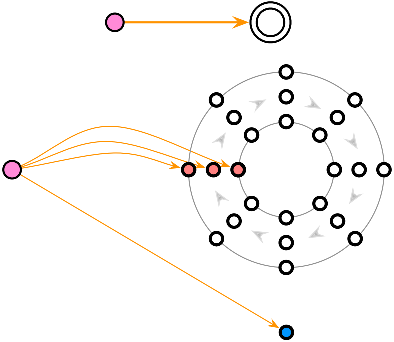


TM and RNN


tic2
writing symbols
& moving heads
from program to
symbol and position
PROGRAM
RINGS
$(|Q| \cdot |\Gamma|^k)$
reading symbols
from cache to program
tic1
tic3
start
'1'
'0'
'b'
CACHE
RINGS
$(3N)$
'1'
'0'
'b'
SYMBOL
RINGS
$(3N)$
'right'
'left'
POSITION
RINGS
$(2N)$

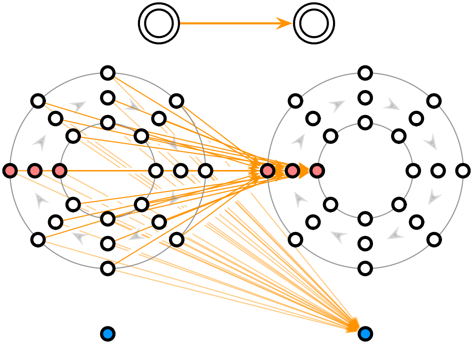
Fibres of connections

- Cell-to-ring one-shot excitation



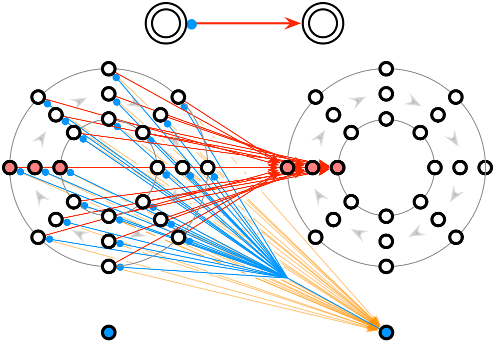
Fibres of connections

- Ring-to-ring constant excitation



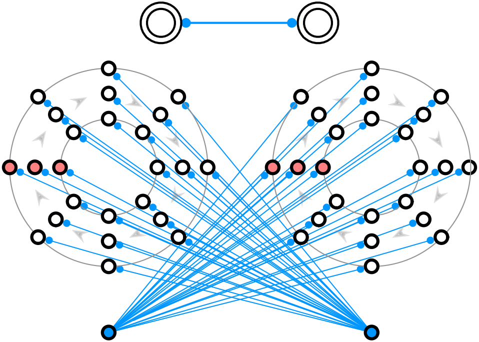
Fibres of connections

- Ring-to-ring constant excitation / one-shot inhibition

