

NEURAL COMPUTATION WITH RECURRENT NEURAL NETWORKS COMPOSED OF SYNFIRE RINGS

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INTRODUCTION

- ▶ We recall important results about the computational power of recurrent neural networks.
- ▶ We show that neural networks represent a natural model for oracle-based computation, beyond the Turing limits.
- ▶ We introduce a bio-inspired paradigm for neural computation based on the concept of *synfire rings*.

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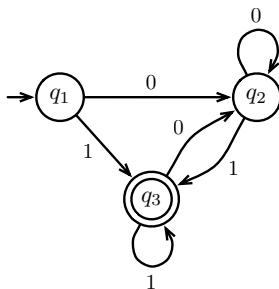
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FINITE STATE AUTOMATON

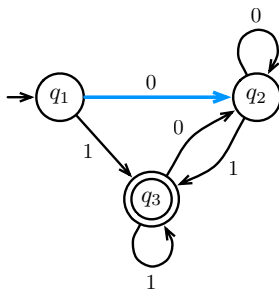
A *finite state automaton* (FSA) can be represented as a graph whose nodes and edges are the computational states and transitions between those.



- ▶ input u is *accepted* by \mathcal{A} if $\mathcal{A}(u)$ reaches a final state
- ▶ input u is *rejected* by \mathcal{A} if $\mathcal{A}(u)$ otherwise

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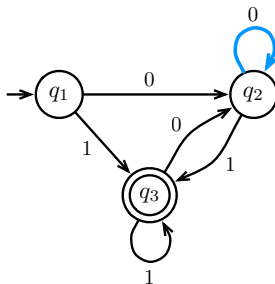
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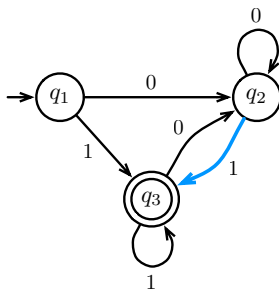
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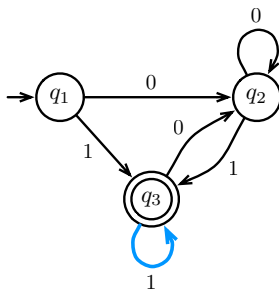
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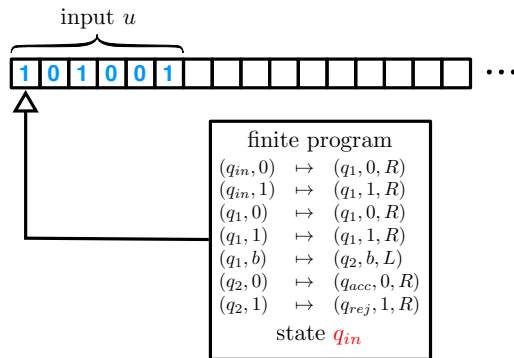
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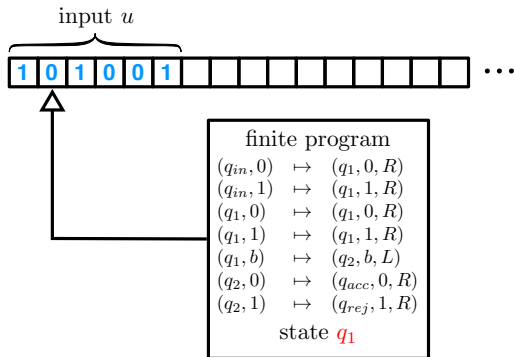
TURING MACHINE

A *Turing machine* (TM) consists of an infinite tape, a read-write head, and a finite program.



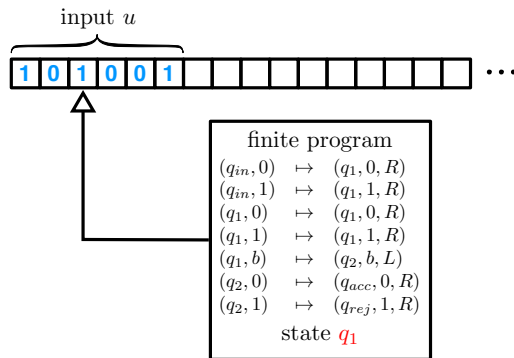
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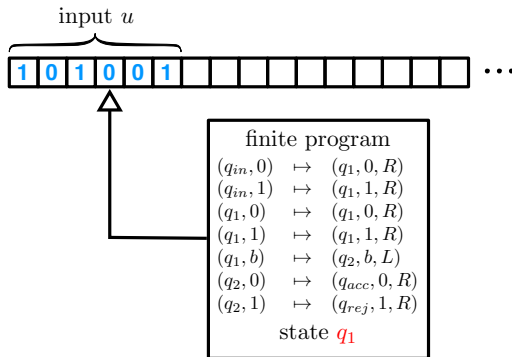
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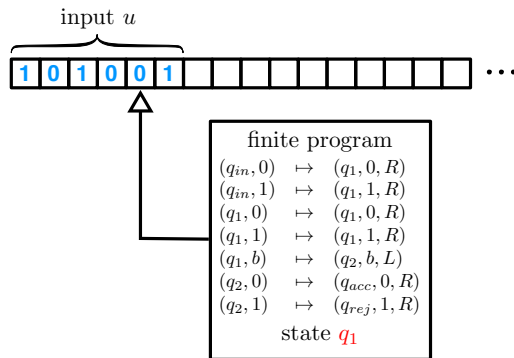
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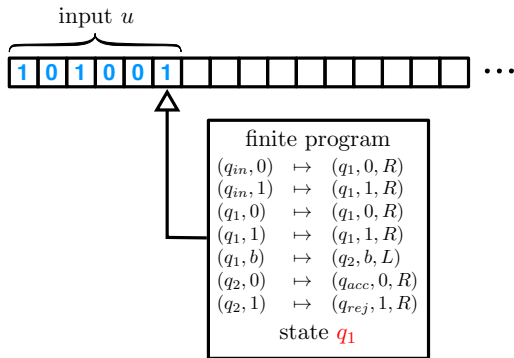
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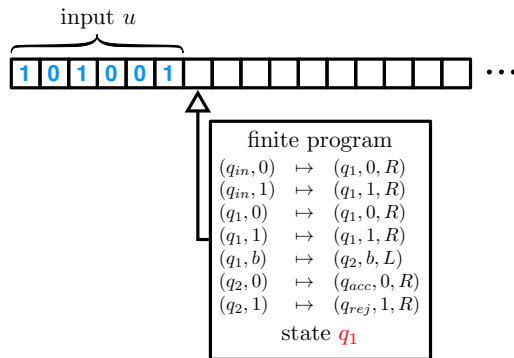
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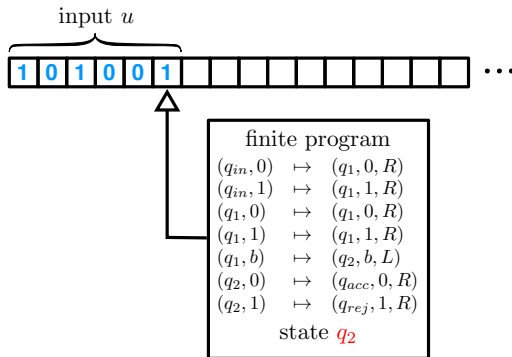
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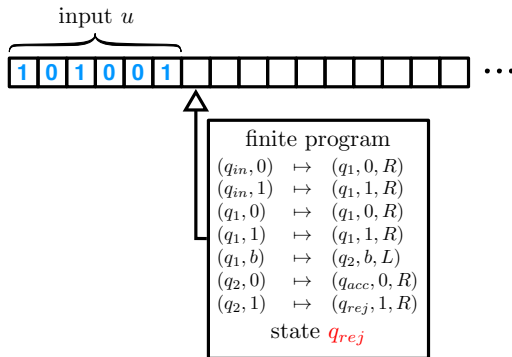
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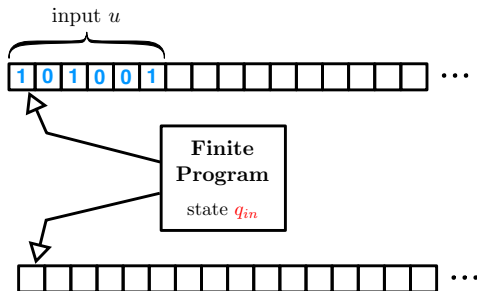
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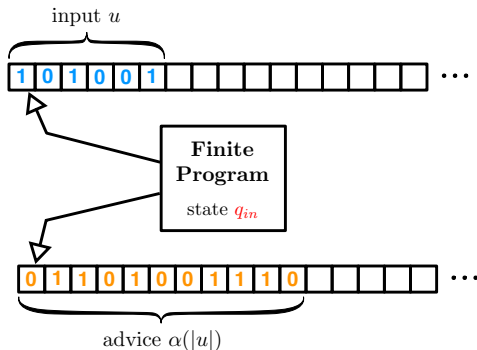
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A *Turing machine with advice* (TM/A) is a TM provided with an additional advice tape and advice function $\alpha : \mathbb{N} \longrightarrow \{0, 1\}^*$.



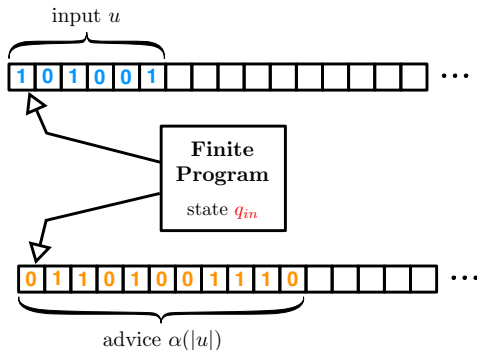
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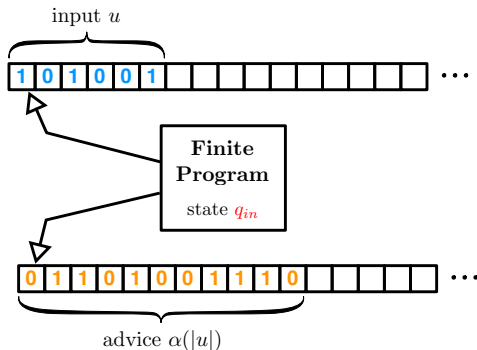
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- ▶ The class of languages recognized in polynomial time by Turing machines with polynomial advices (TM/poly(A)) is **P/poly**.

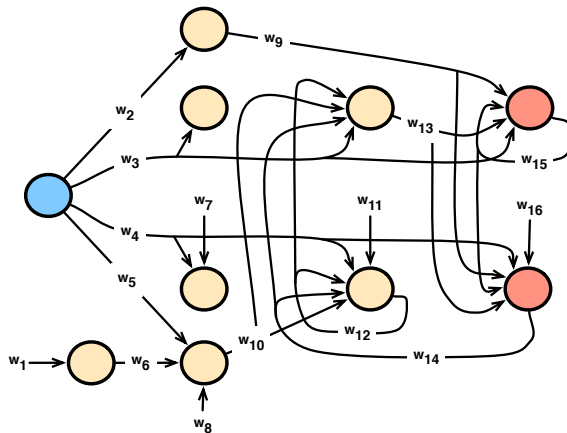
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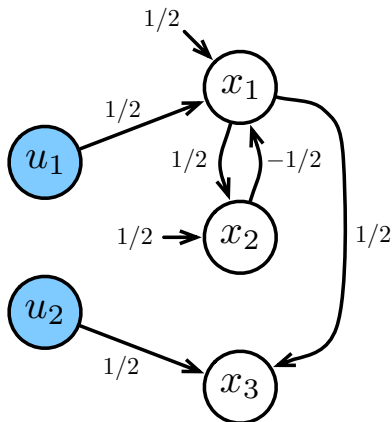


- ▶ TM/As are strictly more powerful than TMs: $\mathbf{P/poly} \supsetneq \mathbf{P}$
They are *super-Turing*...

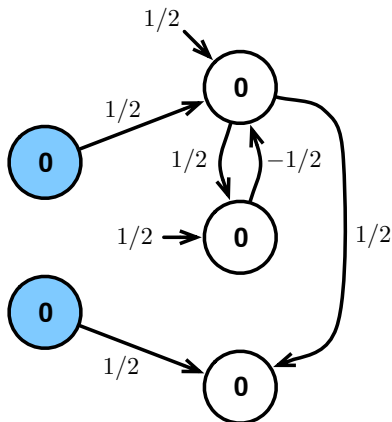
RECURRENT NEURAL NETWORK



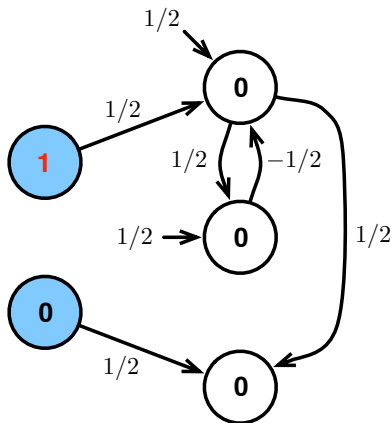
BOOLEAN RECURRENT NEURAL NETWORKS



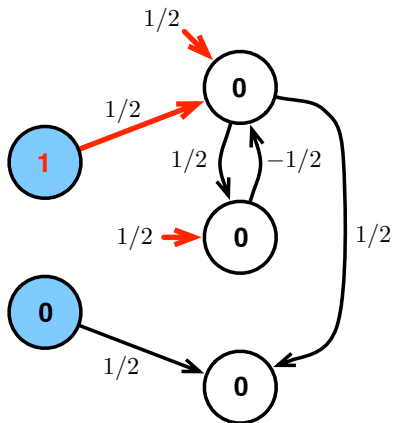
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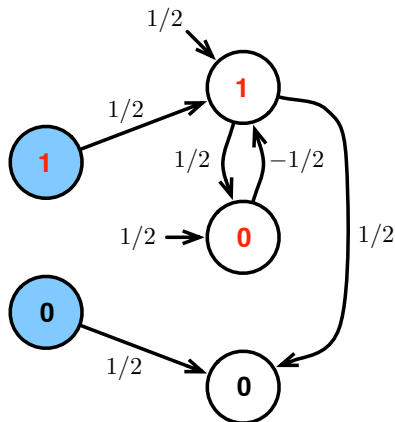
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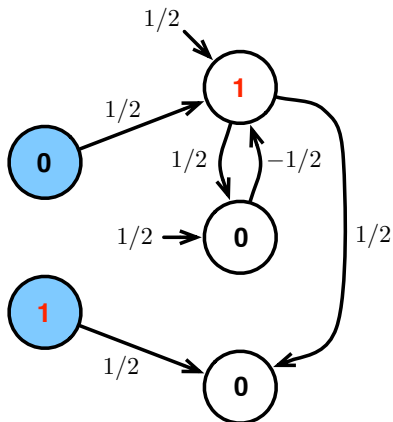
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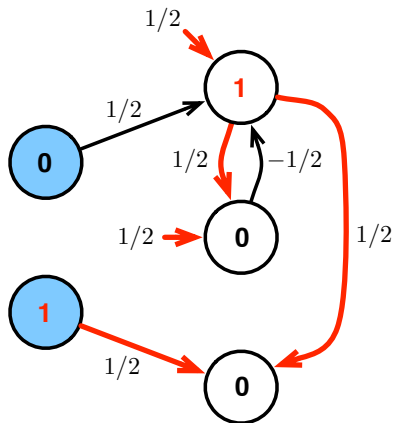
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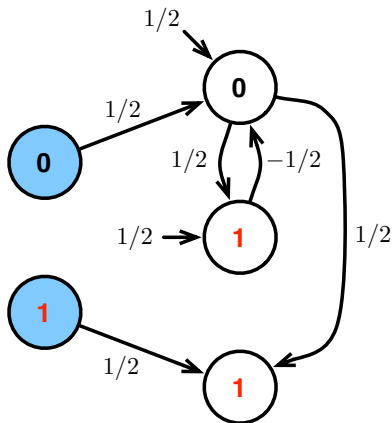
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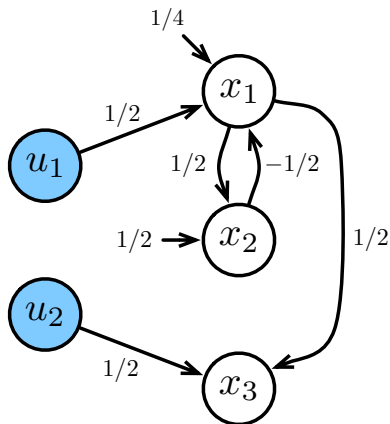
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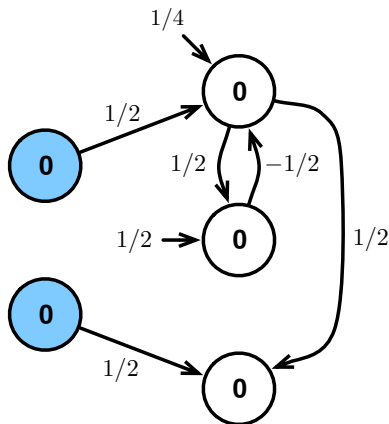
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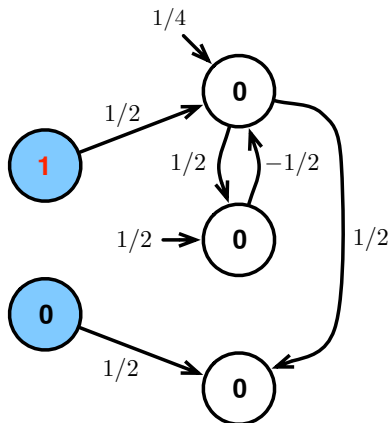
SIGMOID RECURRENT NEURAL NETWORKS



