Turing-Complete Computation with Recurrent Neural Networks Composed of Synfire Rings

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Outline

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Introduction

- We recall important results about the sub-Turing, Turing and super-Turing computational powers of recurrent neural networks.
- We introduce a bio-inspired paradigm for neural computation based on the concept of *synfire rings*.
- We show that finite state machines can be simulated by Boolean recurrent neural networks composed of synfire rings.

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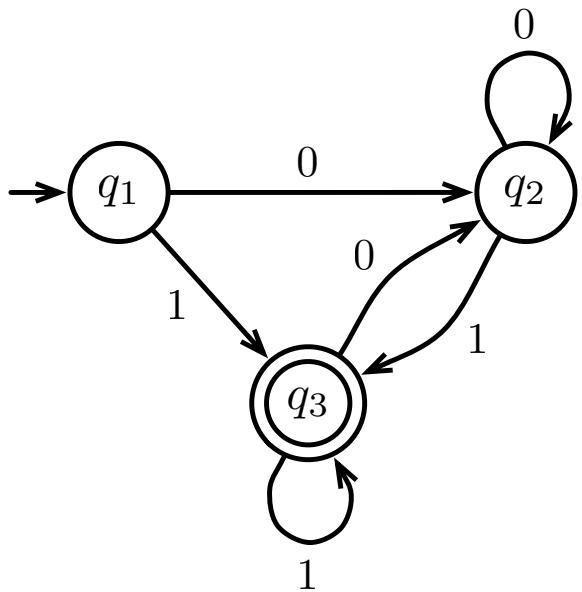
Finite State Automaton

A *finite state automaton* (FSA) can be represented as a graph whose nodes and edges are the computational states and transitions between those.



- input u is *accepted* by $\mathcal{A}$ if $\mathcal{A}(u)$ reaches a final state
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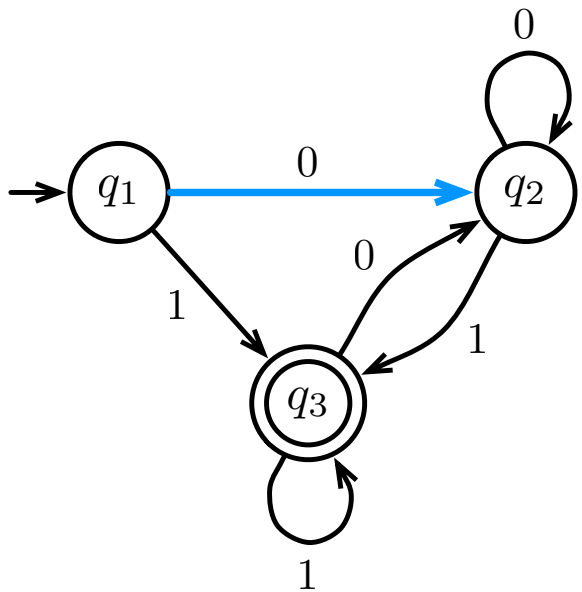

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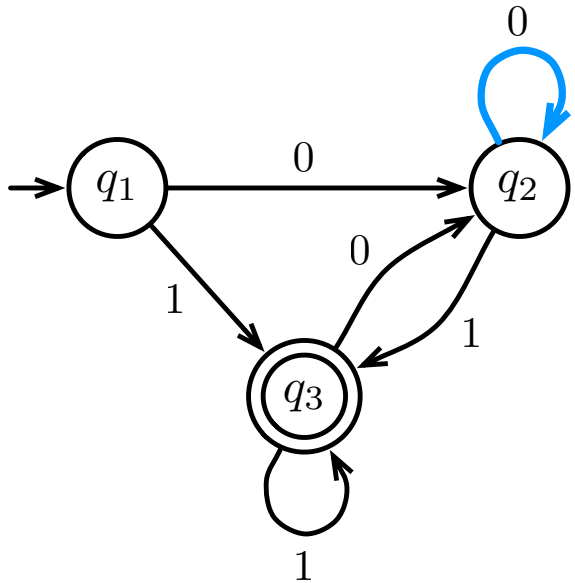

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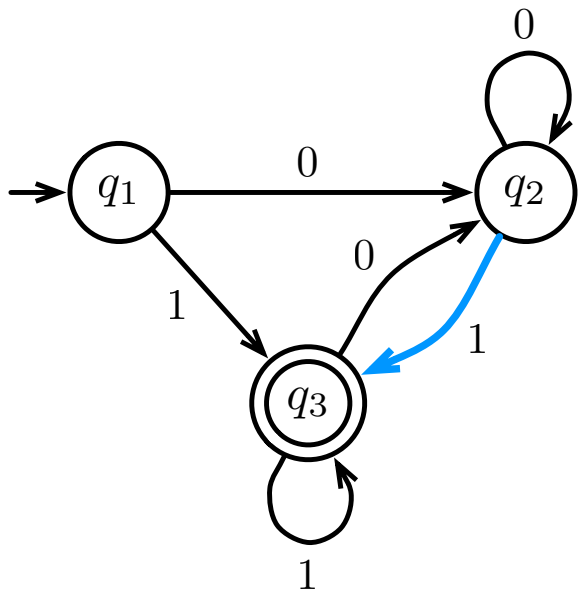

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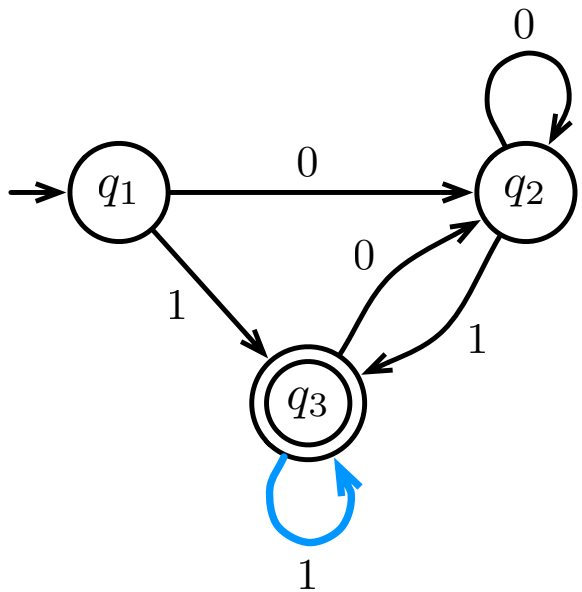