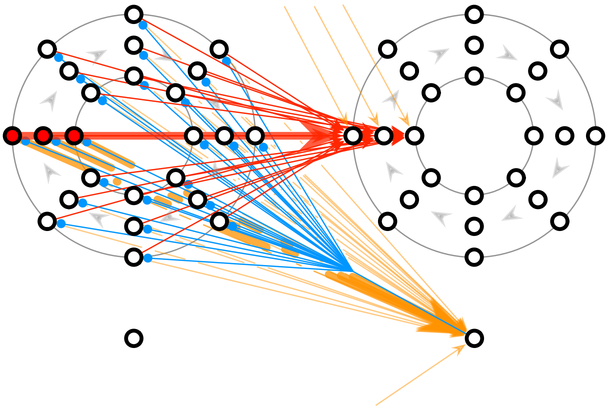


Fibres of connections

- Ring-to-ring bidirectional one-shot inhibition



Transition of ring activities



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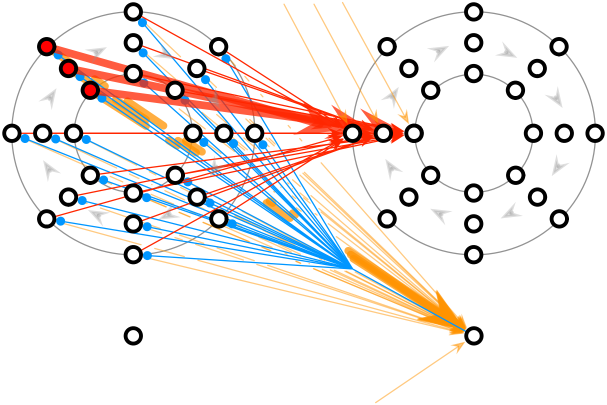

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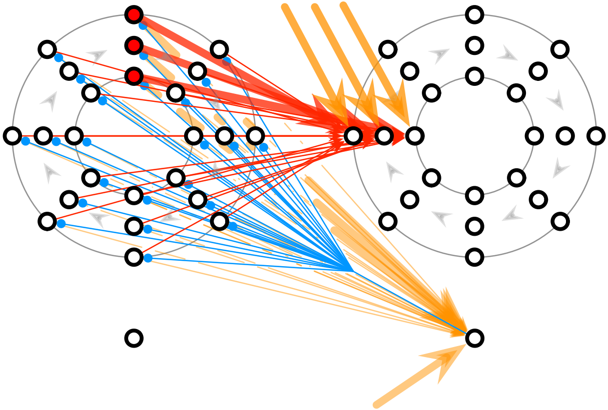

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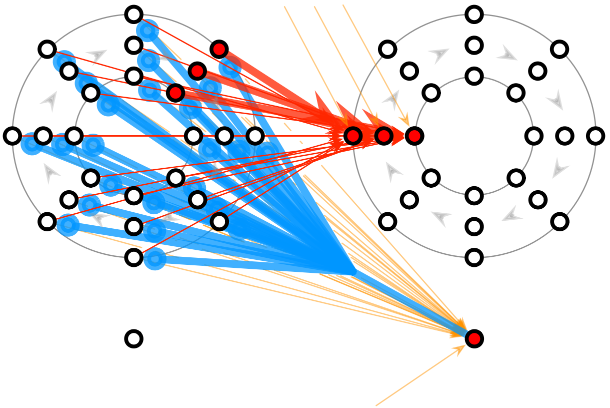

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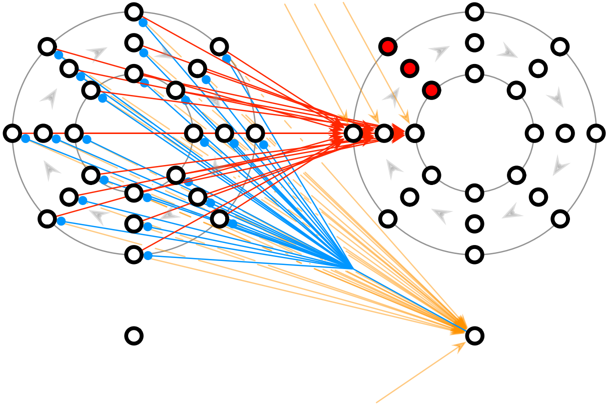

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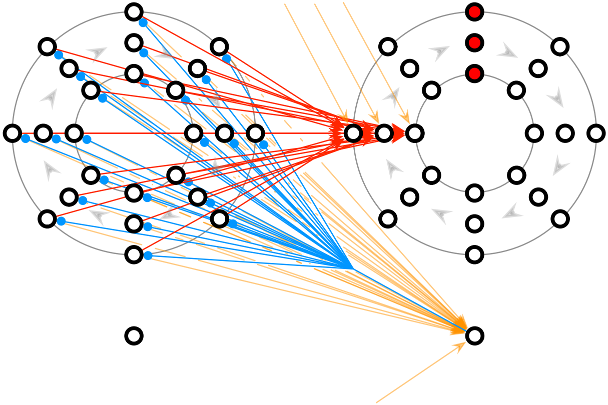