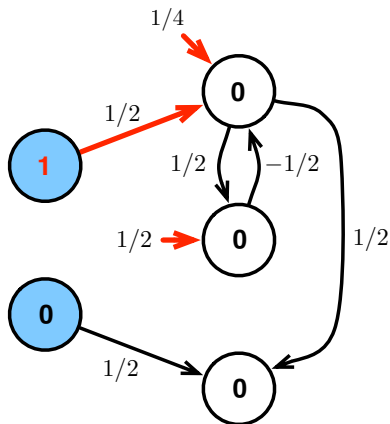
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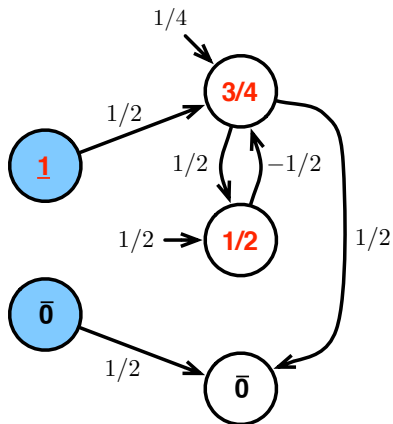
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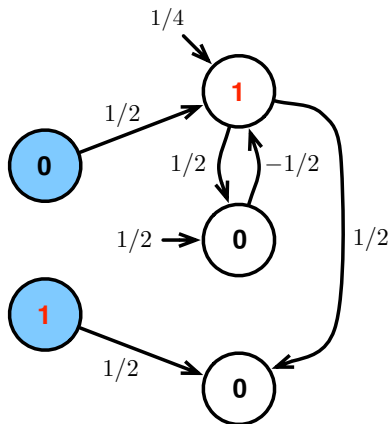
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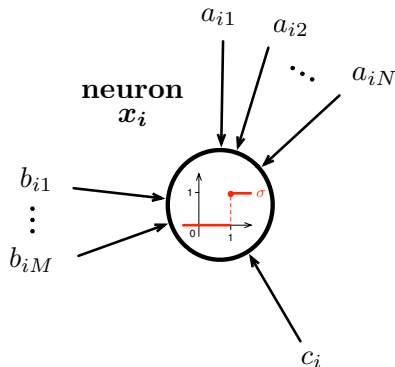
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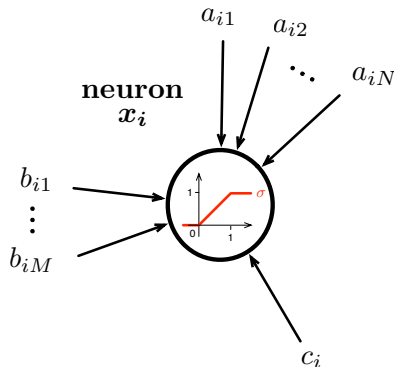


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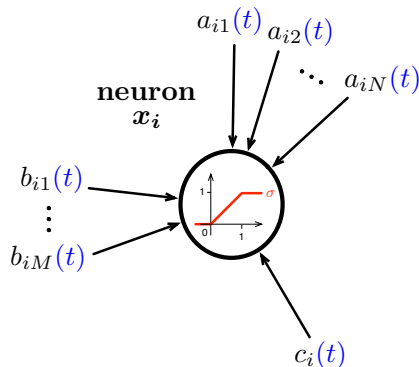
$$x_i(t+1) = \theta \left(\sum_{j=1}^N a_{ij} \cdot x_j(t) + \sum_{j=1}^M b_{ij} \cdot u_j(t) + c_i \right)$$

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RECURRENT NEURAL NETWORKS

1. Boolean rational RNNs:

B-RNN[\mathbb{Q}]_s

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B-RNN[\mathbb{R}]_s

3. Sigmoid static rational RNNs:

St-RNN[\mathbb{Q}]_s

4. Sigmoid static real RNNs:

St-RNN[\mathbb{R}]_s

5. Sigmoid bi-valued evolving rational RNNs:

Ev₂-RNN[\mathbb{Q}]_s

6. Sigmoid bi-valued evolving real RNNs:

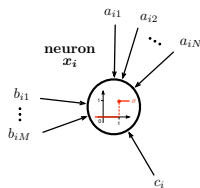
Ev₂-RNN[\mathbb{R}]_s

7. Sigmoid general evolving rational RNNs:

Ev-RNN[\mathbb{Q}]_s

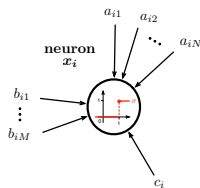
8. Sigmoid general evolving real N-RNNs:

Ev-RNN[\mathbb{R}]_s



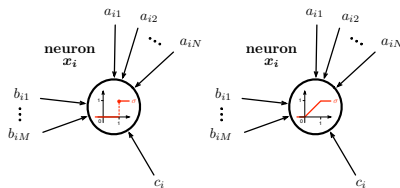
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| 4. Sigmoid static real RNNs: | St-RNN[\mathbb{R}] _s |
| 5. Sigmoid bi-valued evolving rational RNNs: | Ev ₂ -RNN[\mathbb{Q}] _s |
| 6. Sigmoid bi-valued evolving real RNNs: | Ev ₂ -RNN[\mathbb{R}] _s |
| 7. Sigmoid general evolving rational RNNs: | Ev-RNN[\mathbb{Q}] _s |
| 8. Sigmoid general evolving real N-RNNs: | Ev-RNN[\mathbb{R}] _s |



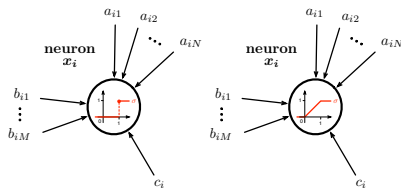
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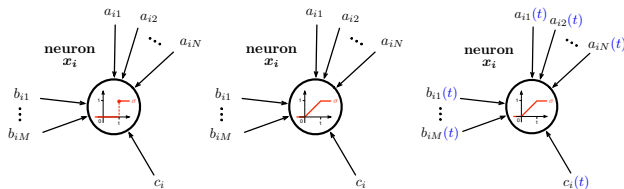
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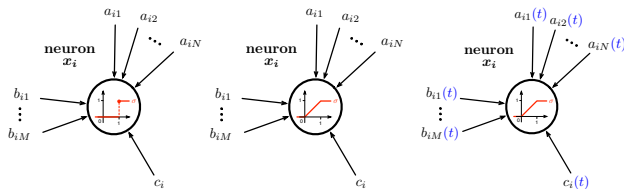
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RESULTS: CLASSICAL COMPUTATION

	BOOLEAN	SIGMOID		
		STATIC	BI-VALUED EVOLVING	EVOLVING
Q	FSA	TM	TM/poly(A)	TM/poly(A)
	REG	P	P/poly	P/poly
	KI 56, Mi 67	Si & So 95	Ca & Si 11,14	Ca & Si 11,14
R	FSA	TM/poly(A)	TM/poly(A)	TM/poly(A)
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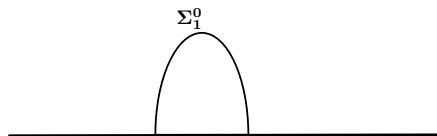
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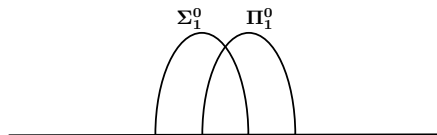
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INFINITE COMPUTATION

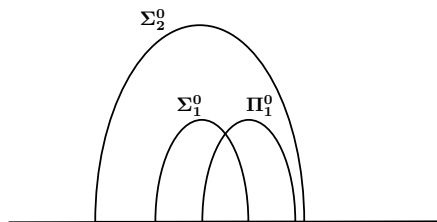
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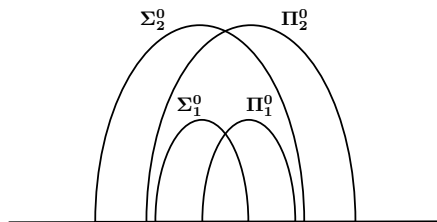
INFINITE COMPUTATION



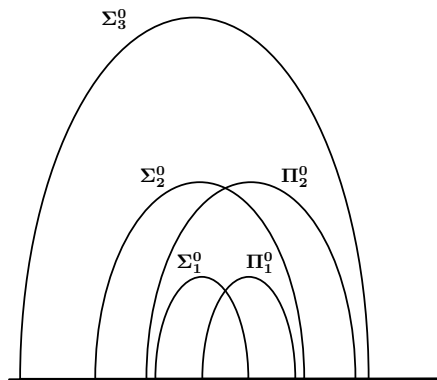
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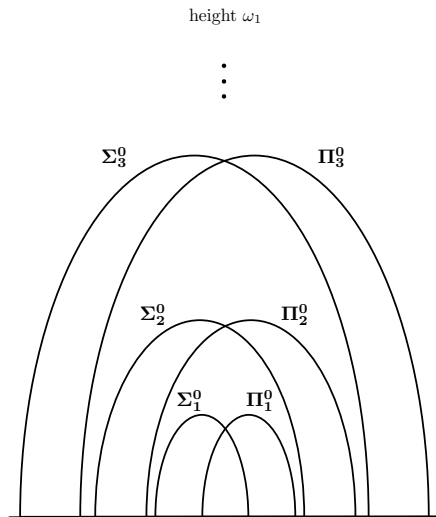
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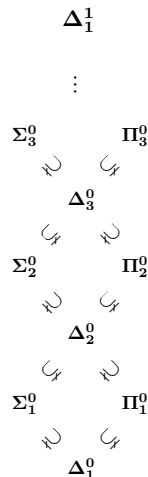
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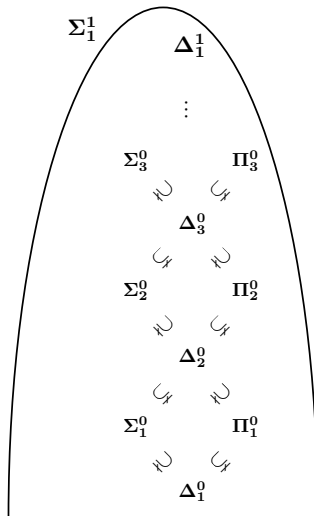
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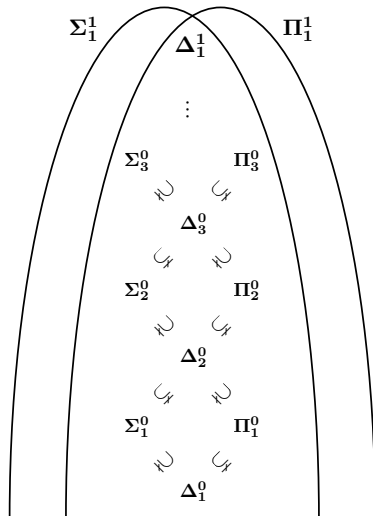
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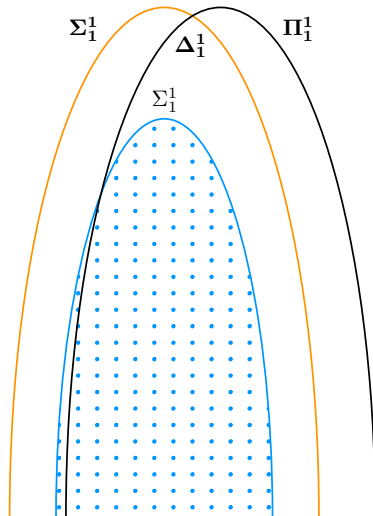


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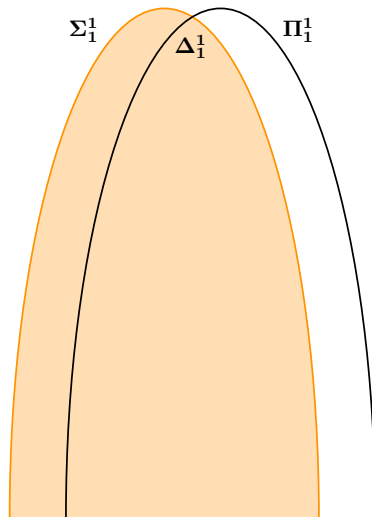


INFINITE COMPUTATION





INFINITE COMPUTATION



RESULTS: INFINITE COMPUTATION / DET.

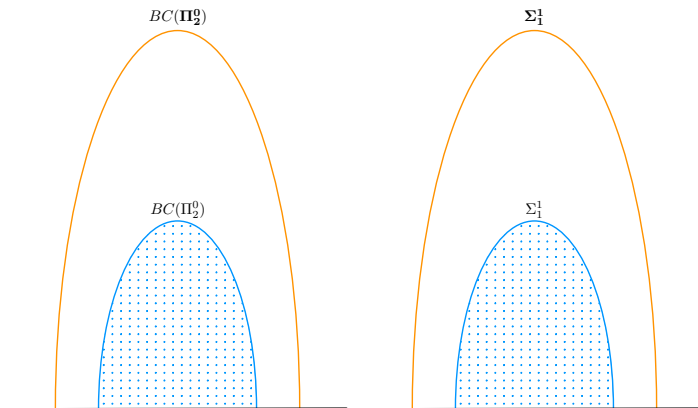
	BOOLEAN	SIGMOID		
		STATIC	BI-VALUED EVOLVING	EVOLVING
Q	Muller FSA	Muller TM	super-Turing	super-Turing
	$\in BC(\Pi_2^0)$	$= BC(\Pi_2^0)$	$= BC(\Pi_2^0)$	$= BC(\Pi_2^0)$
	Ca & Vi 10,14	Ca & Vi 15,16	Ca & Vi 15,16	Ca & Vi 15,16
R	Muller FSA	super-Turing	super-Turing	super-Turing
	$\in BC(\Pi_2^0)$	$= BC(\Pi_2^0)$	$= BC(\Pi_2^0)$	$= BC(\Pi_2^0)$
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		STATIC	BI-VALUED EVOLVING	EVOLVING
Q	Muller FSA	Muller TM	super-Turing	super-Turing
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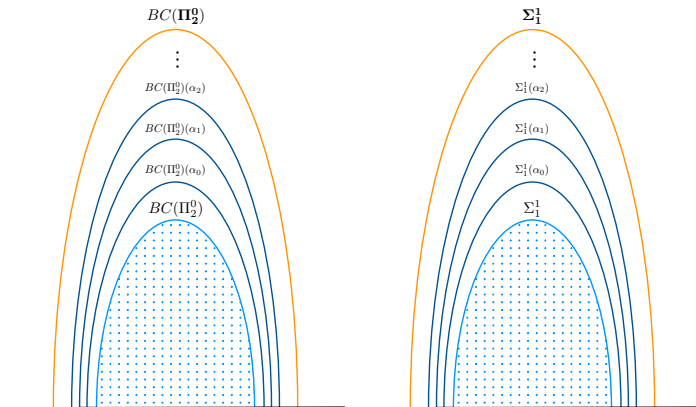
HIERARCHY THEOREMS: STRATIFICATION OF THE SUPER-TURING “WORLD”

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- ▶ Recurrent neural networks is a natural model for *oracle-based (super-Turing) computation*.
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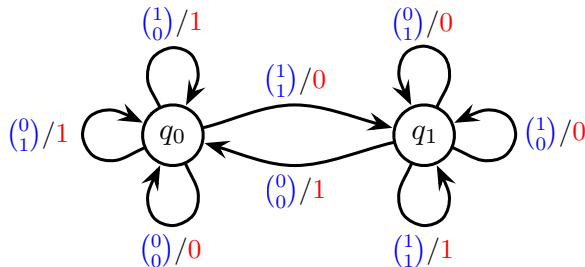
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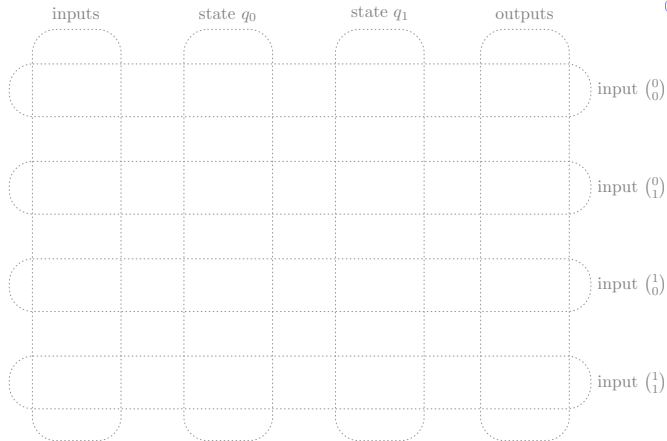
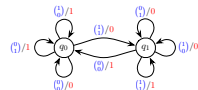
JÉRÉMIE CABESSA