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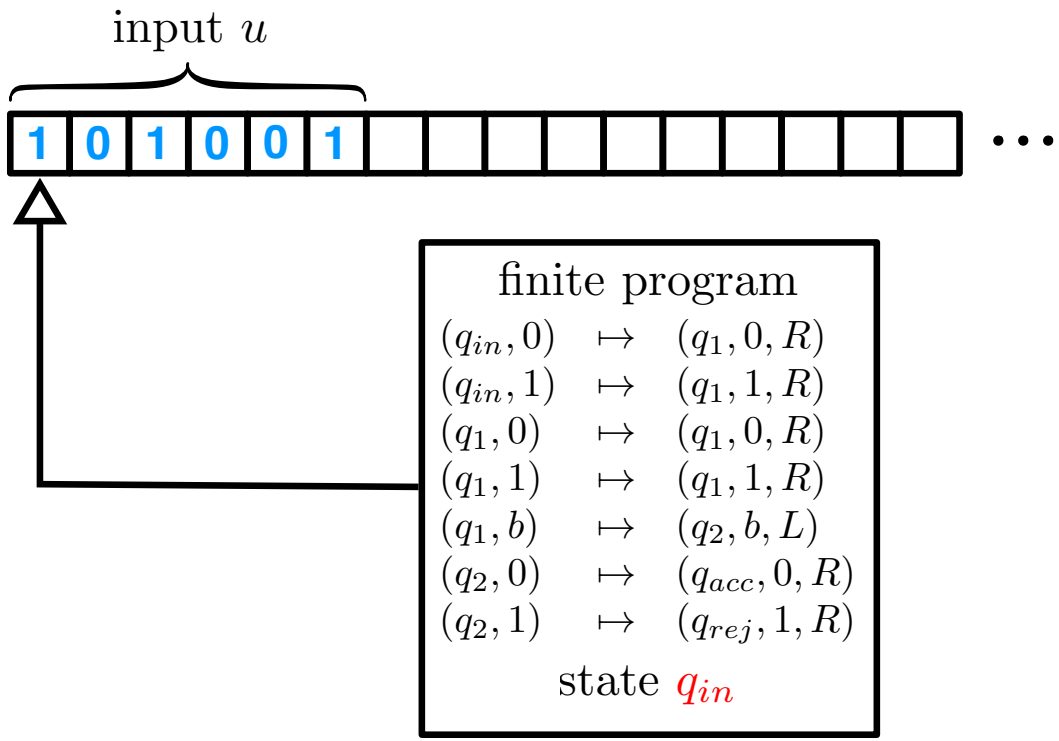

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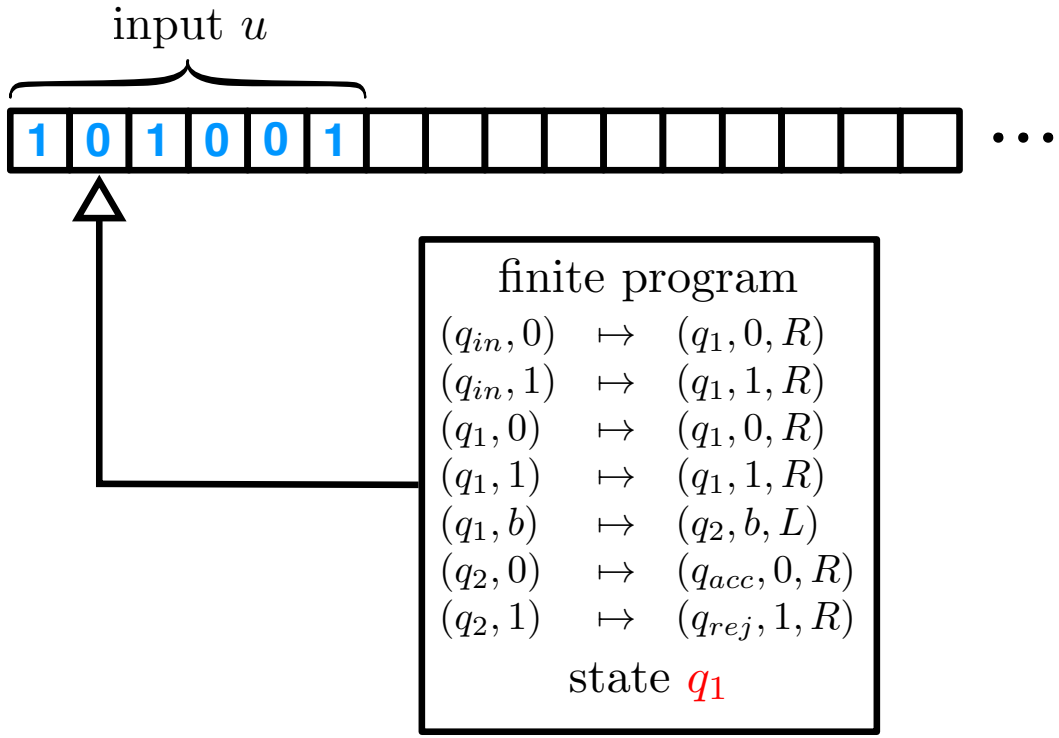

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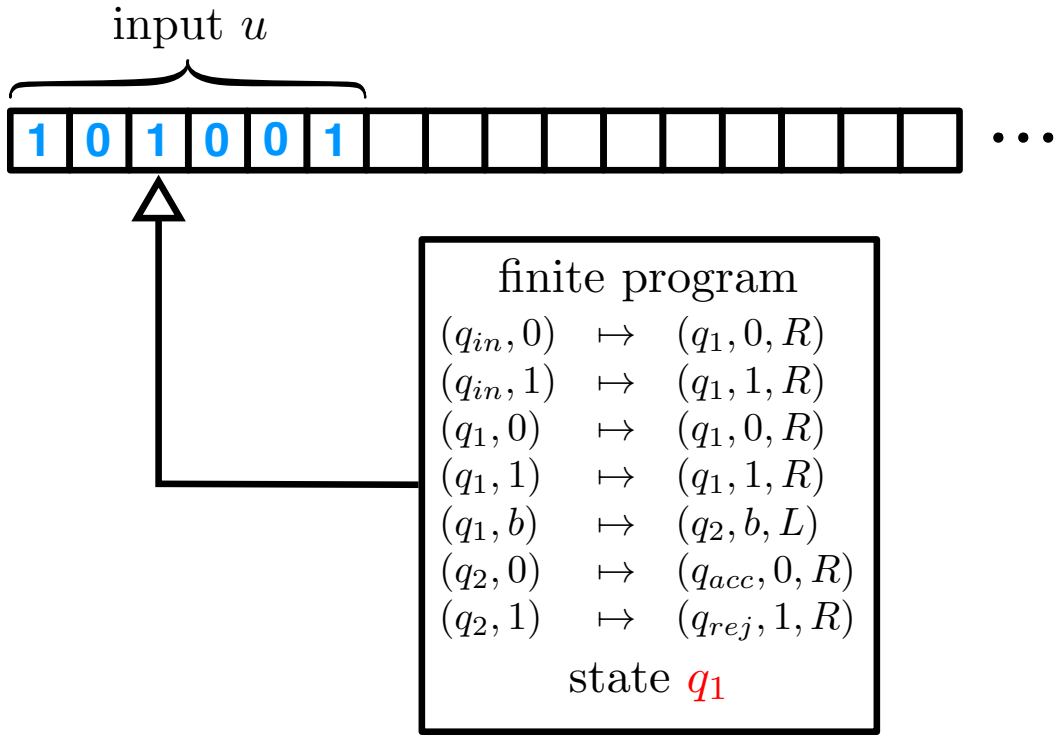