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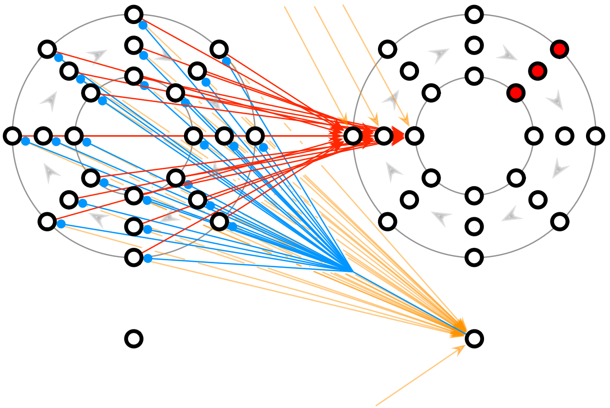
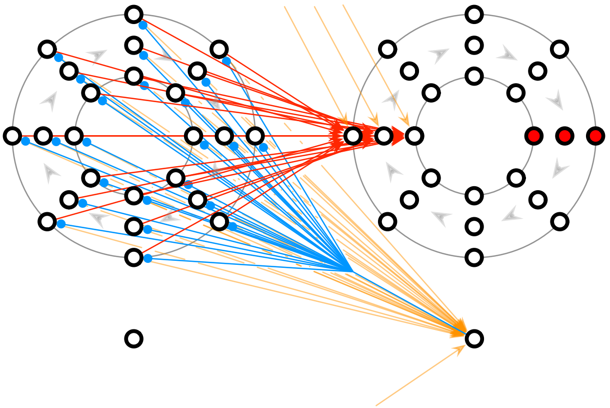
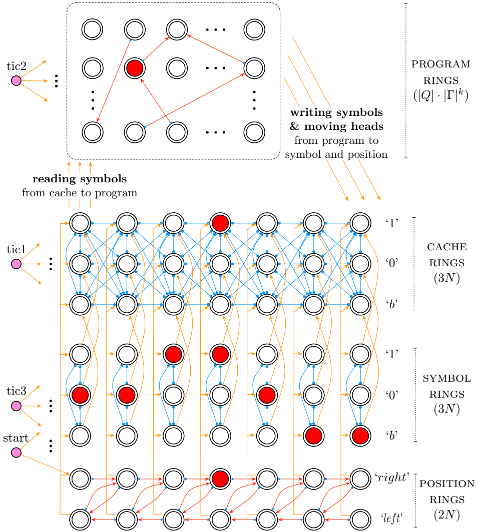