BINARY ADDER AUTOMATON

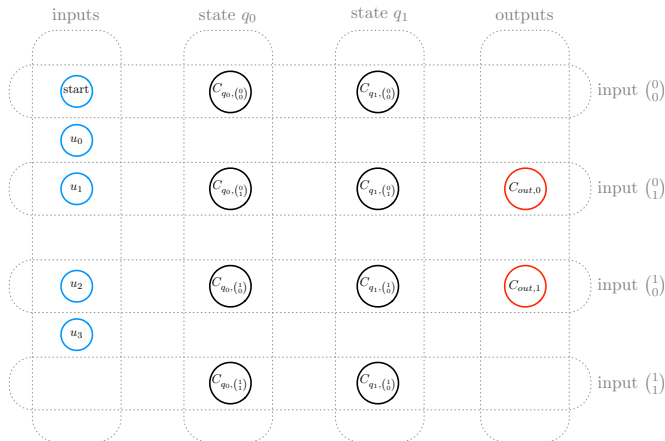
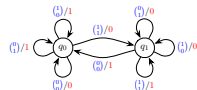


$$\begin{array}{rcccccccc}
 & & 1 & 1^1 & 0^1 & 1 & 1 & 0^1 & 1 \\
 + & & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\
 \hline
 & & 1 & 1 & 0 & 0 & 1 & 1 & 0
 \end{array}$$

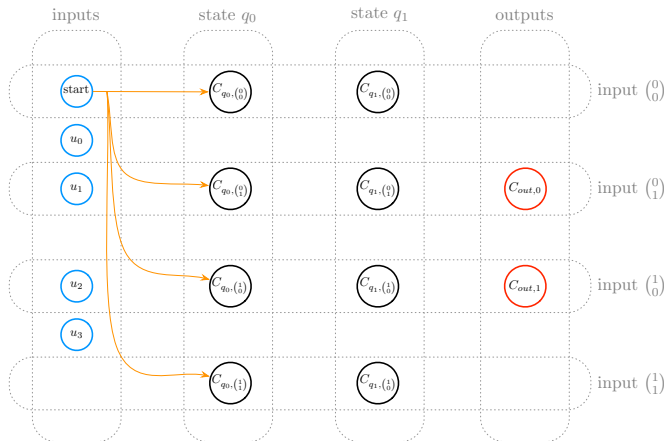
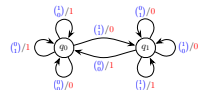
BINARY ADDER BOOLEAN NEURAL NETWORK



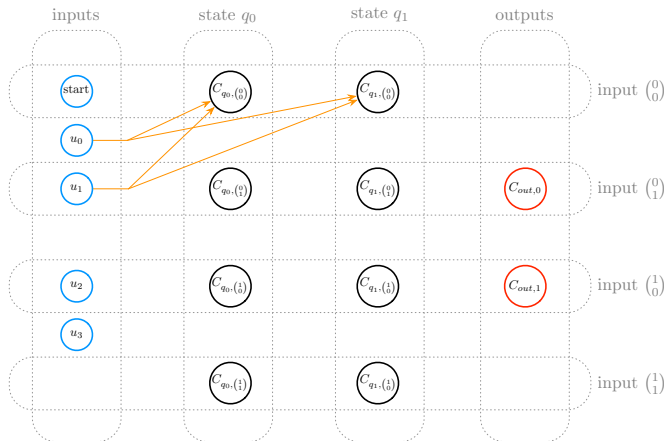
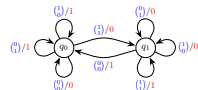
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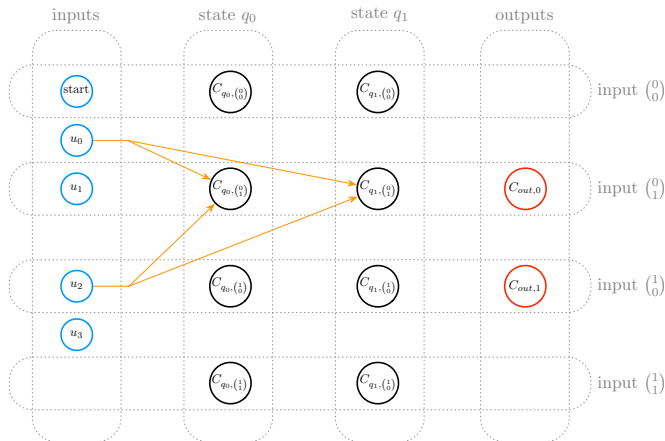
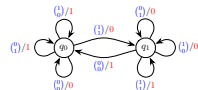
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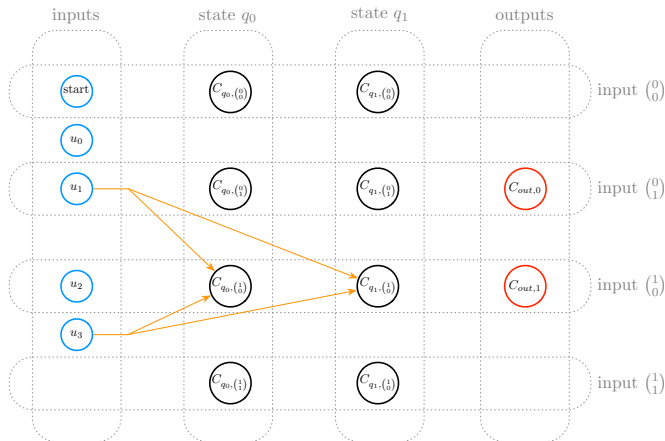
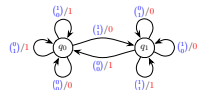
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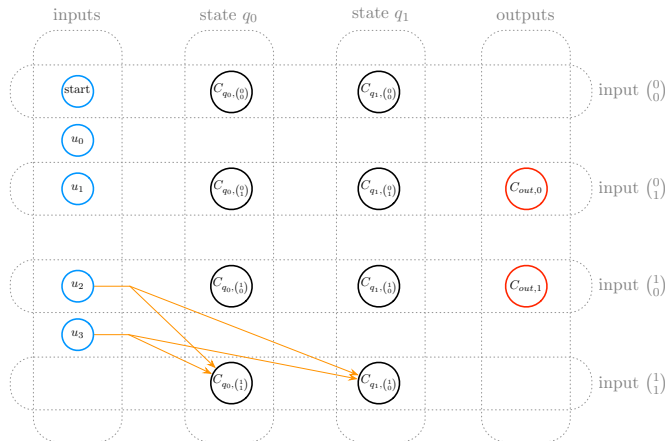
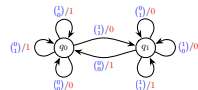
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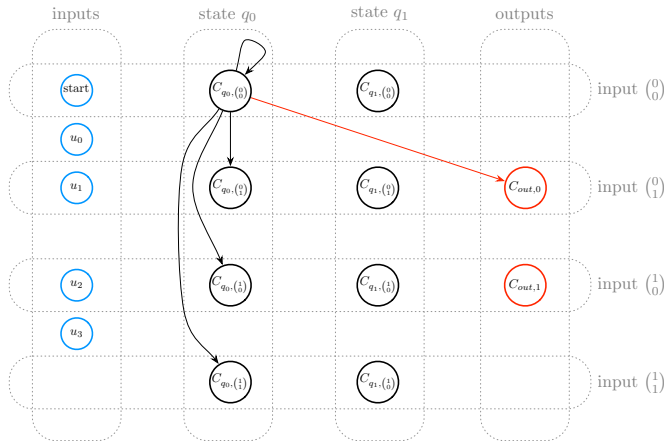
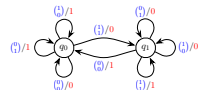
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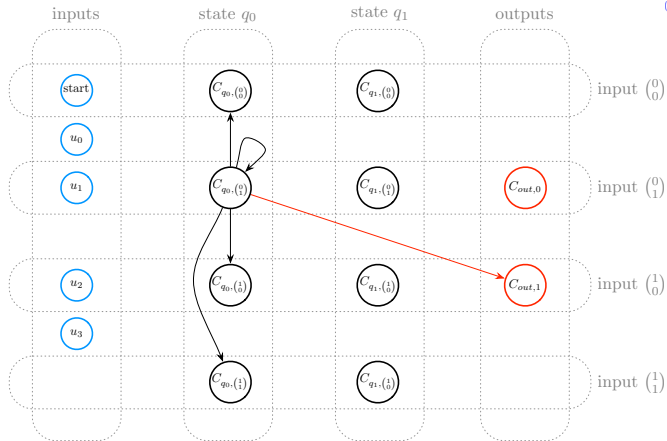
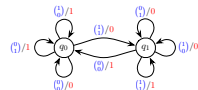
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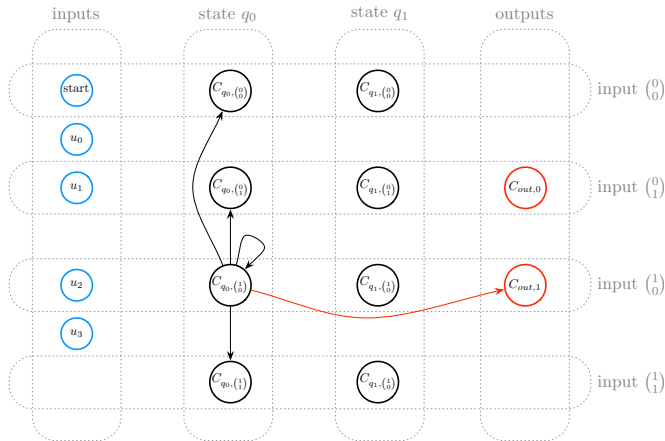
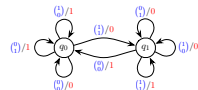
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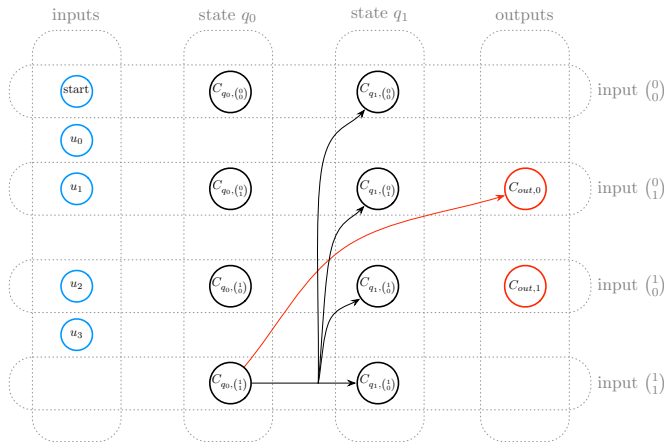
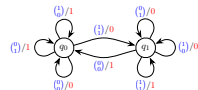
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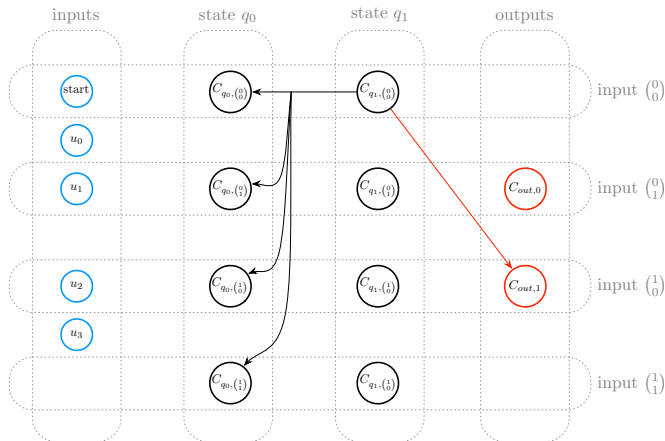
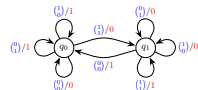
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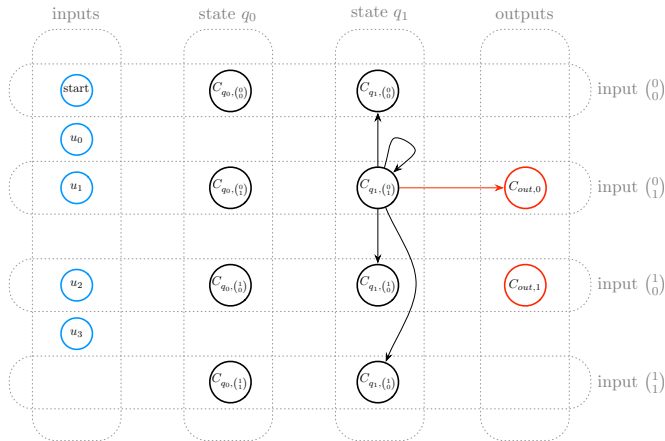
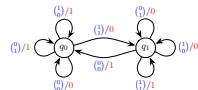
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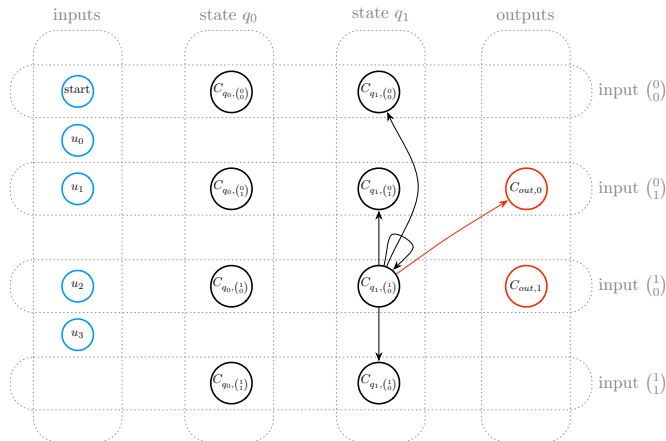
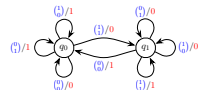
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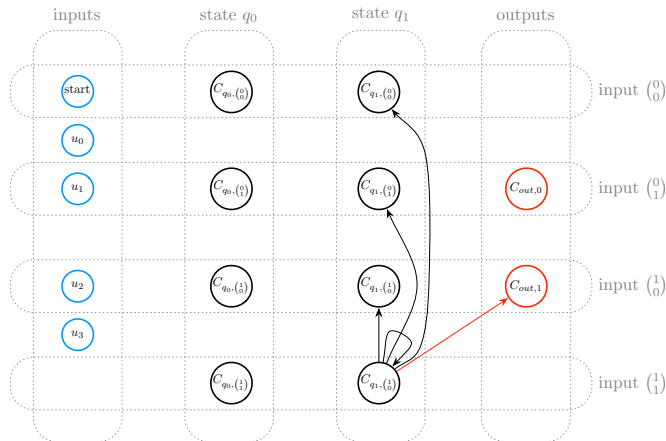
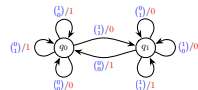
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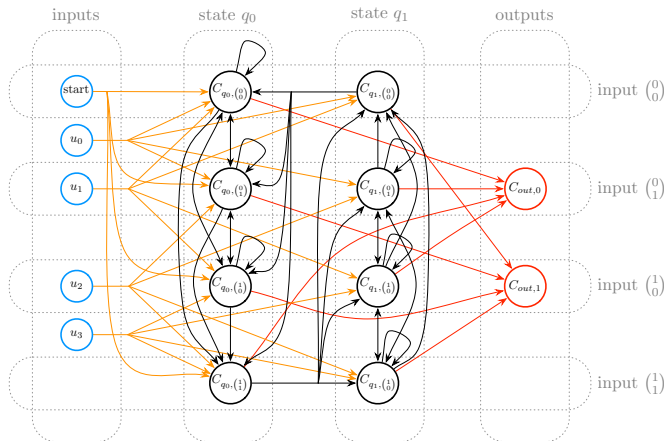
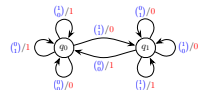
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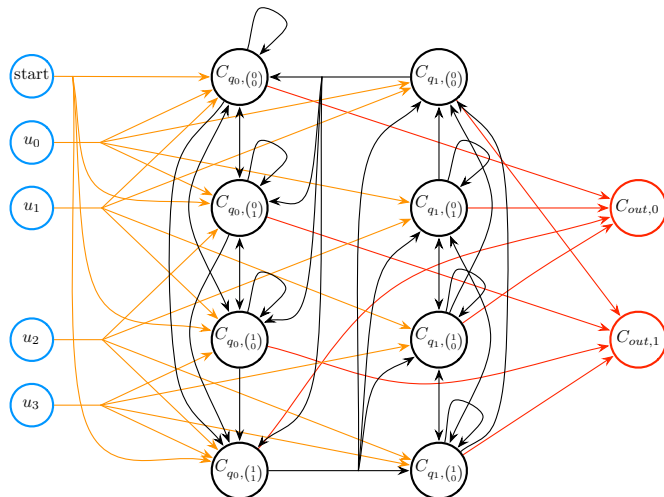
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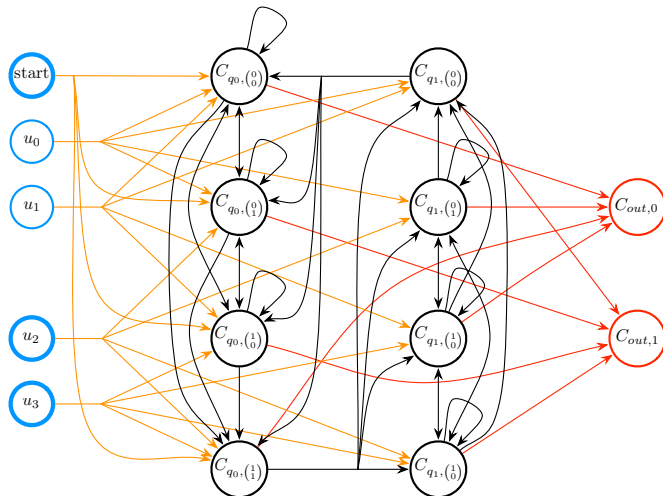
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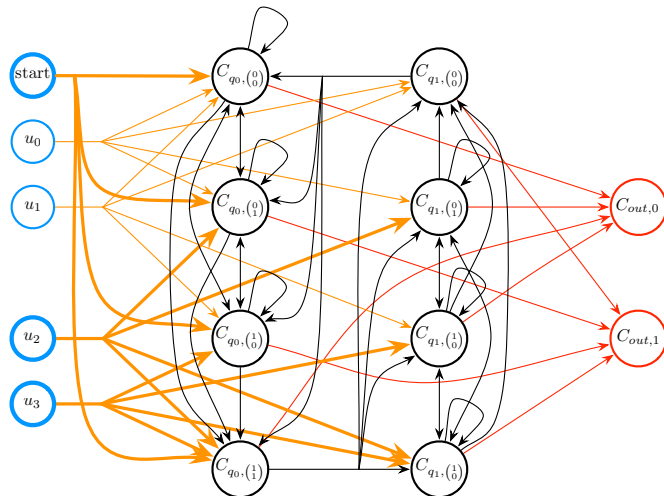
SIMULATION