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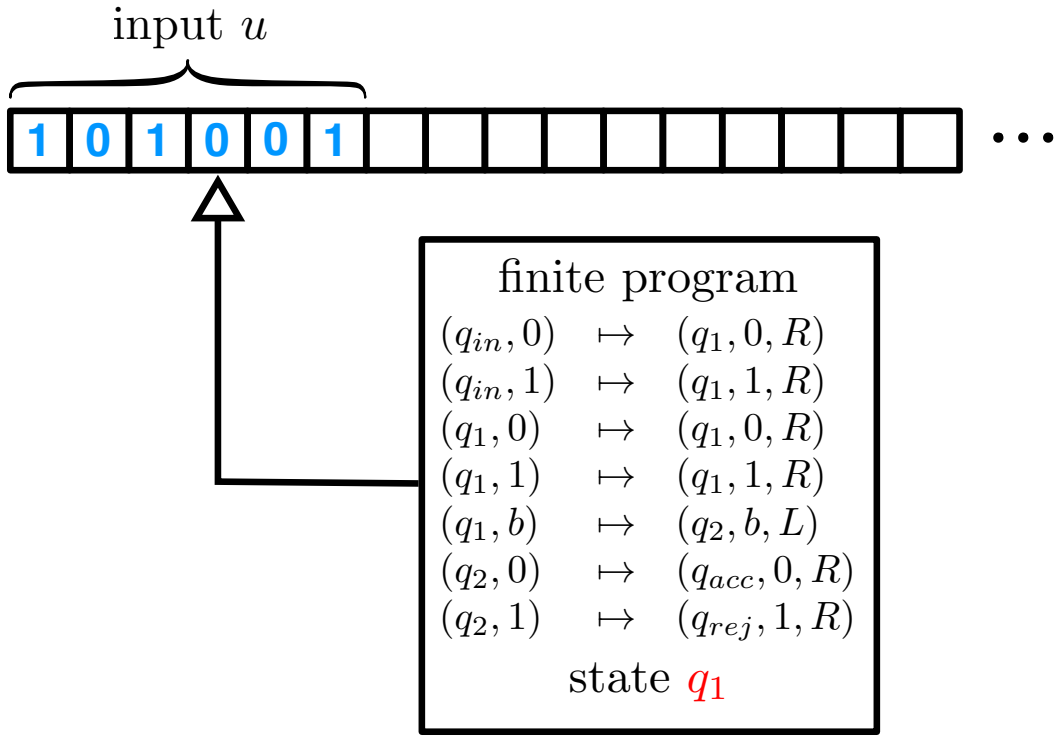

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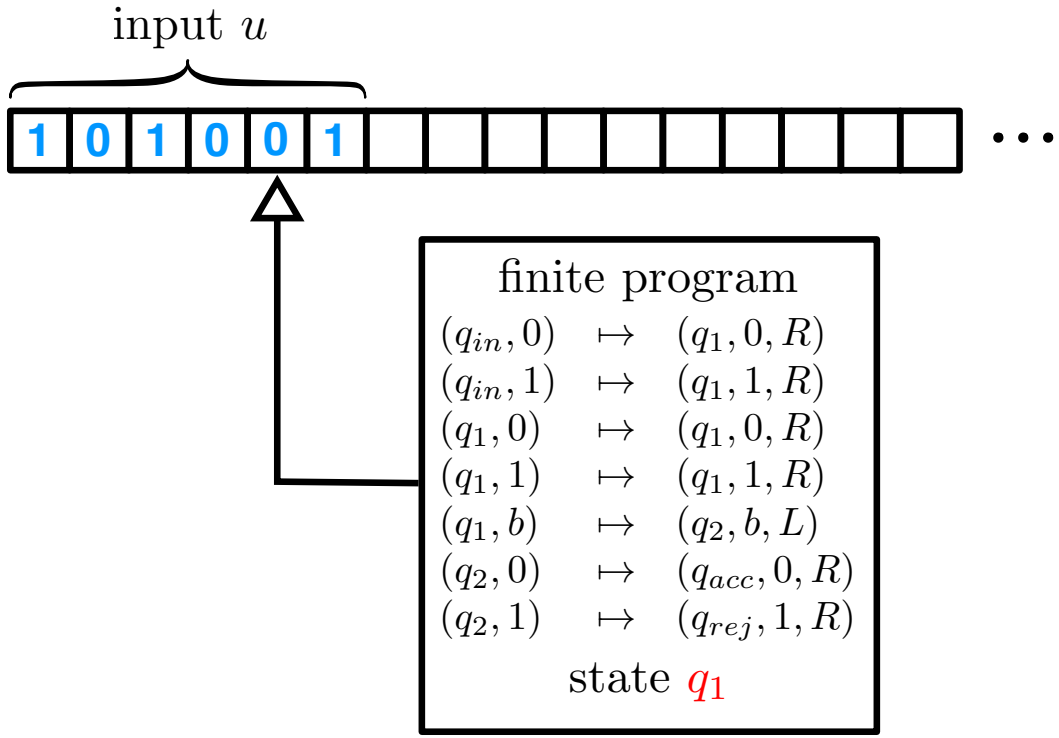