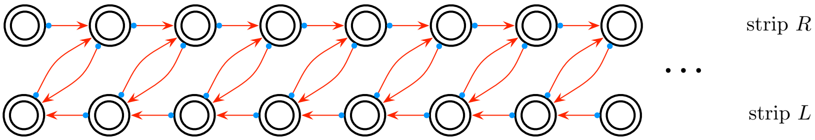
Transition of ring activities



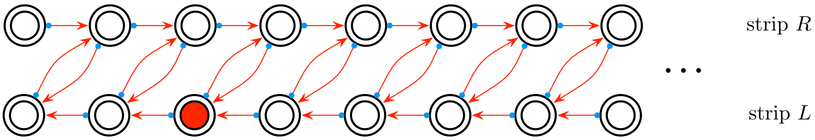
Position Rings



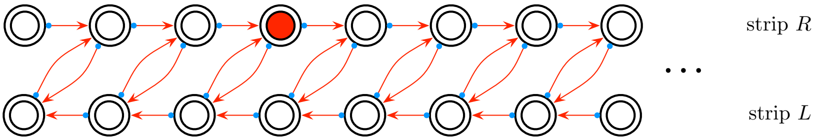
Position Rings



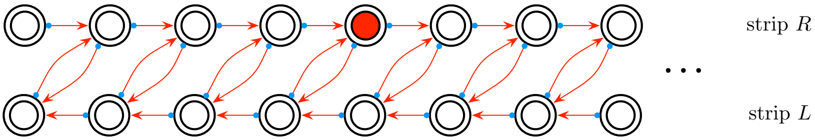
Position Rings



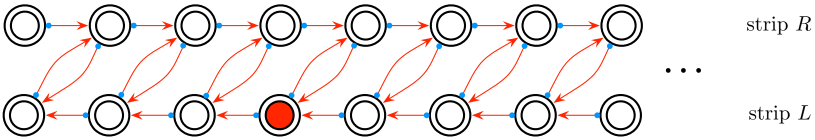
Position Rings



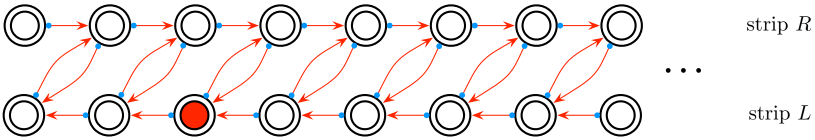
Position Rings



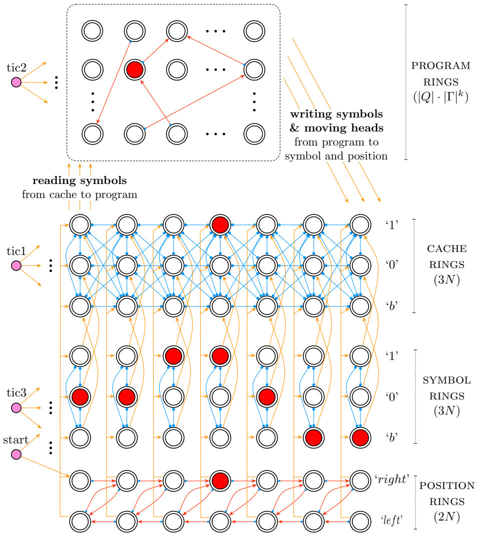
Position Rings



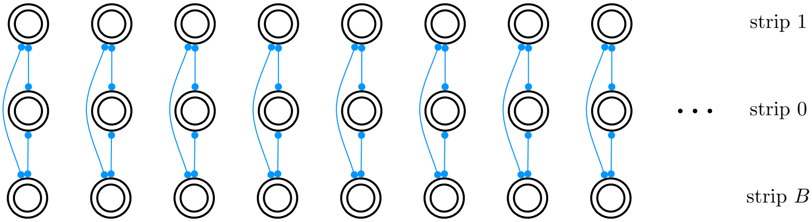
Position Rings



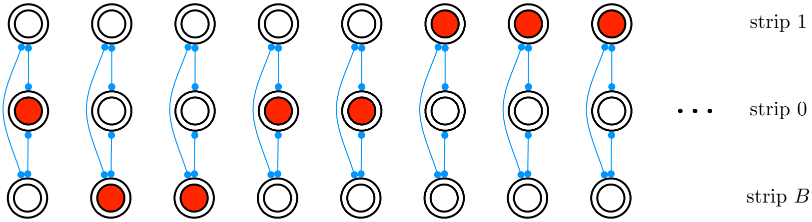
Symbol Rings



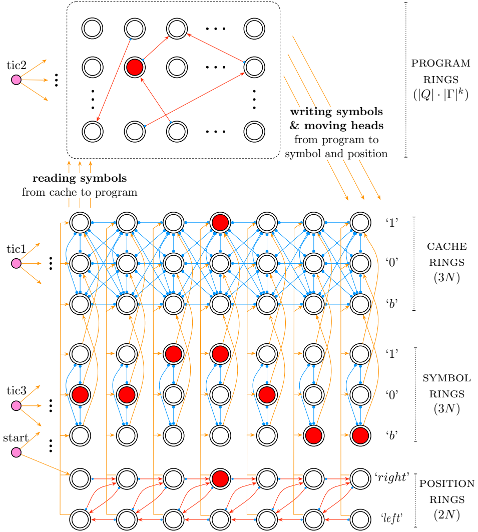
Symbol Rings



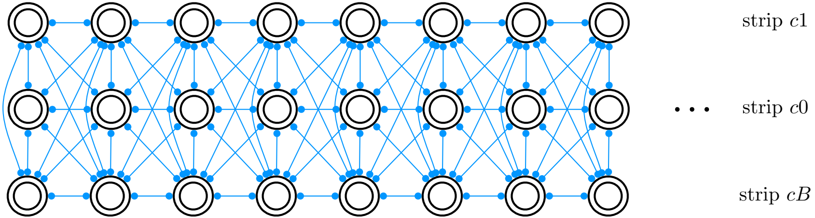