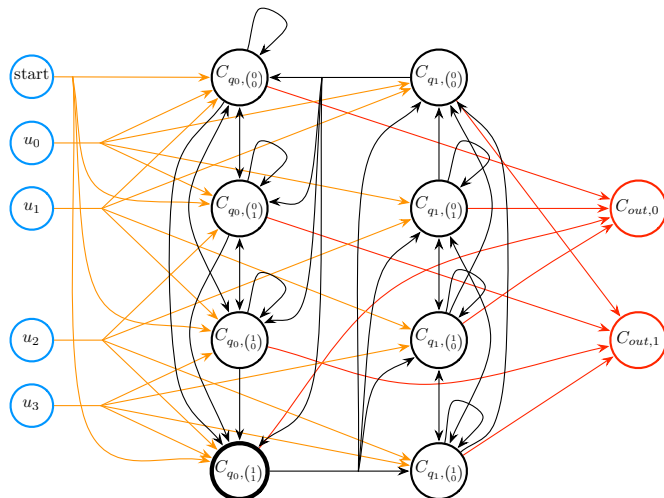
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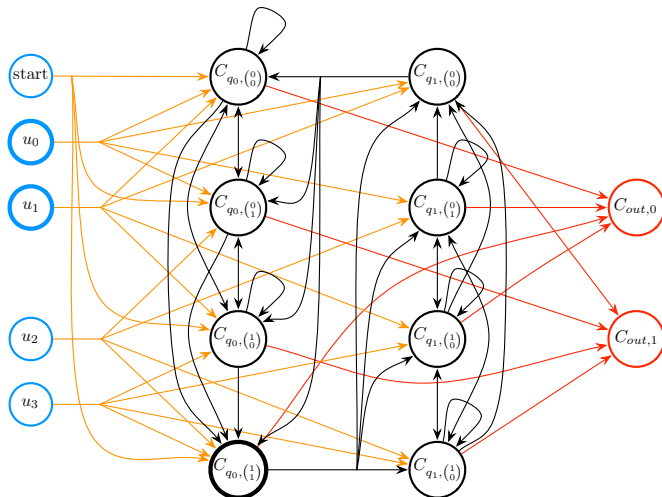
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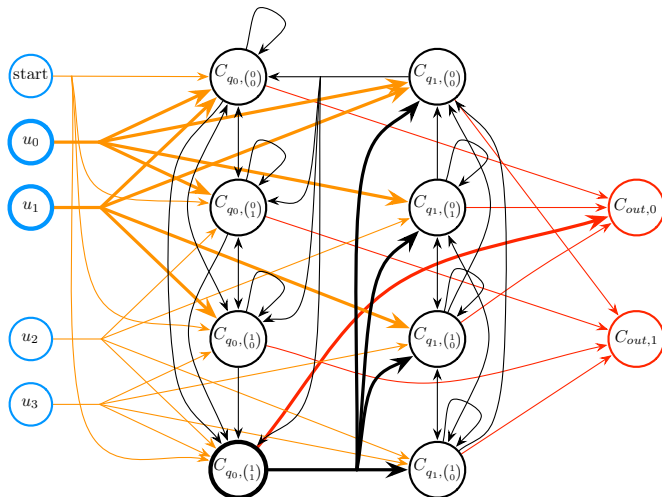
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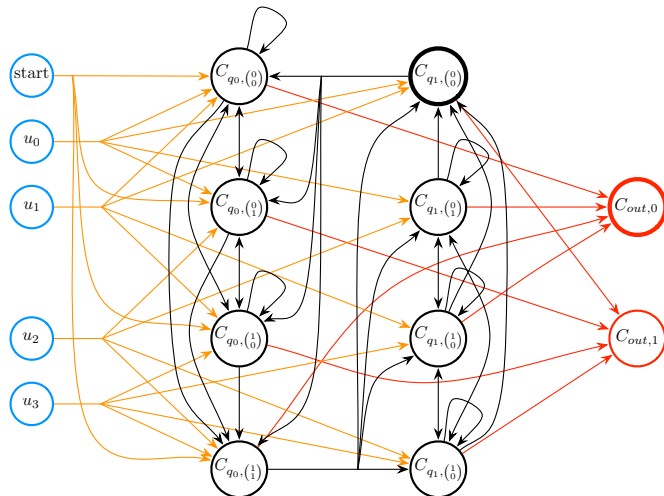
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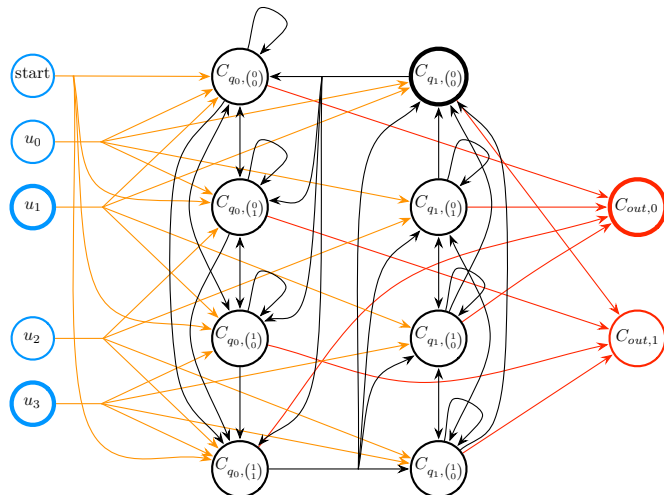
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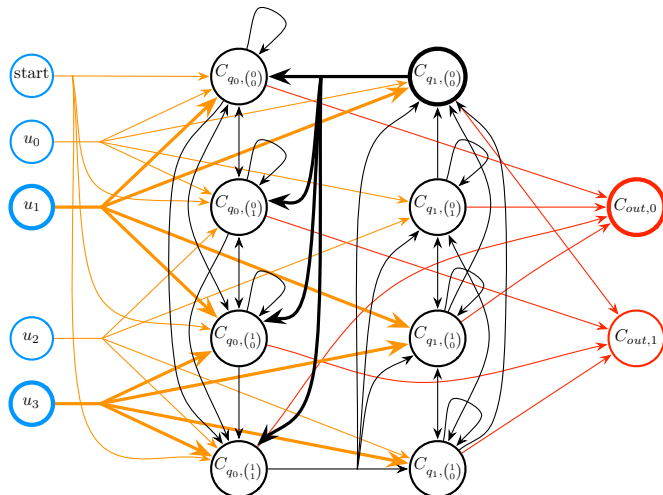
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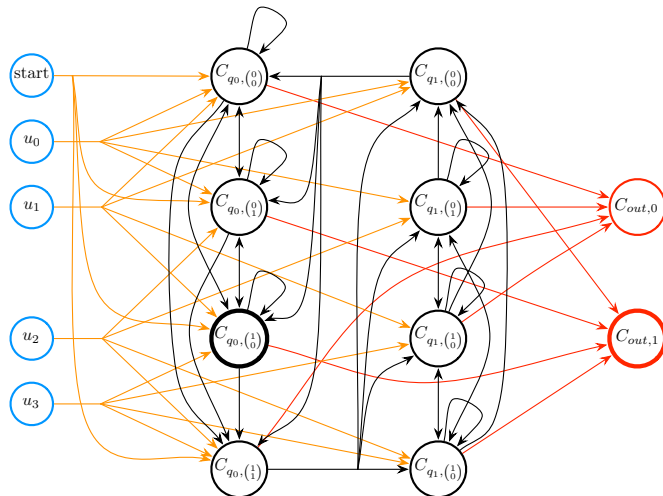
SIMULATION



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SIMULATION

time	0	1	2	3	4	5	6	7	8
states	q_0	q_1	q_0	q_0	q_1	q_1	q_1	q_0	–
inputs	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	–	–
outputs	0	1	1	0	0	1	1	–	–
$start$	1	0	0	0	0	0	0	0	0
u_0	0	1	0	0	1	0	1	0	0
u_1	0	1	1	0	0	0	1	0	0
u_2	1	0	0	1	1	1	0	0	0
u_3	1	0	1	1	0	1	0	0	0
$C_{s,i}$	–	$q_0, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$q_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$q_0, \begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$q_0, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$q_1, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$q_1, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$q_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	–
$C_{out,0}$	0	0	1	0	0	1	1	0	0
$C_{out,1}$	0	0	0	1	1	0	0	1	1

DRAWBACKS OF THE CONSTRUCTION

- ▶ Computational states of the automaton are represented as Boolean states, i.e., spiking configurations of the network.
- ★ Computational states should rather be represented by *sustained activities of neural assemblies*, e.g., by *cyclic attractors*.
- ▶ Network is not robust to cell death, synaptic plasticity, architectural plasticity in general.
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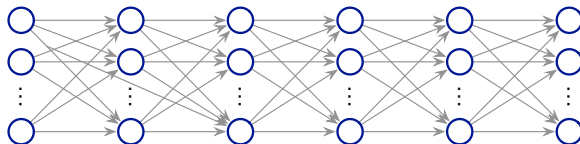
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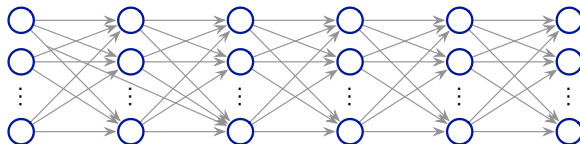
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- ▶ *Synfire chains* allow for robust and highly precise transmission of information in neural networks.
- ▶ *Synfire chains* are likely to be crucially involved in the processing and coding of information in neural networks.
- ▶ *Synfire chains* have been theorized as fundamental neuronal structures (ABELES 82).



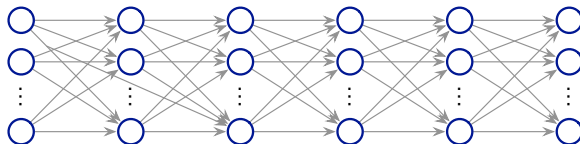
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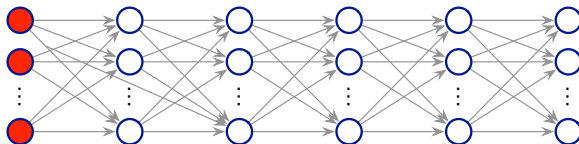
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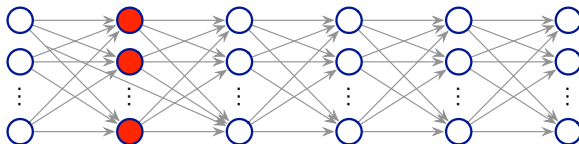
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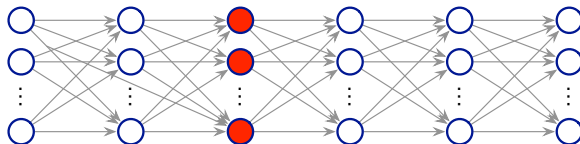
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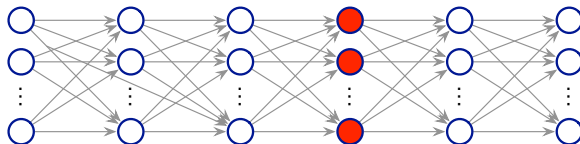
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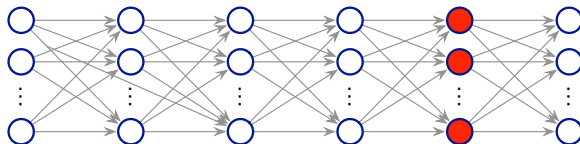
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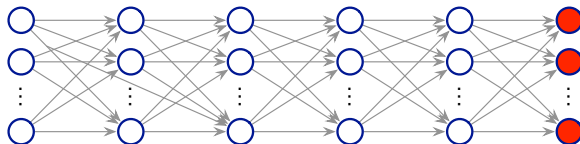
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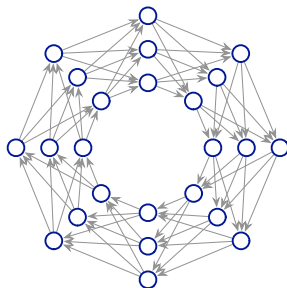
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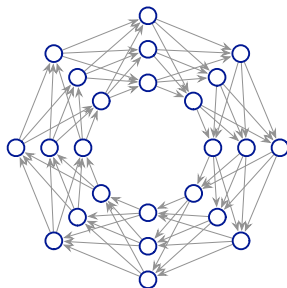
SYNFIRE RINGS

- ▶ *Synfire rings* are looping synfire chains that have been observed in self-organizing neural networks (ZHENG & TRIESCH 14).
- ▶ *Synfire rings* allow for robust and temporally precise *self-sustained activities*.



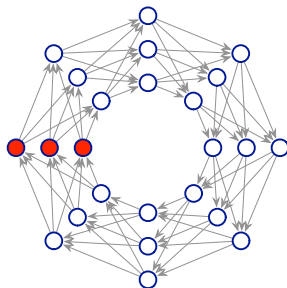
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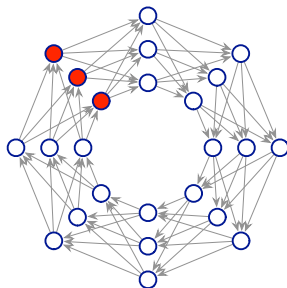
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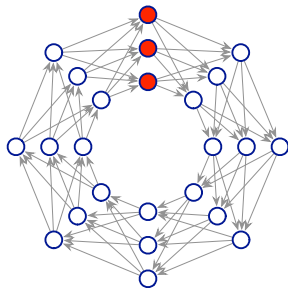
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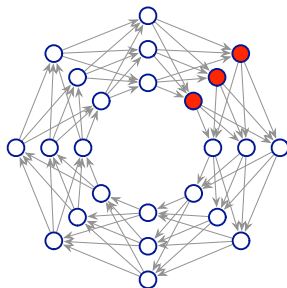
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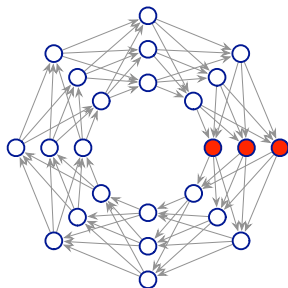
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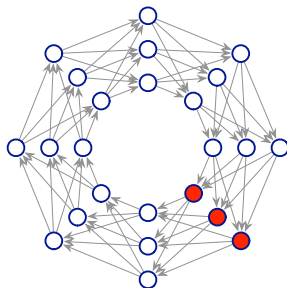
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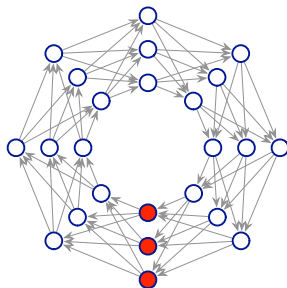
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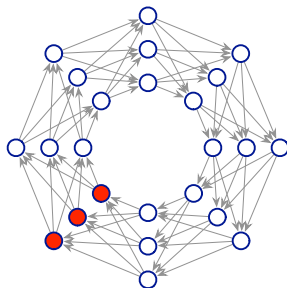
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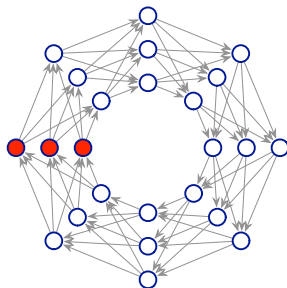
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NEURAL COMPUTATION WITH SYNFIRE RINGS