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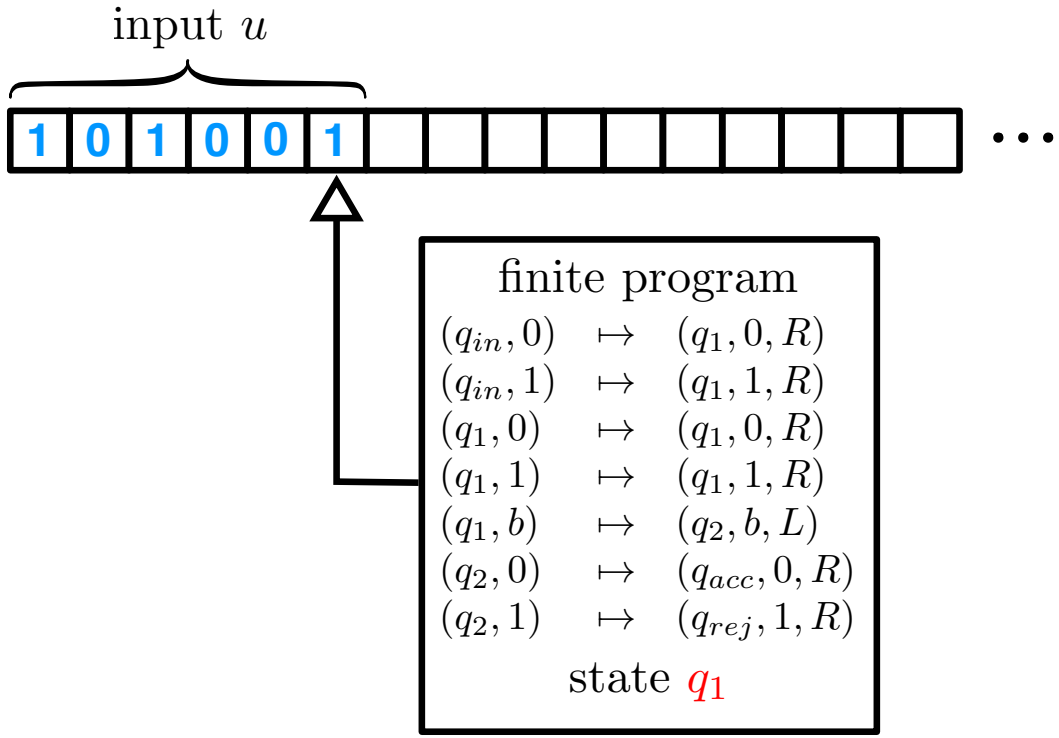

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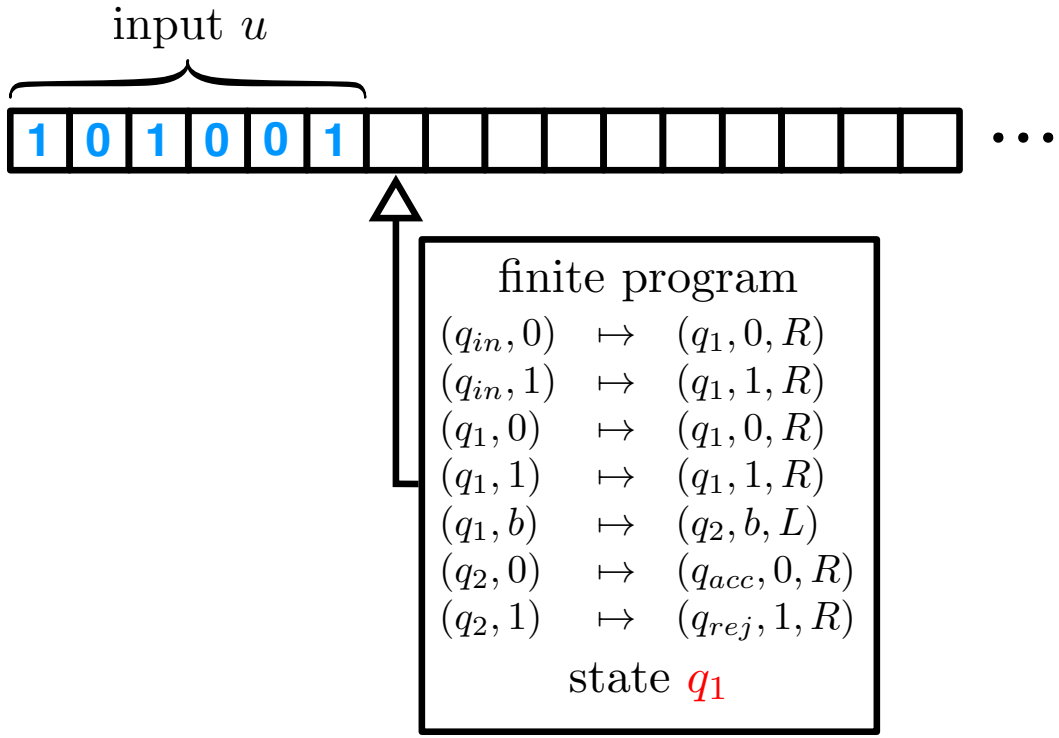

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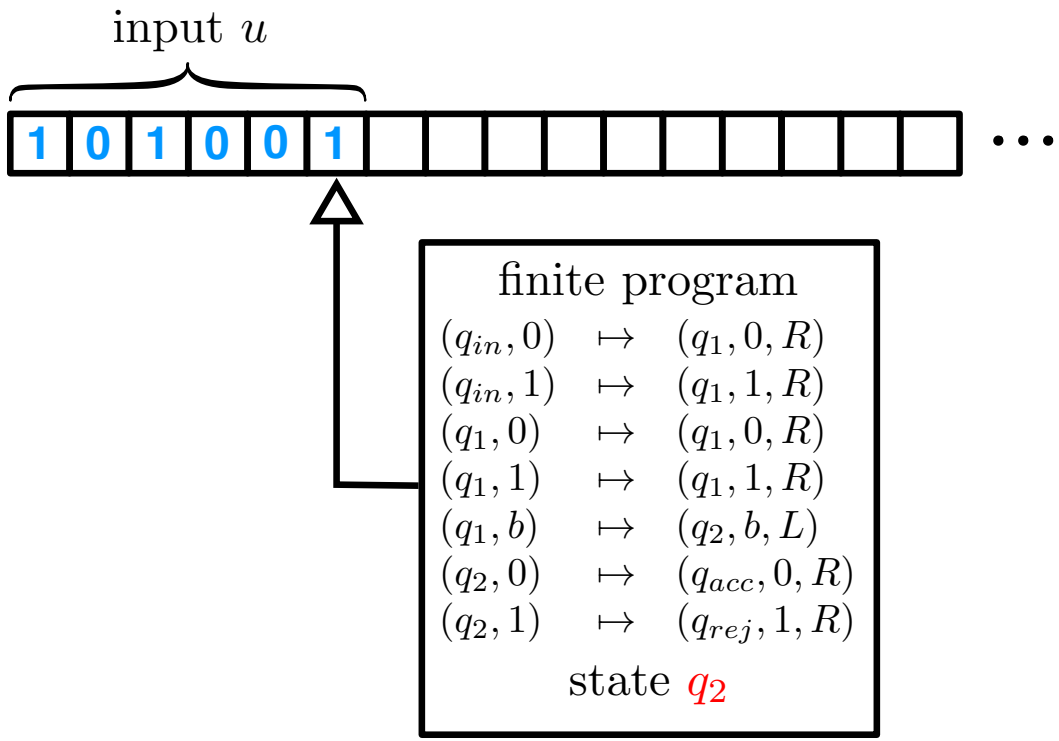