Symbol Rings



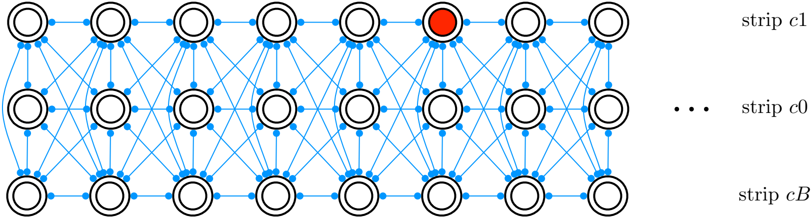
Cache Rings


tic2
tic1
tic3
start
PROGRAM
RINGS
$(|Q| \cdot |\Gamma|^k)$
writing symbols
& moving heads
from program to
symbol and position
reading symbols
from cache to program
'1'
'0'
'b'
CACHE
RINGS
$(3N)$
'1'
'0'
'b'
SYMBOL
RINGS
$(3N)$
'*right*'
'*left*'
POSITION
RINGS
$(2N)$

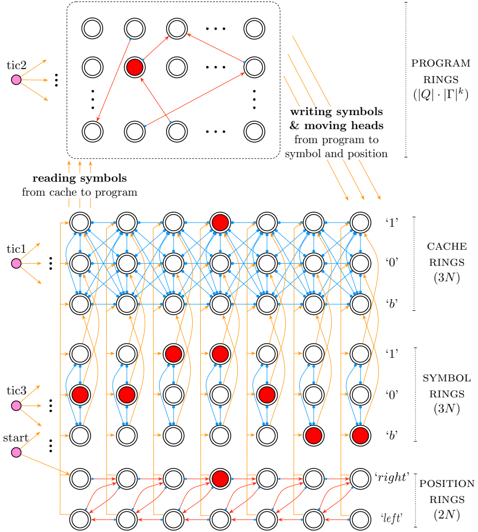
Cache Rings



Cache Rings



Position-Symbol-Cache Connections



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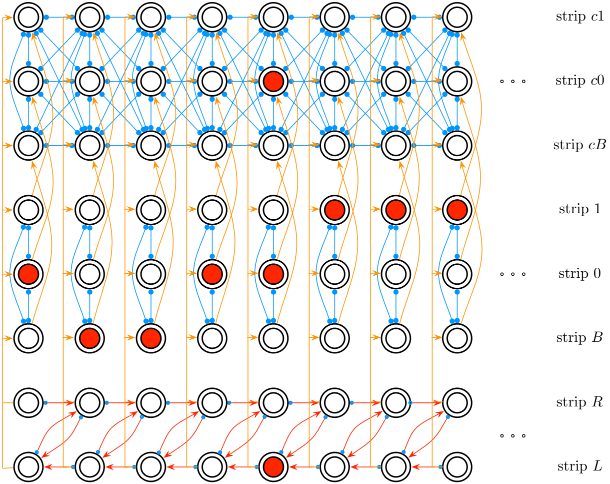

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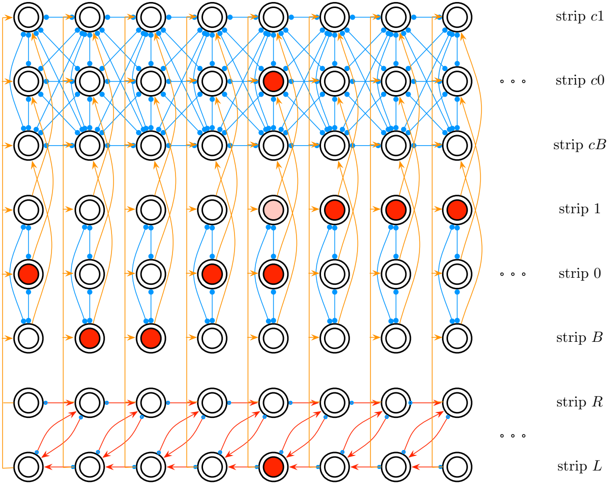