- ▶ We introduce a paradigm of neural computation based on *synfire rings*.
- ▶ Computational states are represented by sustained activities within synfire rings.
- ▶ Hence, the successive computational states are encoded into cyclic attractors.
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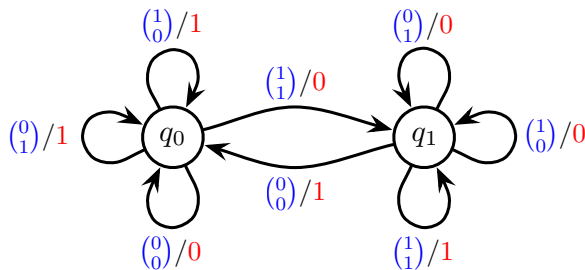
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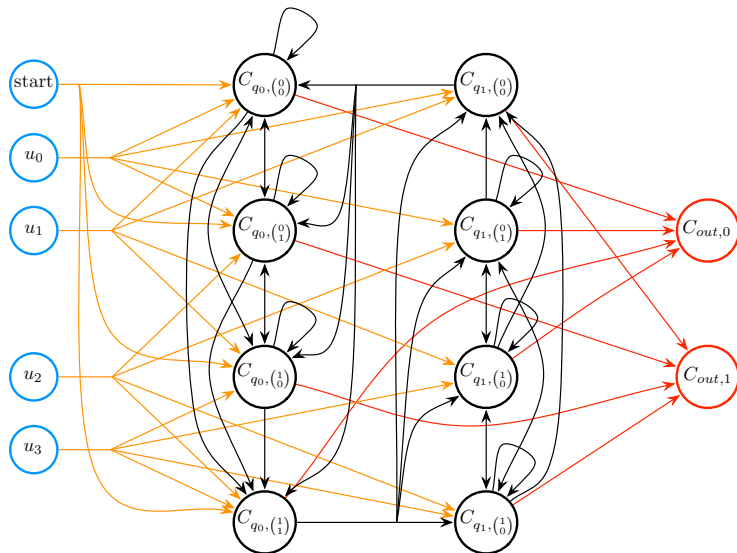
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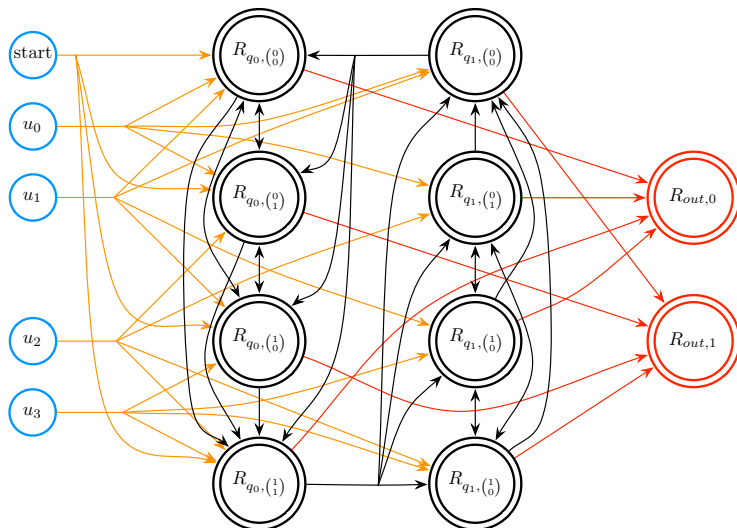
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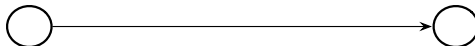
GENERAL CONSTRUCTION



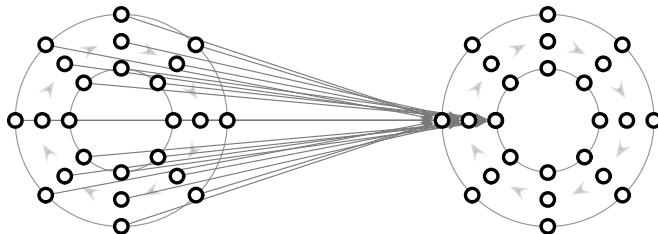
GENERAL CONSTRUCTION



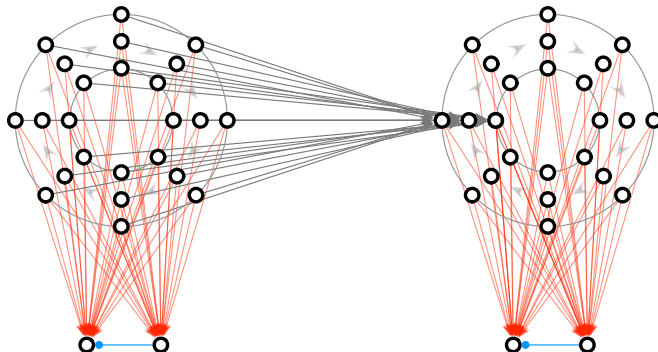
FIBRES OF CONNECTIONS & INHIBITORY SYSTEM



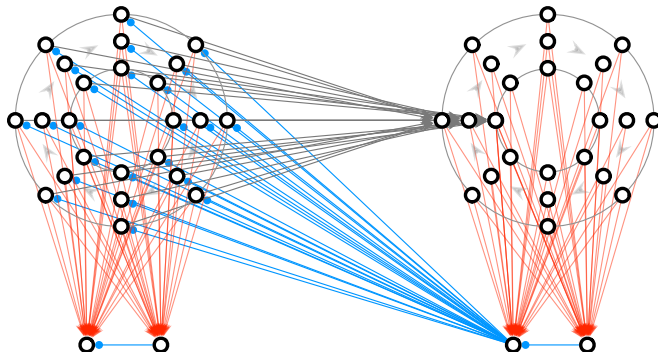
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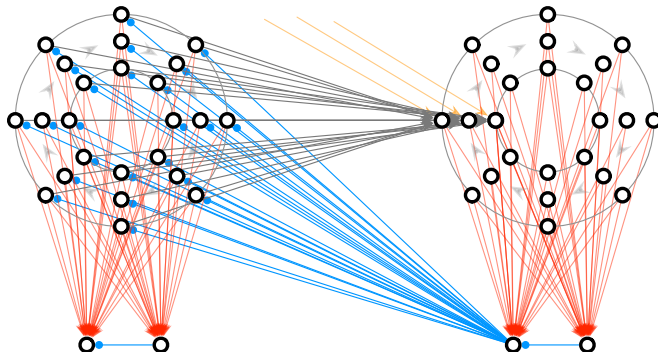
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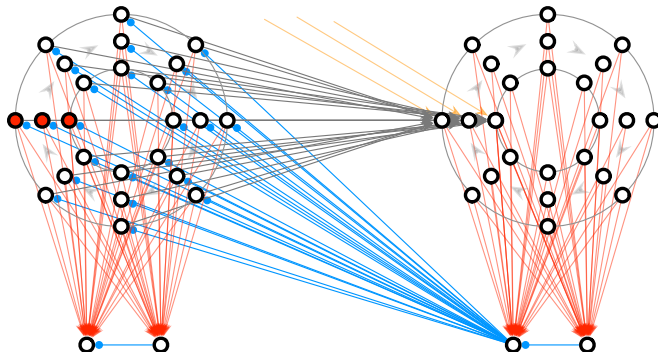
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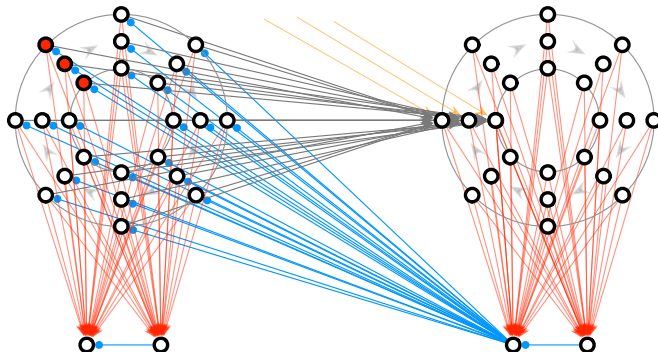
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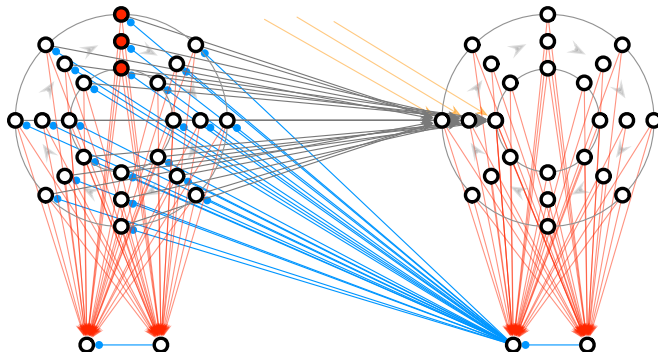
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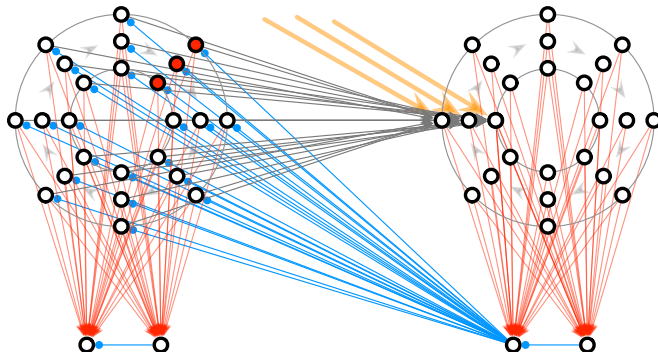
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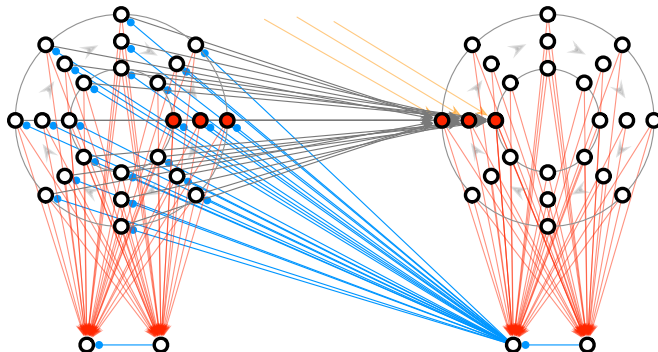
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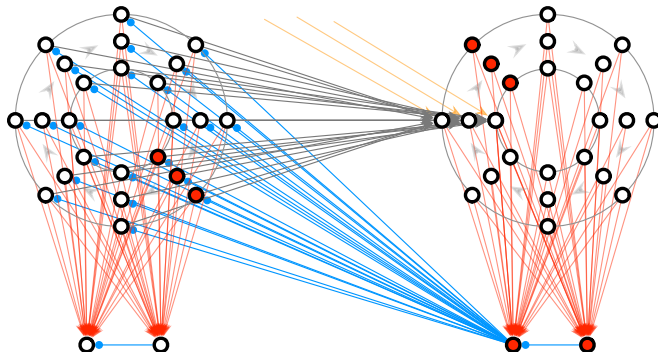
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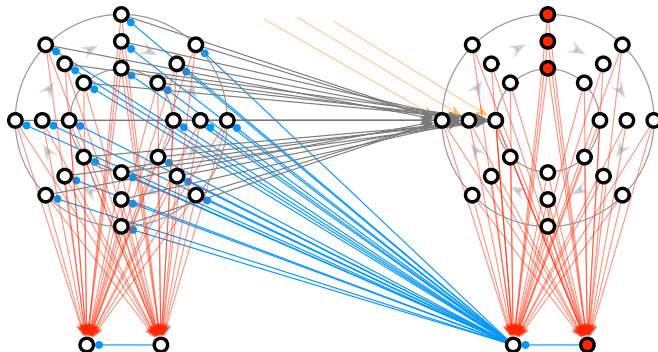
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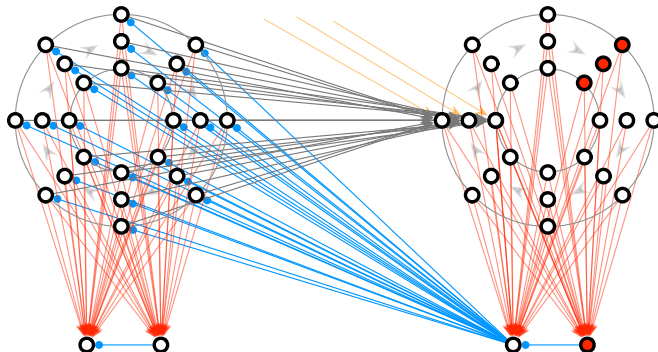
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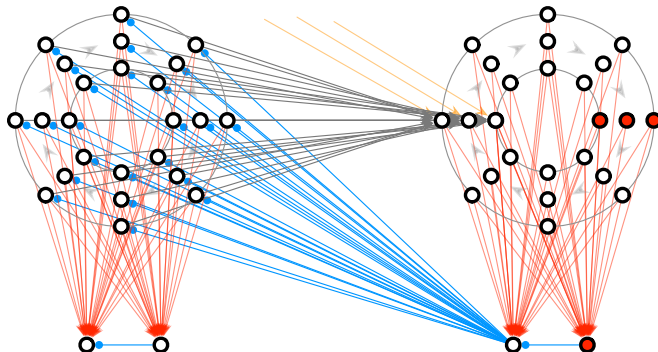
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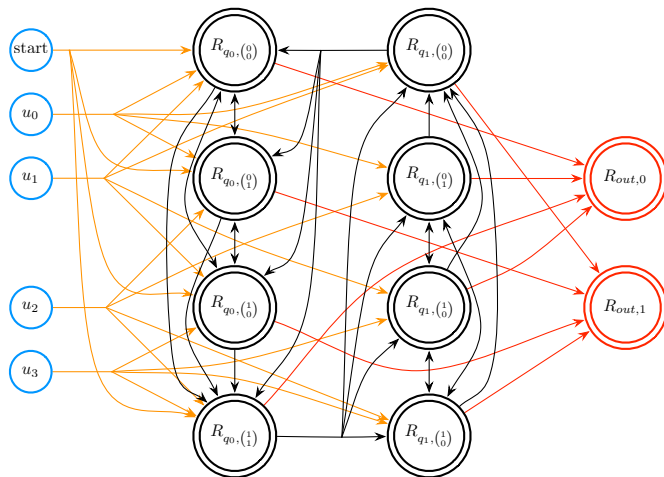
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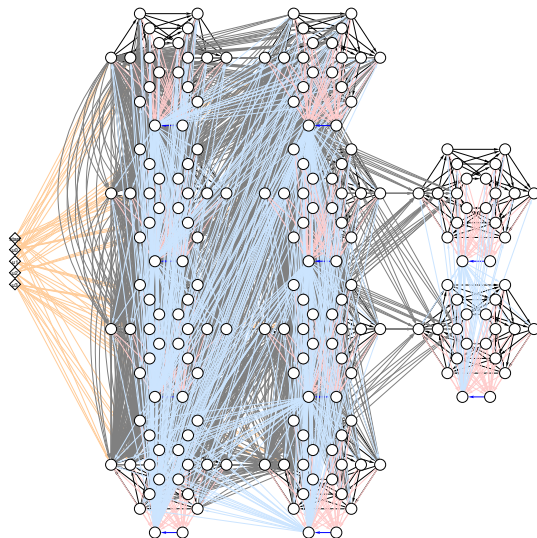
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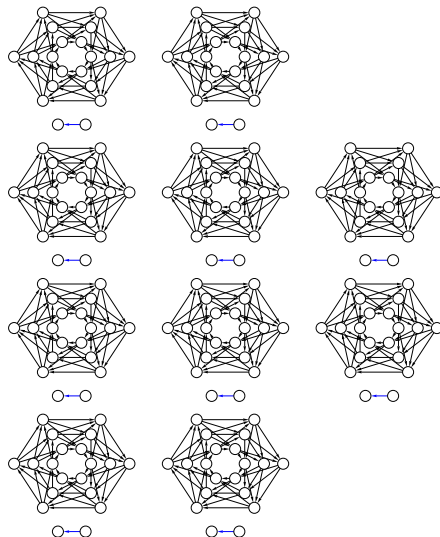
SIMULATION