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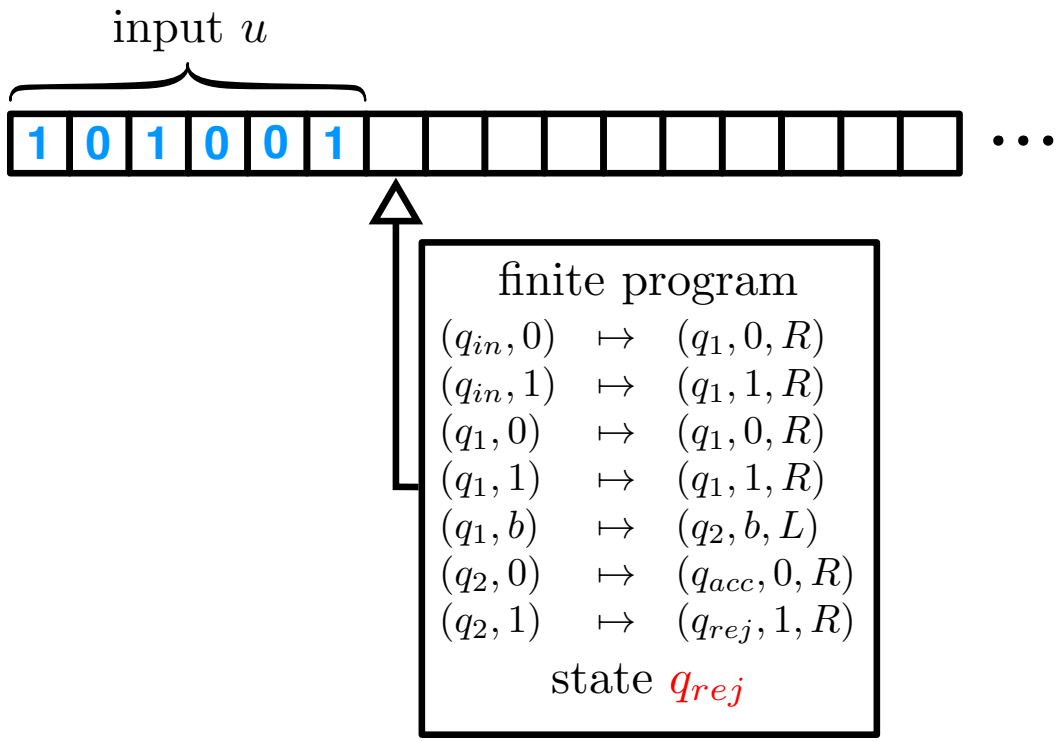

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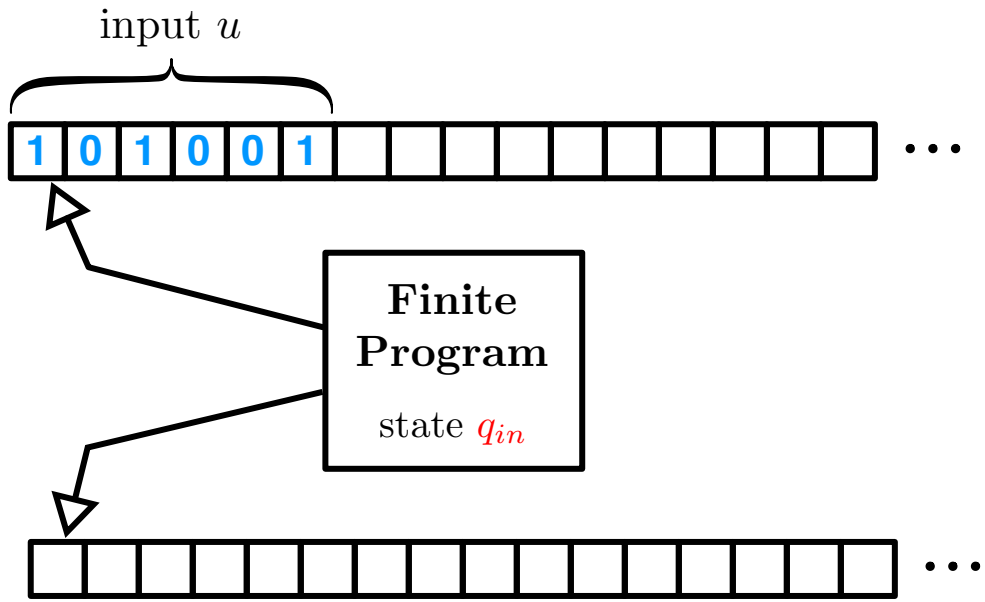
Turing Machine with Advice

A *Turing machine with advice* (TM/A) is a TM provided with an additional advice tape and advice function $\alpha : \mathbb{N} \longrightarrow \{0,1\}^*$.



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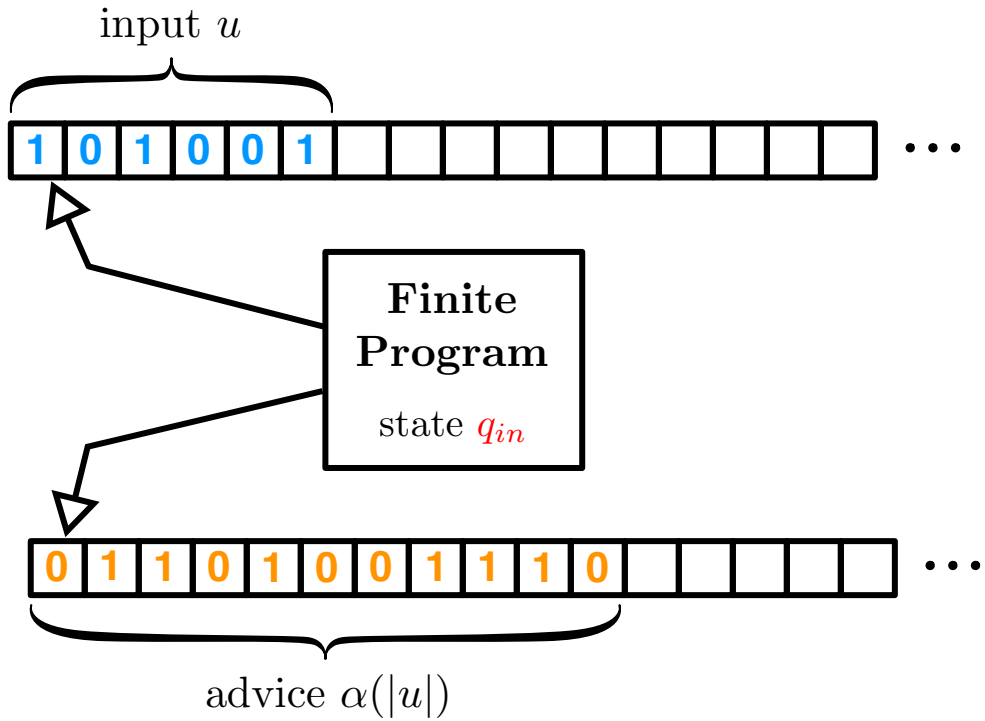