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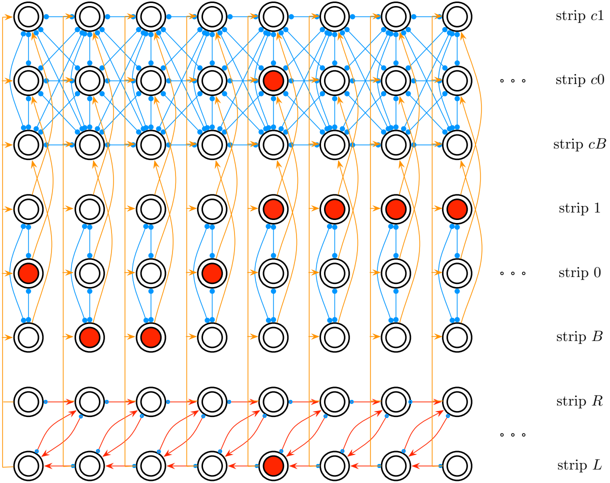

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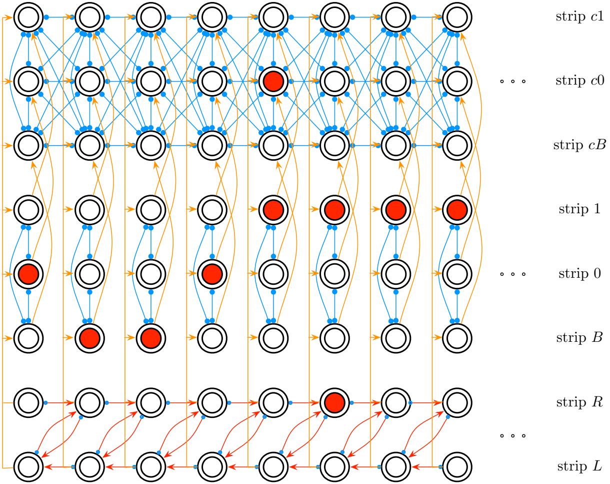

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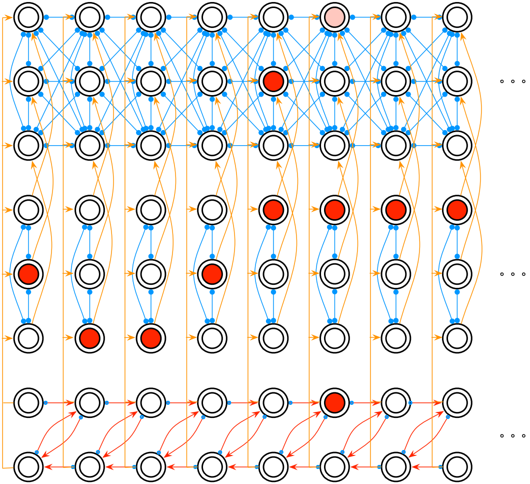