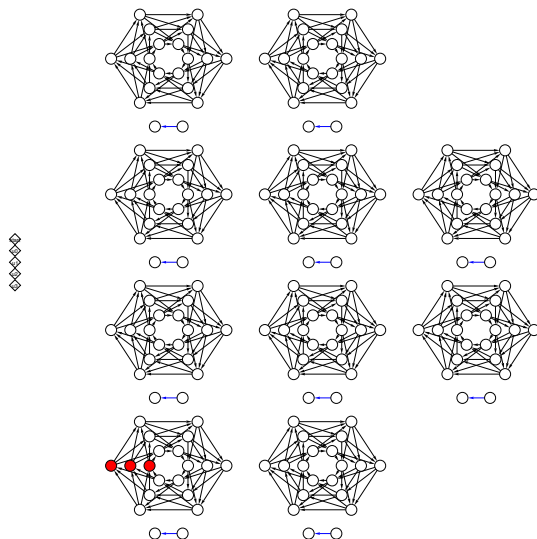
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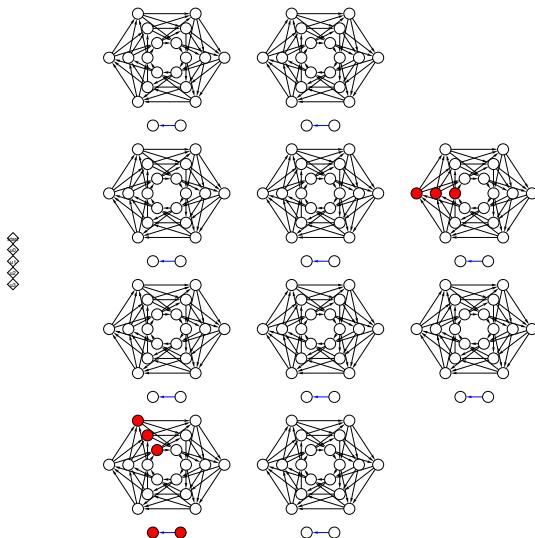
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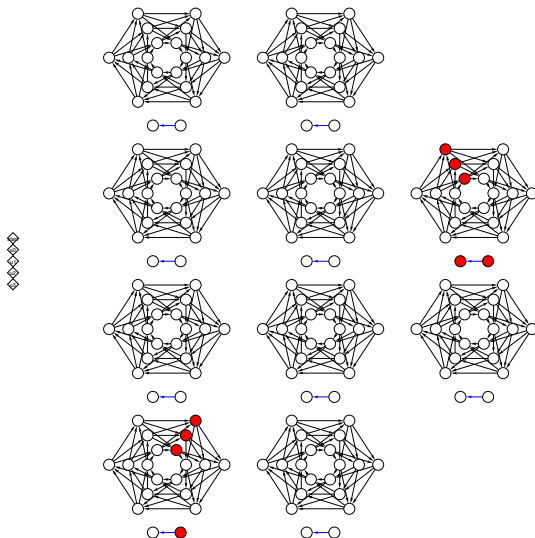
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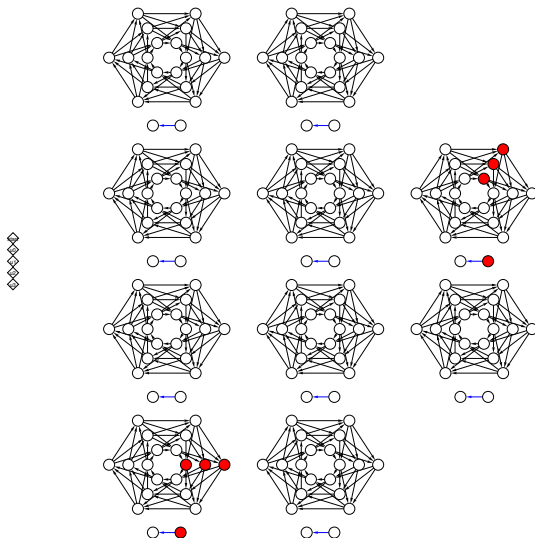
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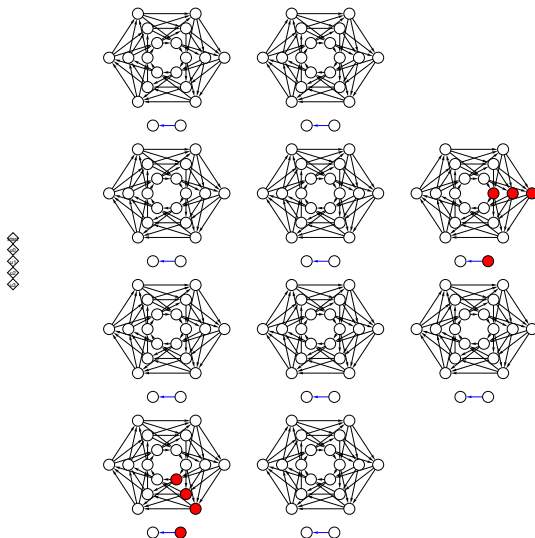
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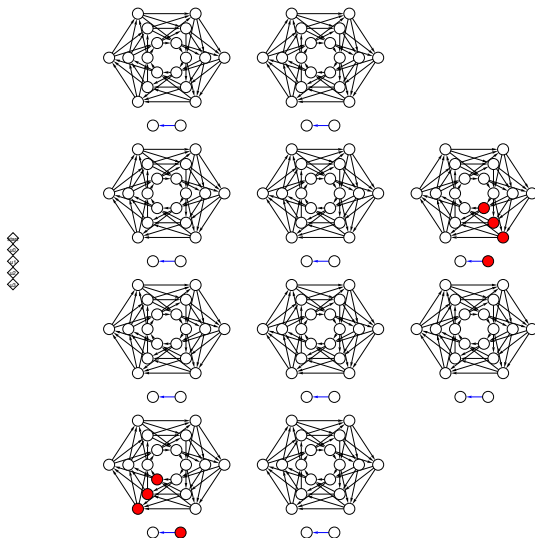
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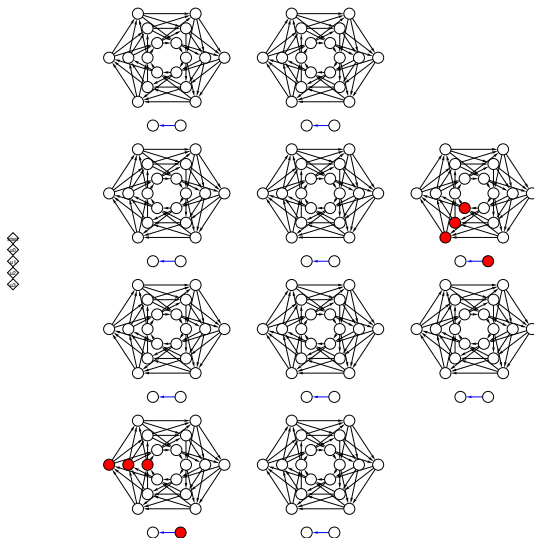
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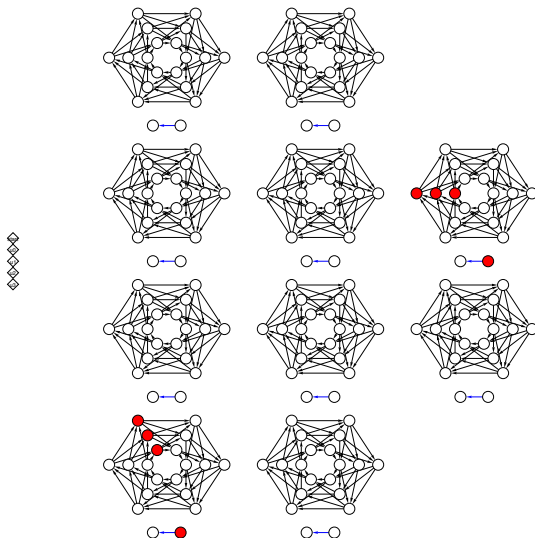
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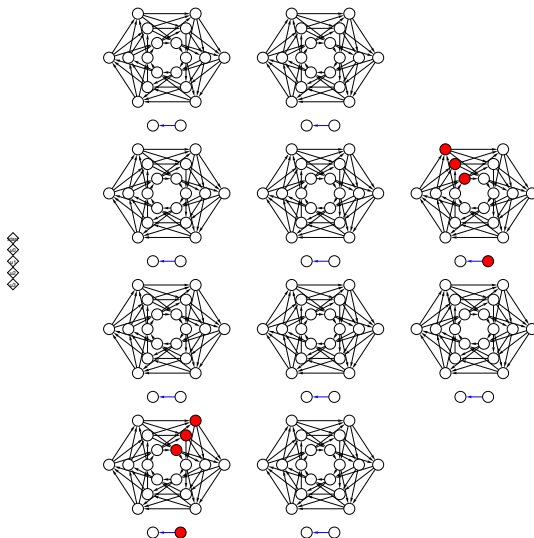
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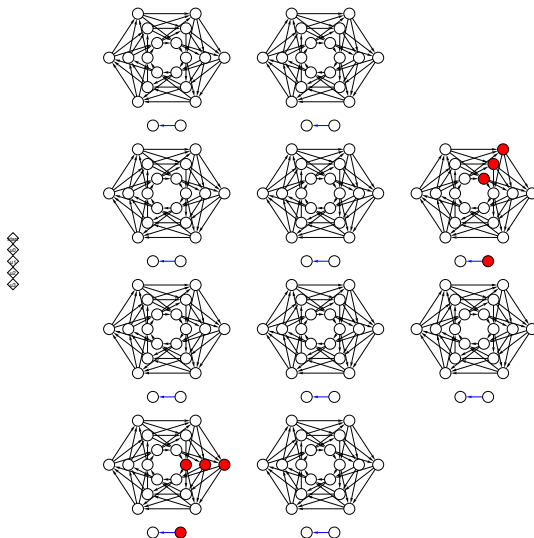
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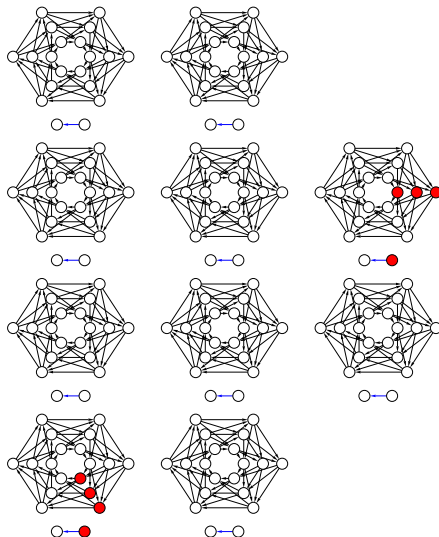
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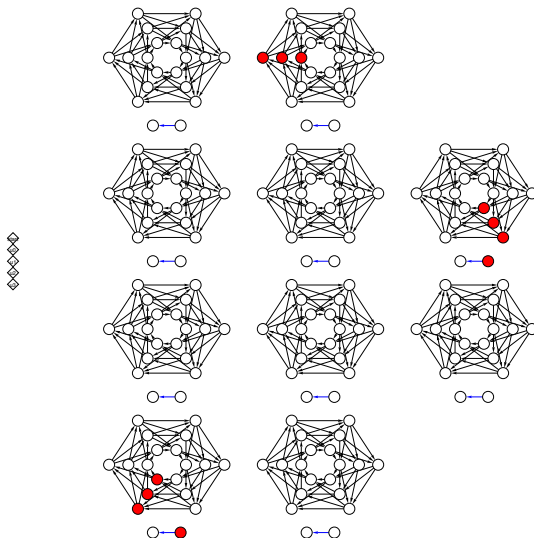
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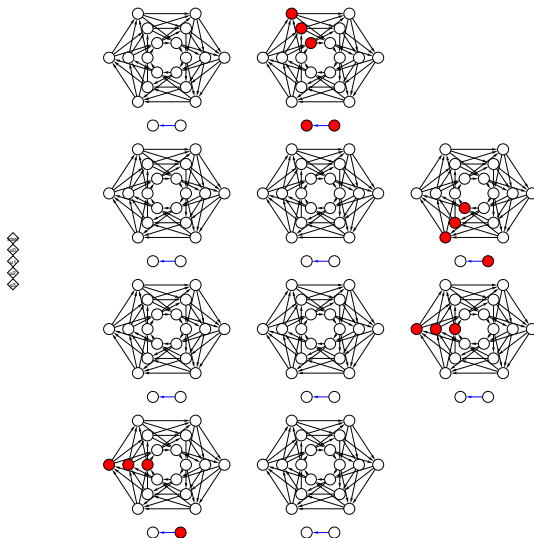
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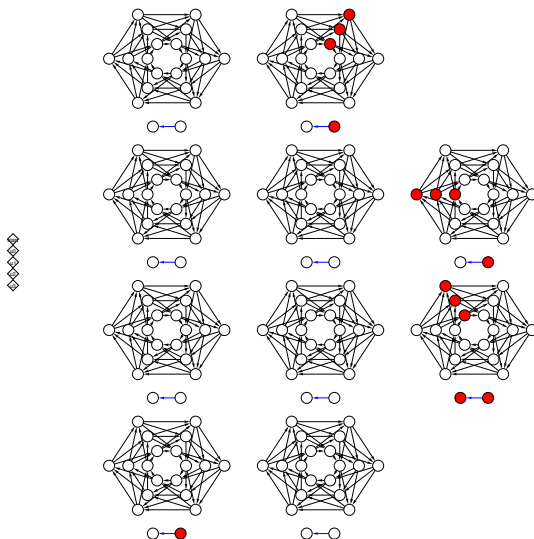
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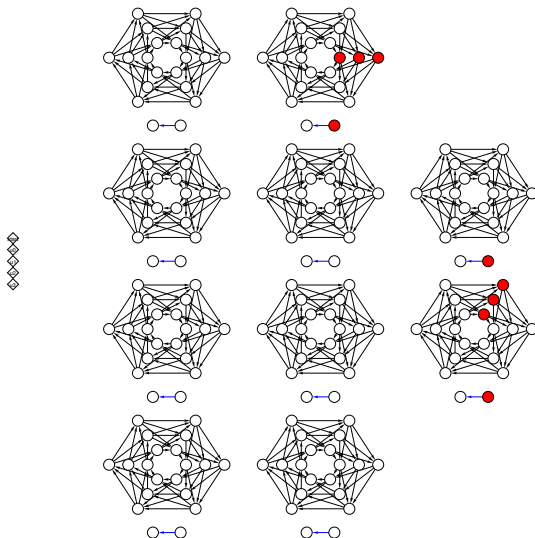
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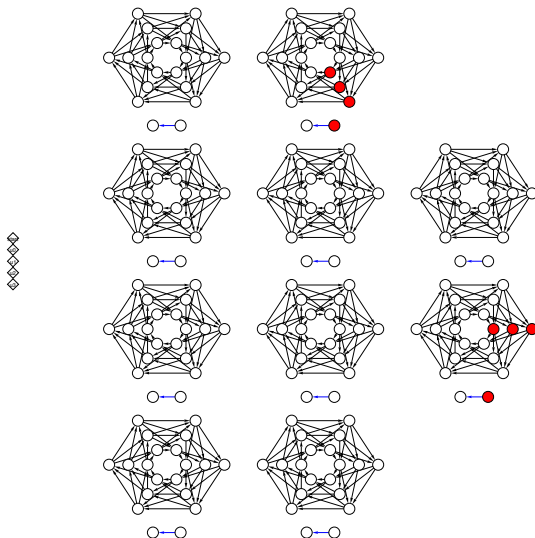
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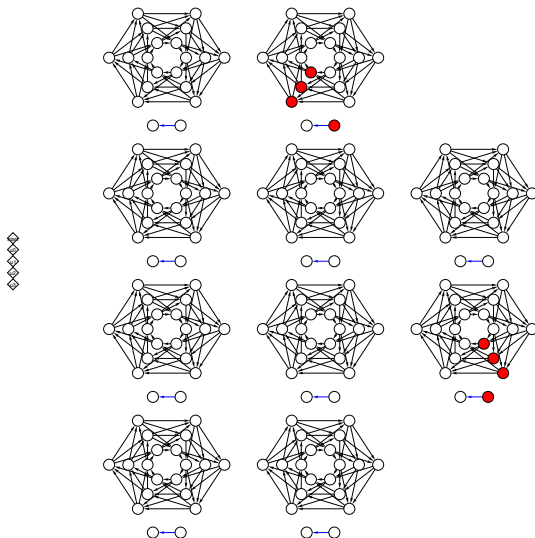
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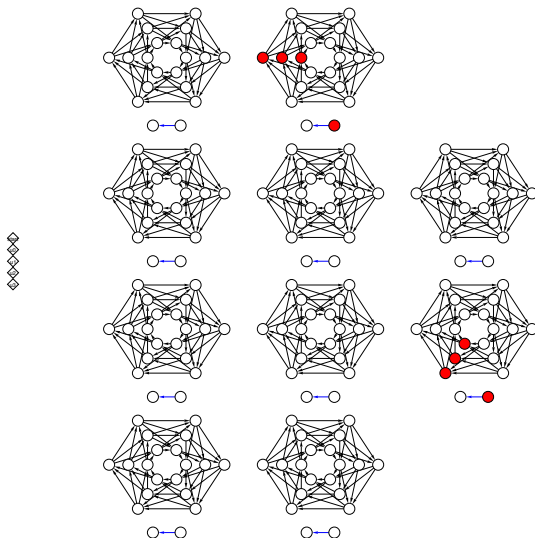
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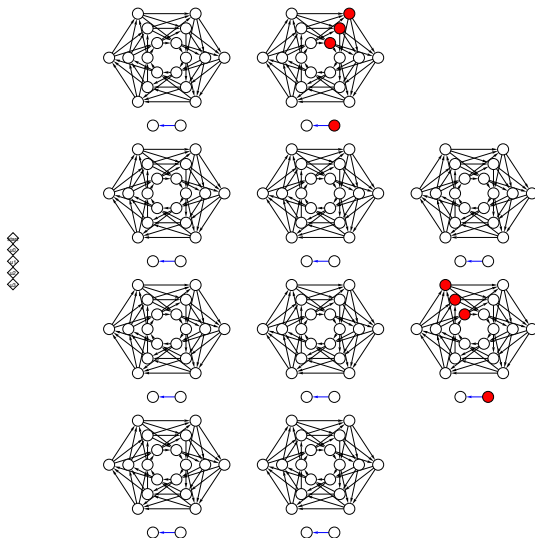
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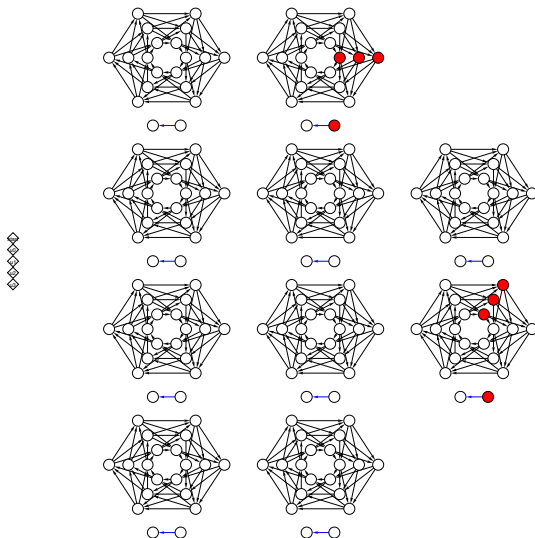
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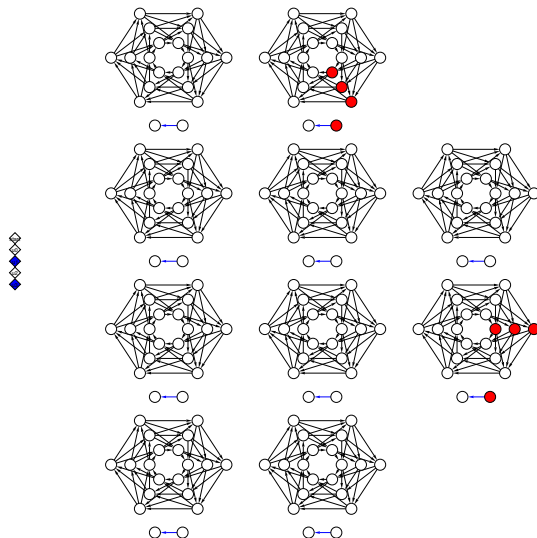
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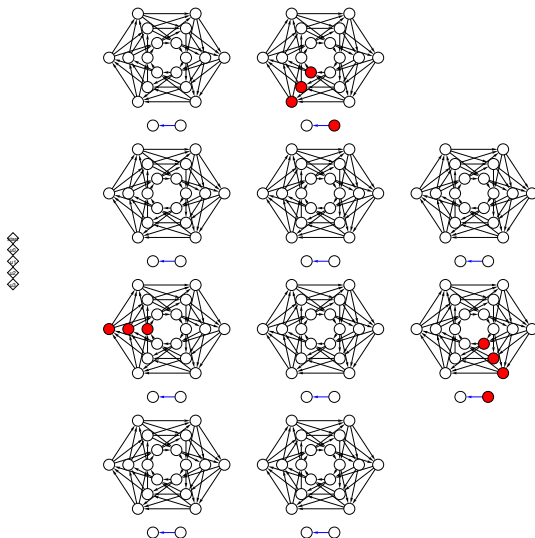
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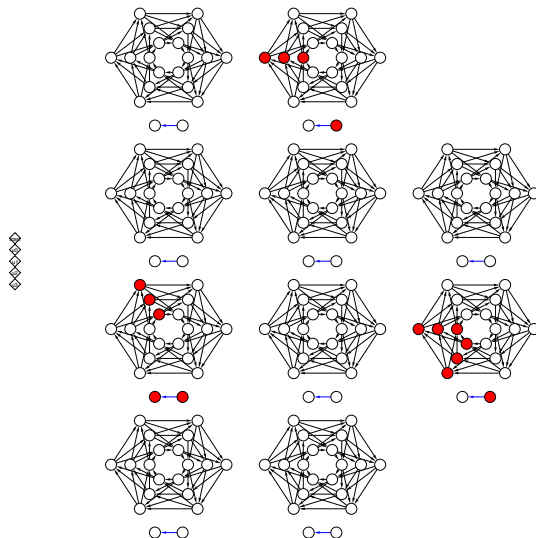
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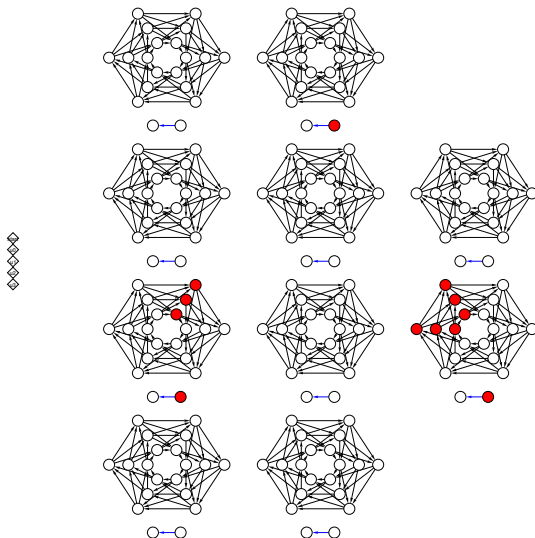
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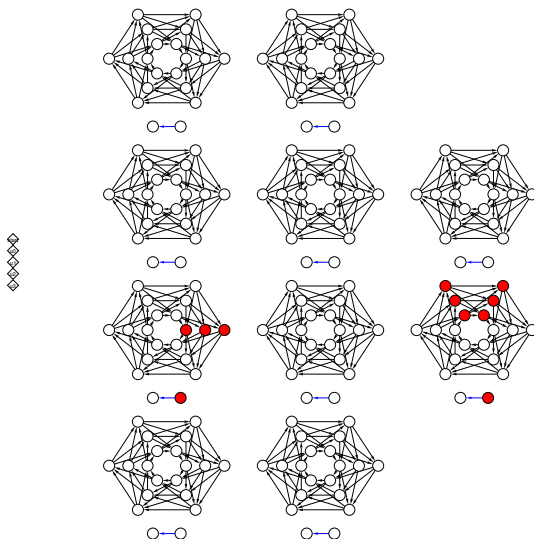
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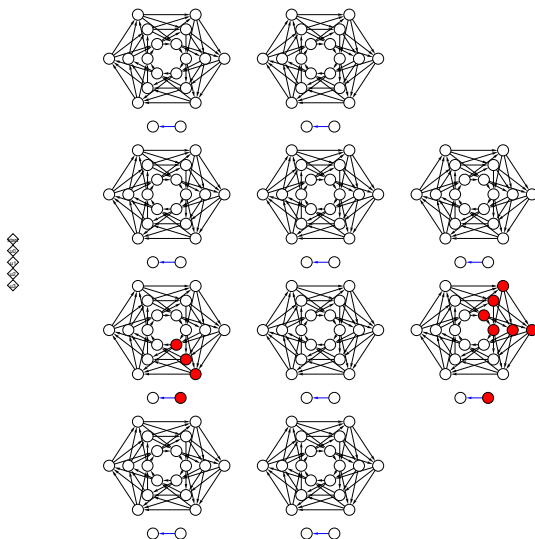
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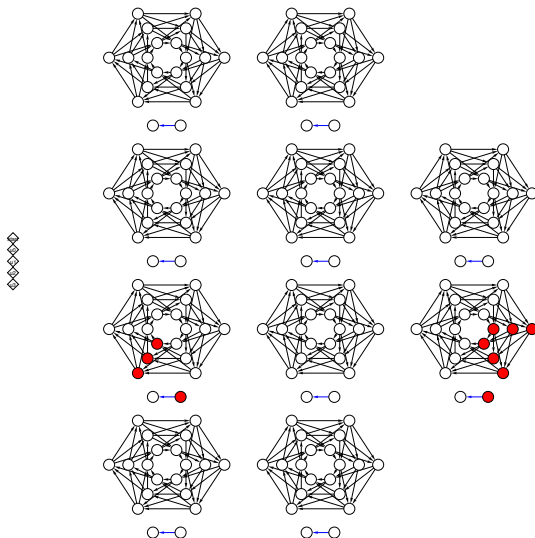
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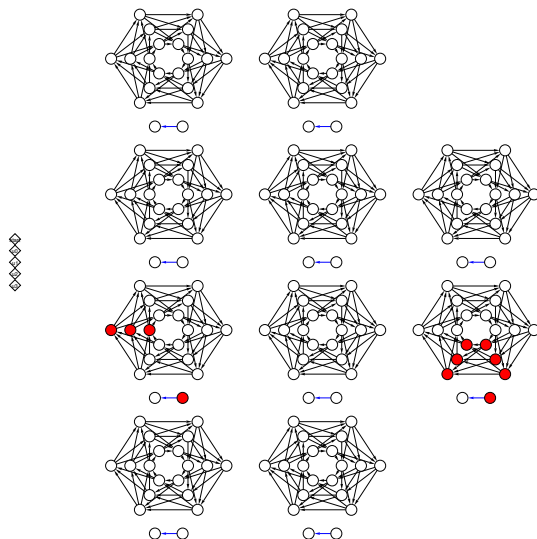
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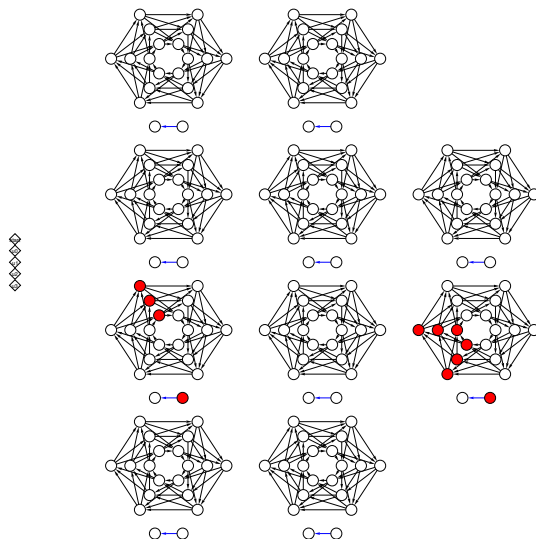
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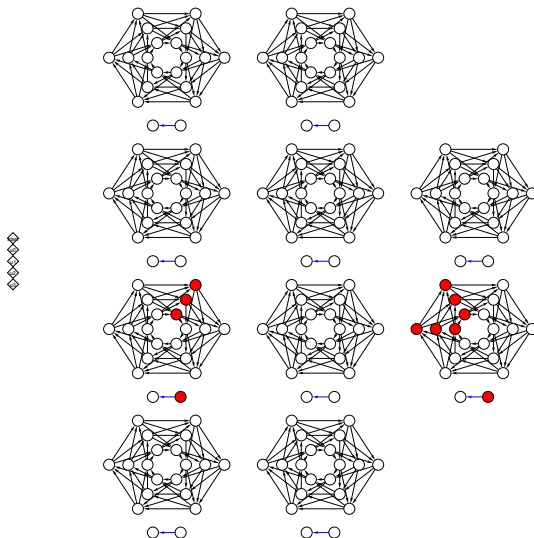
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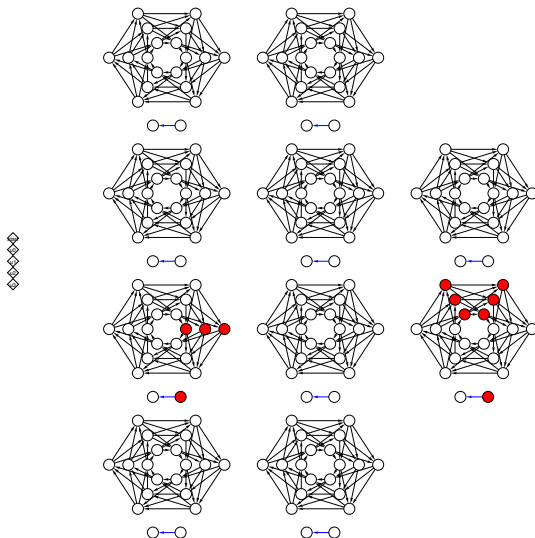
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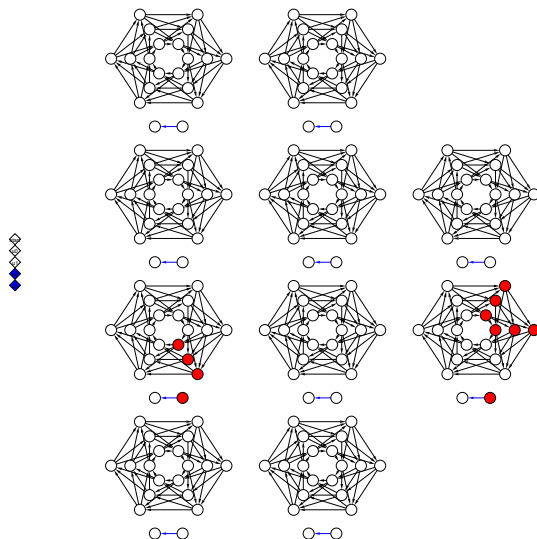
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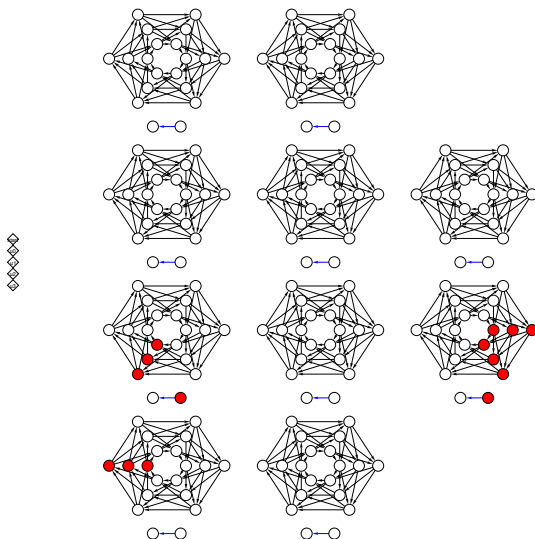
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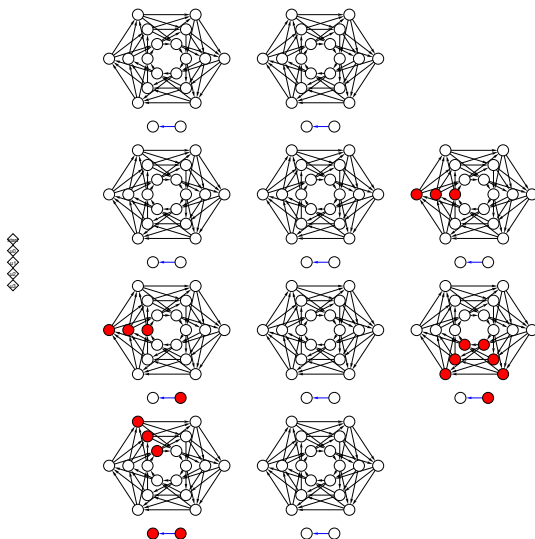
SIMULATION