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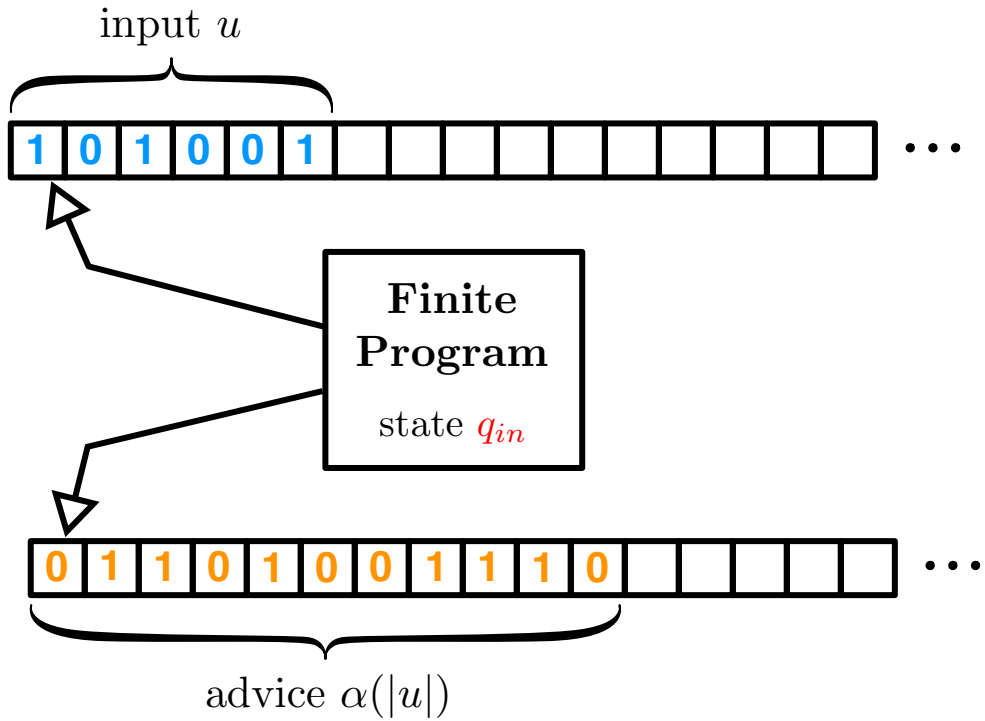

- The class of languages recognized in polynomial time by Turing machines with polynomial advices (TM/poly(A)) is **P/poly**.

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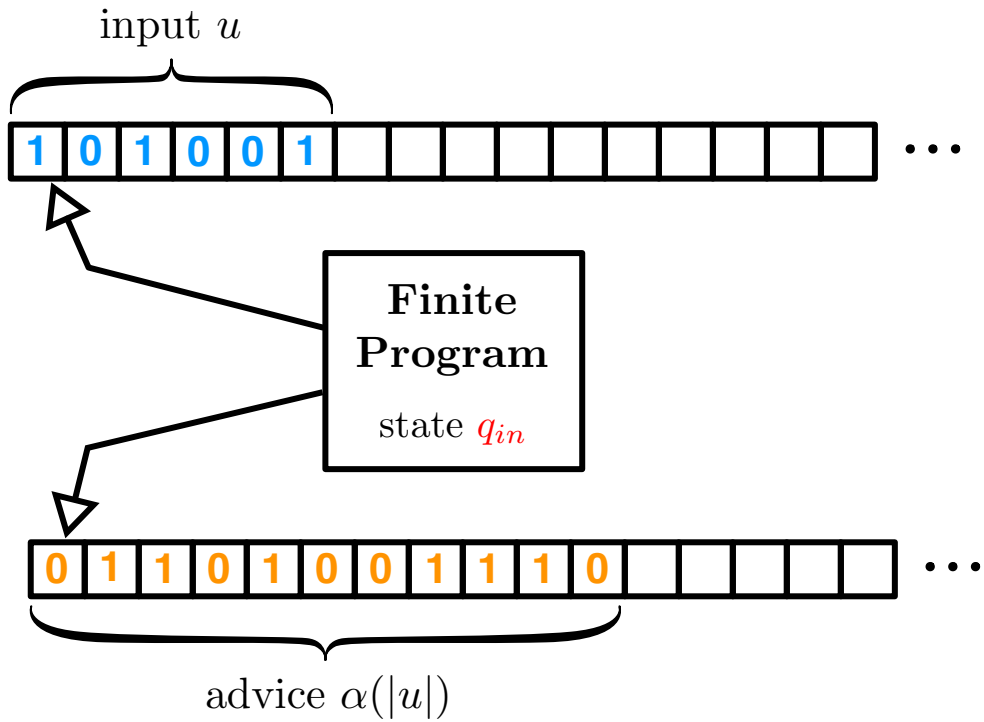
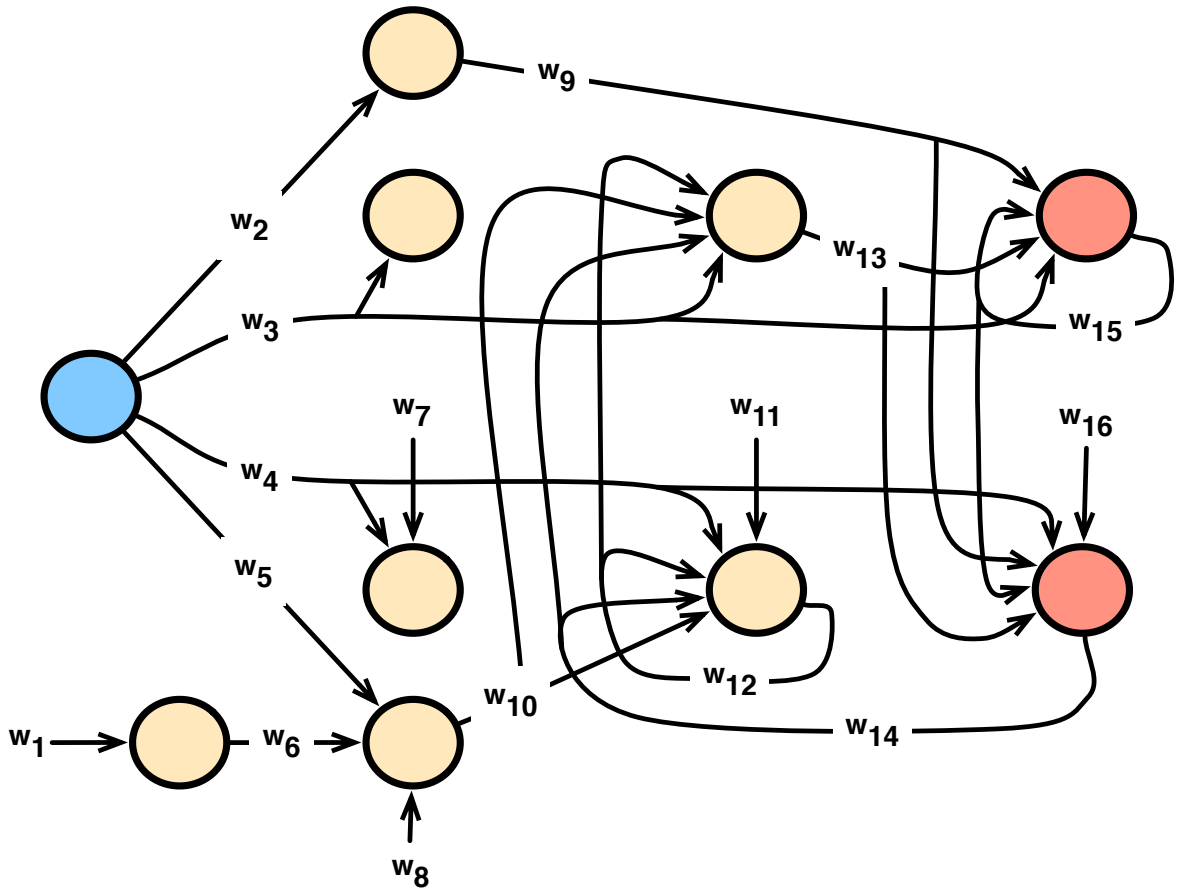
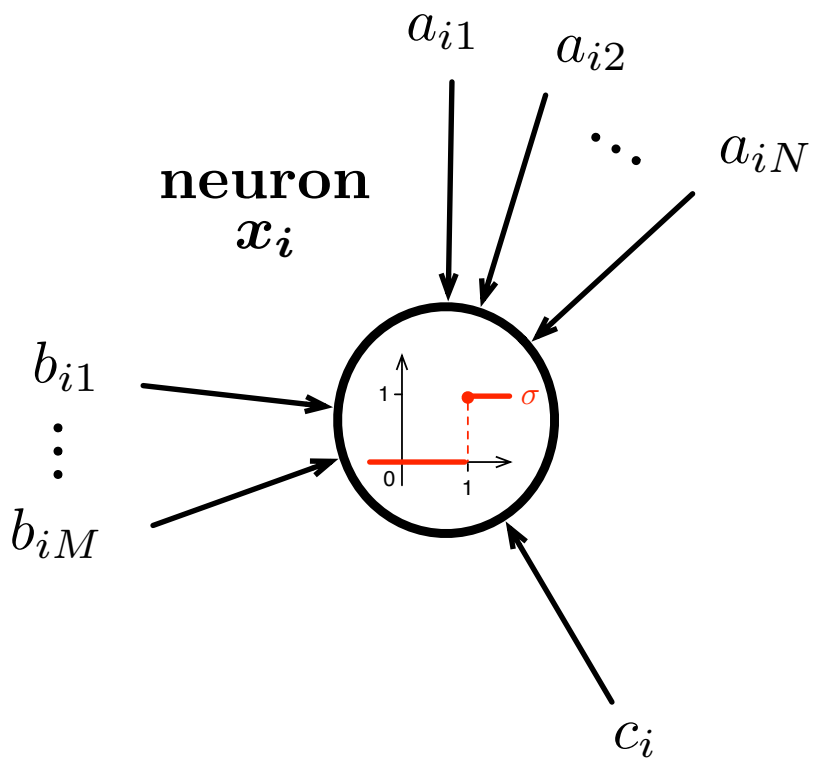