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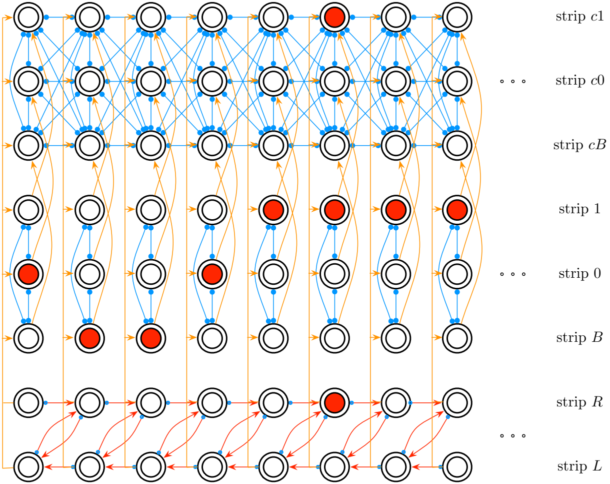
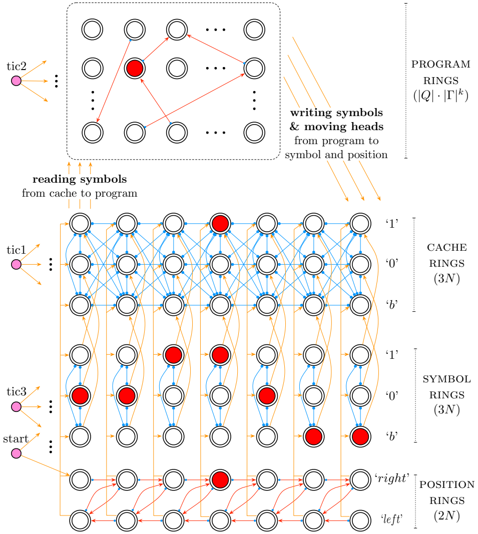
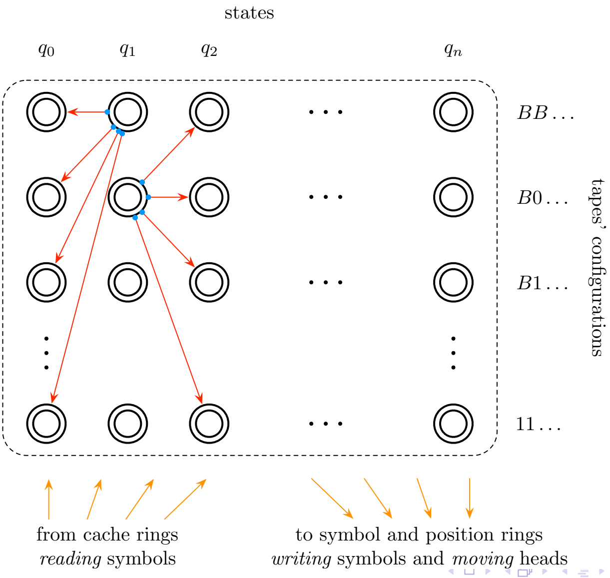


Program Rings



Program Rings



Turing machines & Boolean RNNs with Synfire Rings

Since the construction is generic, the following results hold:

Theorem

- *Let $\mathcal{M}$ be a fixed-space k-tape TM whose every tape is of length N. Then, there exists some B-RNN composed of $\mathcal{O}(N)$ synfire rings and cells that simulates $\mathcal{M}$ in linear time.*
- *Let $\mathcal{M}$ be a k-tape TM. Then, there exists a B-RNN composed of infinitely many synfire rings that simulates $\mathcal{M}$ in linear time.*

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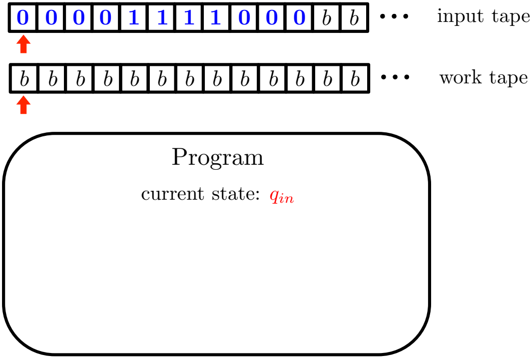
Simulation

- TM recognizing the non-regular language $L = \{0^n 1^n 0^n : n \geq 0\}$.



Play movie

Future Works

1. Reservoir computing: echo state networks (ESN) / liquid state machines (LSM)
   - Introduce *learning* – via synaptic plasticity, intrinsic plasticity, etc. – within this bio-inspired neural architecture.
2. Neuromorphic computing
   - Implement bio-inspired into neuromorphic hardwares.
3. Biology
   - Implement this architecture into cultured neural networks (*in vitro*): towards neuronal computers. . .

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