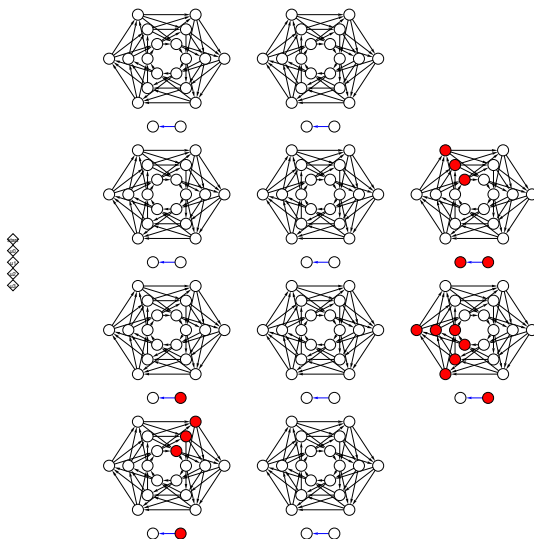
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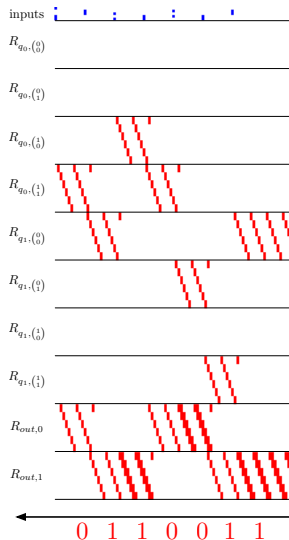
SIMULATION



SIMULATION



SIMULATION



AUTOMATA & BOOLEAN RNNs WITH SYNFIRE RINGS

Since the construction is generic, one has the following result:

THEOREM

Any finite state automaton can be simulated by some Boolean neural network composed of synfire rings.