- TM/As are strictly more powerful than TMs: **P/poly** $\supsetneq$ **P**
  They are *super-Turing*...

Recurrent Neural Network

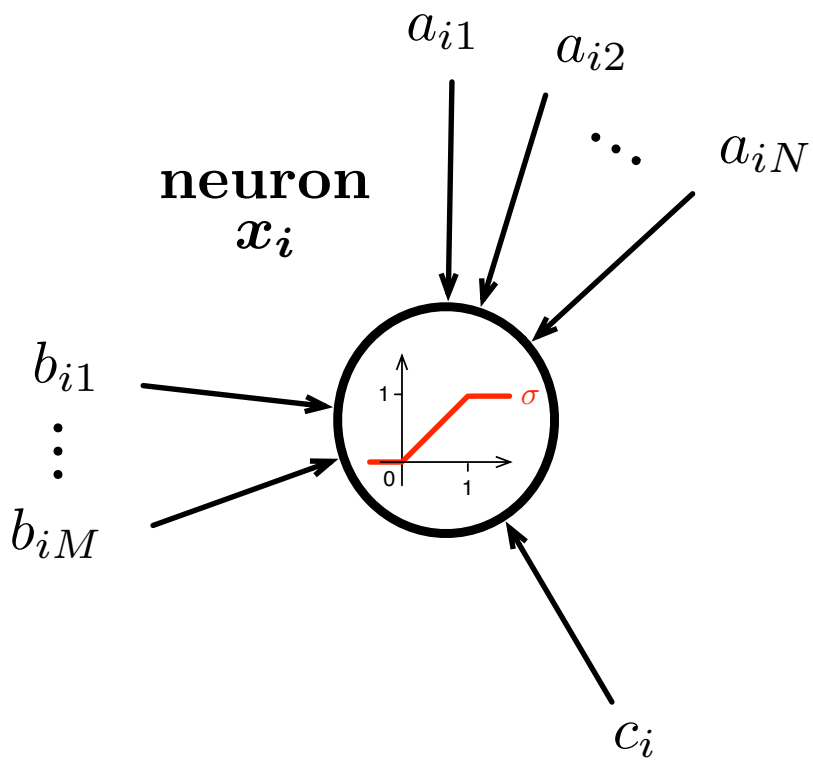


Boolean Recurrent Neural Networks



$$x_i(t+1) = \theta\left(\sum_{j=1}^{N} a_{ij} \cdot x_j(t) + \sum_{j=1}^{M} b_{ij} \cdot u_j(t) + c_i\right)$$

Sigmoid Recurrent Neural Networks



$$x_i(t+1) = \sigma\left(\sum_{j=1}^{N} a_{ij} \cdot x_j(t) + \sum_{j=1}^{M} b_{ij} \cdot u_j(t) + c_i\right)$$