HODGKIN-HUXLEY NEURONS (SOFTWARE DEMO)

$$\alpha_n(V_m) = \frac{0.01(10 - V_m)}{\exp(\frac{10 - V_m}{10}) - 1}$$

$$\beta_n(V_m) = 0.125 \exp(\frac{-V_m}{80})$$

$$\alpha_m(V_m) = \frac{0.1(25 - V_m)}{\exp(\frac{25 - V_m}{10}) - 1}$$

$$\beta_m(V_m) = 4 \exp(\frac{-V_m}{18})$$

$$\alpha_h(V_m) = 0.07 \exp(\frac{-V_m}{20})$$

$$\beta_h(V_m) = \frac{1}{\exp(\frac{30 - V_m}{10}) + 1}$$

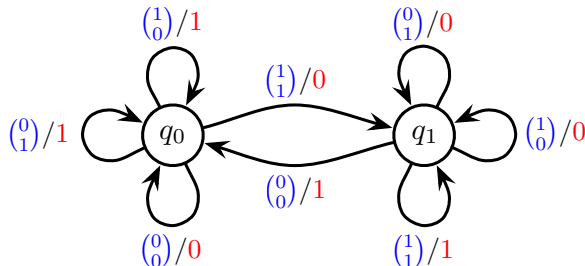
$$\frac{dn}{dt} = \alpha_n(V_m)(1 - n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1 - m) - \beta_m(V_m)m$$

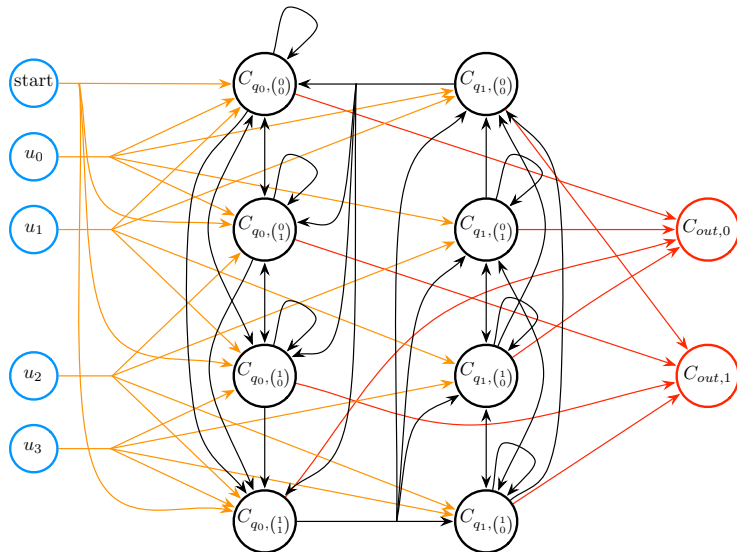
$$\frac{dh}{dt} = \alpha_h(V_m)(1 - h) - \beta_h(V_m)h$$

$$C_m \frac{dV_m}{dt} = I - \bar{g}_K n^4 (V_m - V_K) - \bar{g}_{Na} m^3 h (V_m - V_{Na}) - \bar{g}_l (V_m - V_l)$$

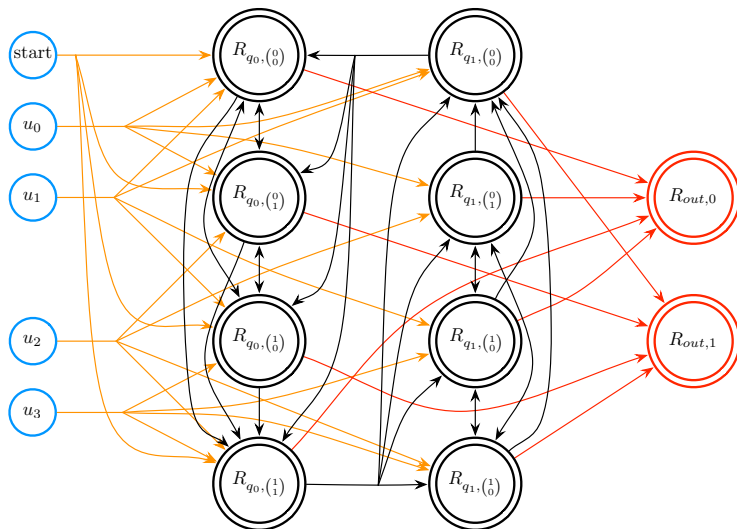
GENERAL CONSTRUCTION



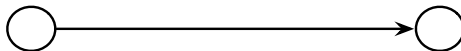
GENERAL CONSTRUCTION



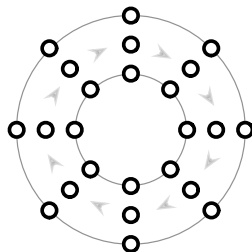
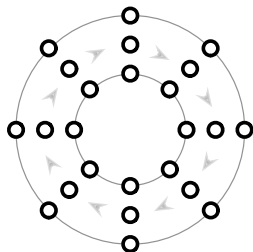
GENERAL CONSTRUCTION



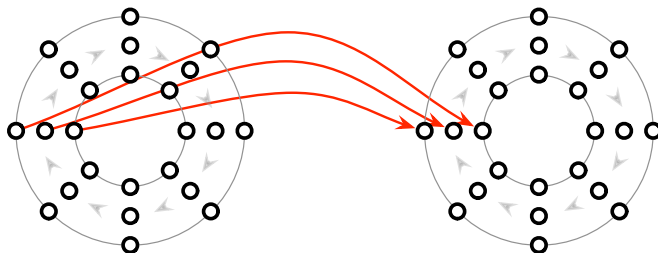
FIBRES OF CONNECTIONS & INHIBITORY SYSTEM



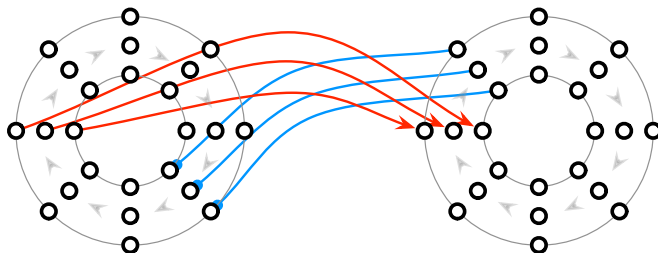
FIBRES OF CONNECTIONS & INHIBITORY SYSTEM



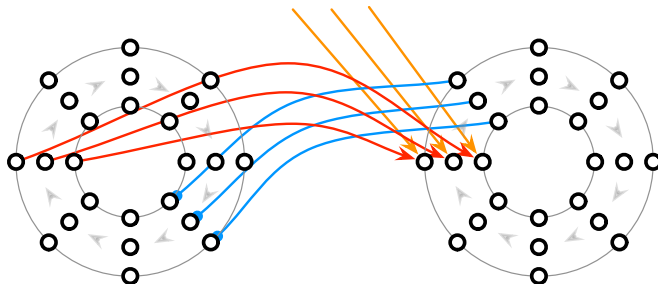
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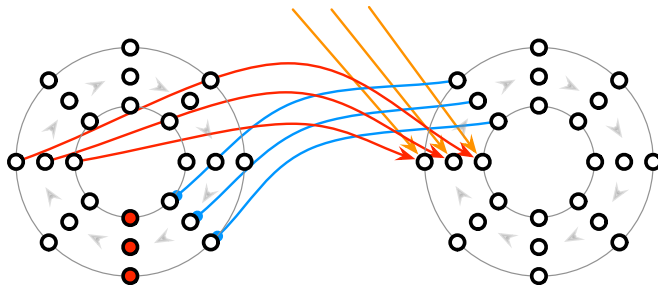
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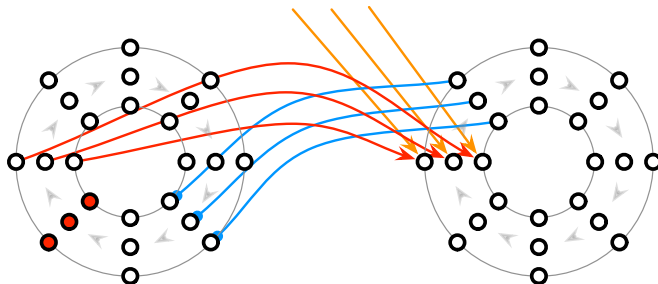
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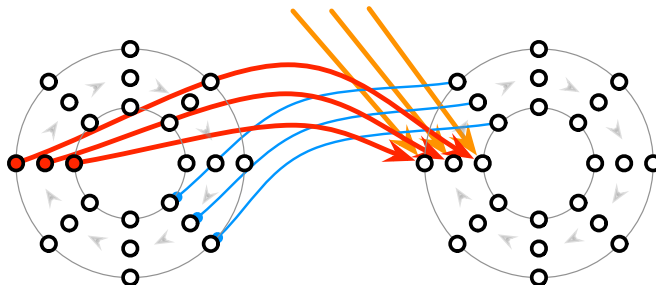
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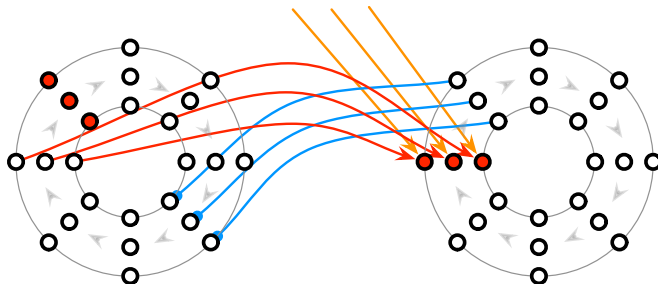
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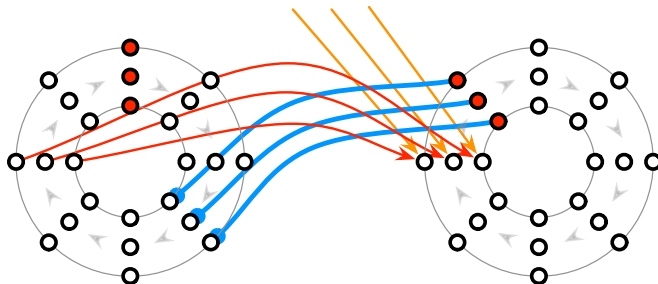
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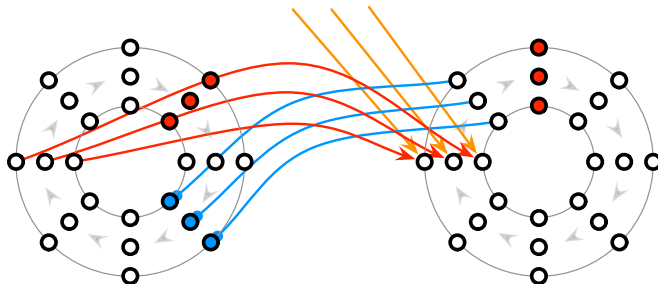
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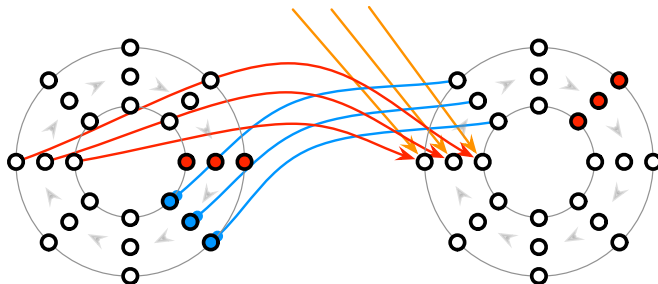
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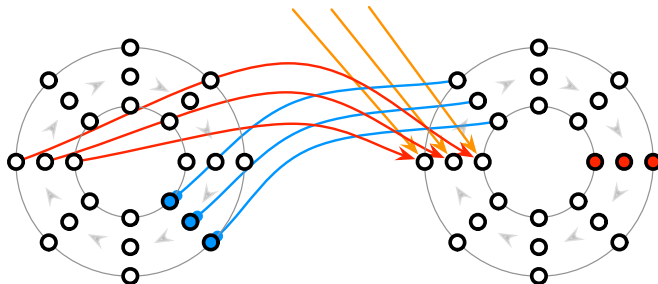
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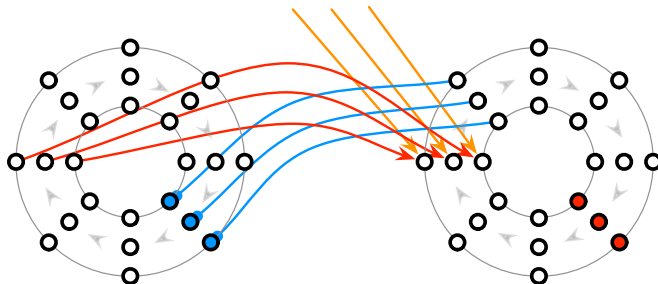
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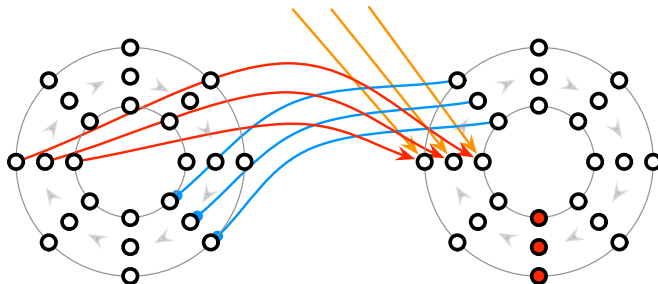
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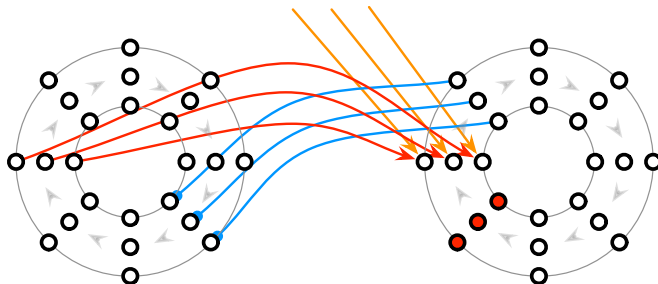
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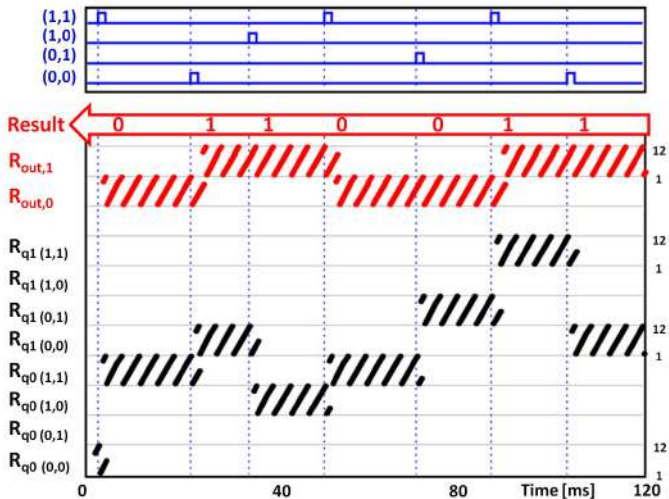
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SIMULATION

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SIMULATION



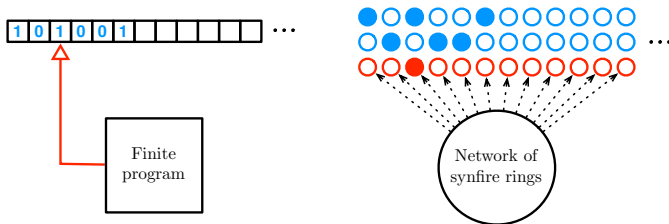
AUTOMATA & HODGKIN-HUXLEY RNNs WITH SYNFIRE RINGS

THEOREM

Any finite state automaton can be simulated by some Hodgkin-Huxley based neural network composed of synfire rings.

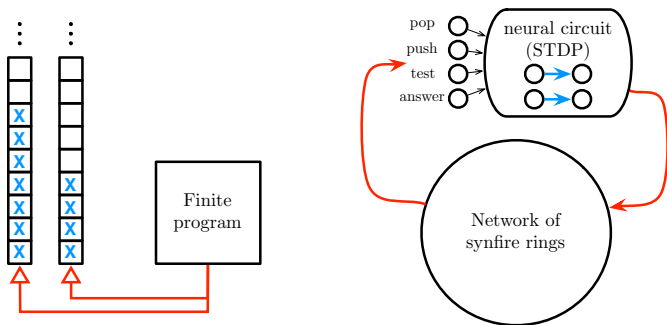
FUTURE WORK: TOWARDS TURING COMPLETENESS

We intend to simulate Turing machines with recurrent neural networks composed of synfire rings.

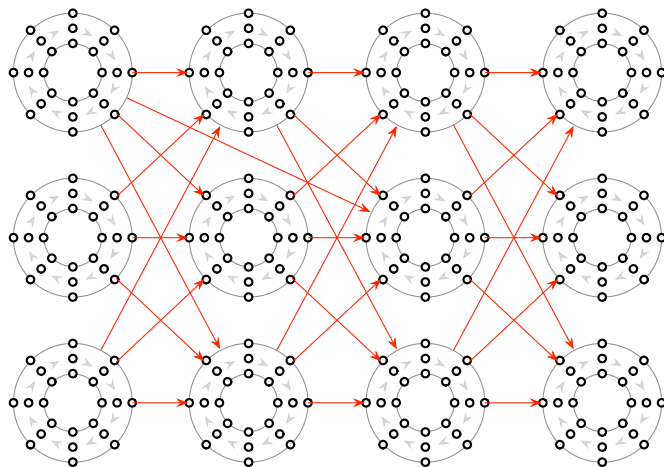


FUTURE WORK: TOWARDS TURING COMPLETENESS

We intend to simulate Turing machines with recurrent neural networks composed of synfire rings.



FUTURE WORK: LEARNING WITHIN THE SYNFIRE RING ARCHITECTURE



CONCLUSIONS

- ▶ Recurrent neural networks represent a natural models for oracle-based computation, beyond the Turing limits.
- ▶ We introduced a new paradigm of neural computation based on the concept of synfire rings.
- ▶ We intend to study the issue of *learning* within the synfire ring architecture.
- ▶ Utopia: achieve the realization of neural computers. By growing cultures of neurons according to the synfire ring architecture, one could in principle realize neural computers.

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