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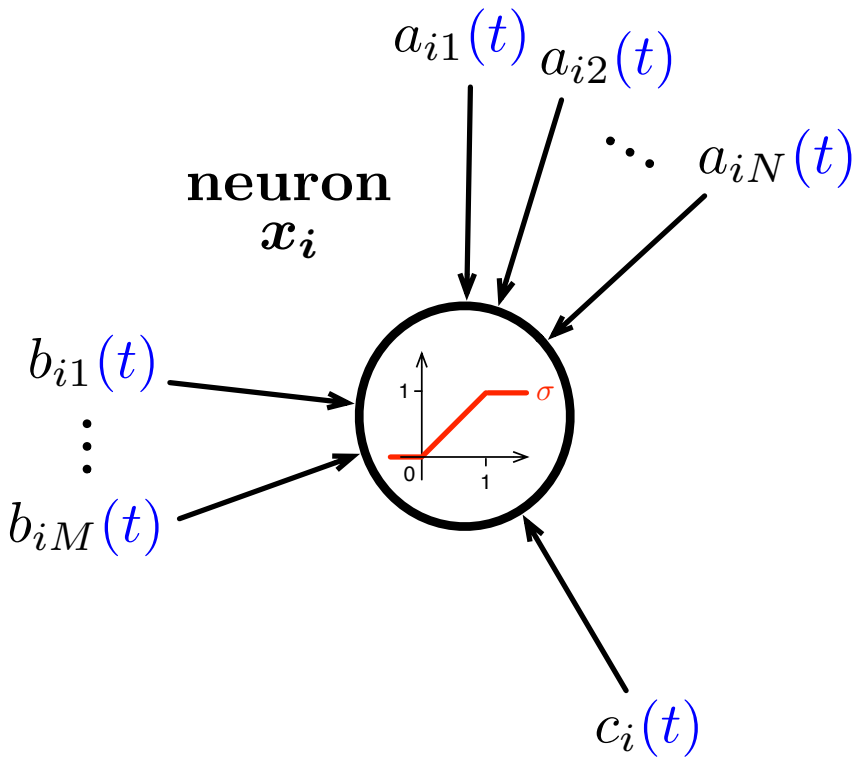

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Recurrent Neural Networks

1. Boolean rational RNNs: B-RNN[$\mathbb{Q}$]s
2. Boolean real RNNs: B-RNN[$\mathbb{R}$]s
3. Sigmoid static rational RNNs: St-RNN[$\mathbb{Q}$]s
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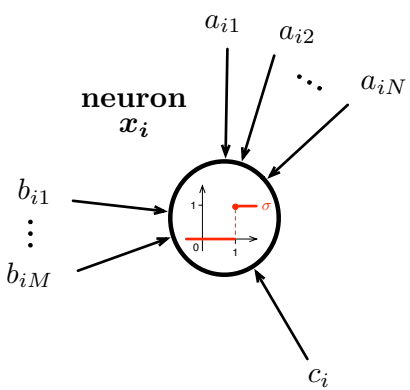

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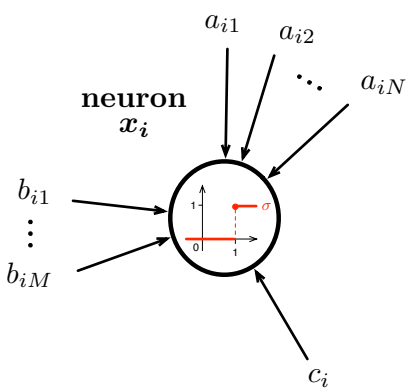

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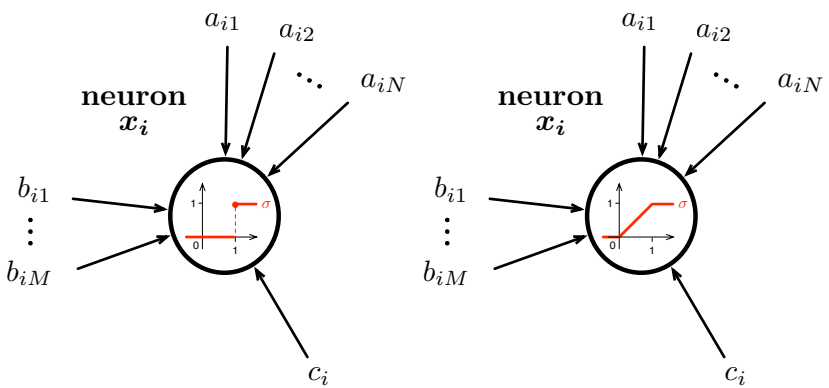

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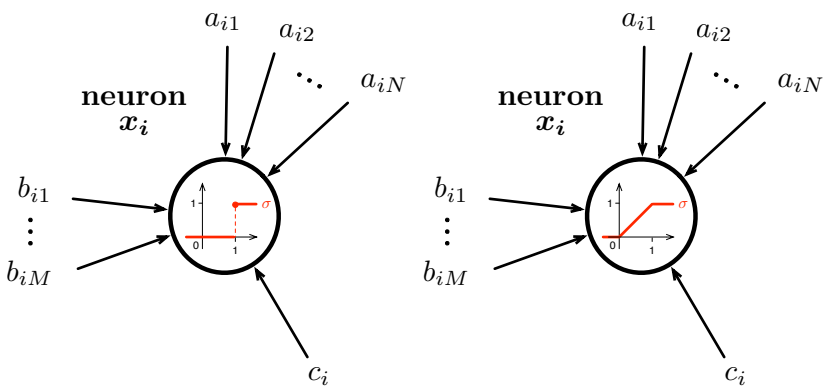

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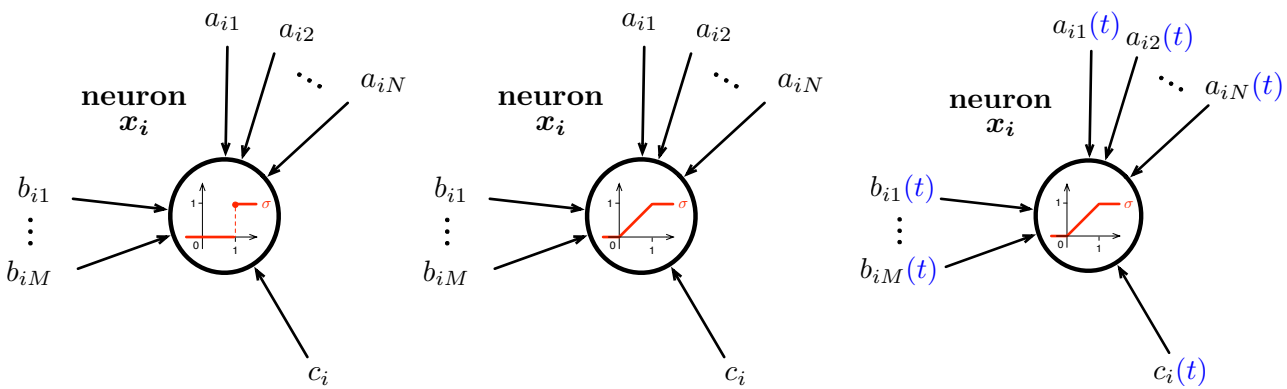

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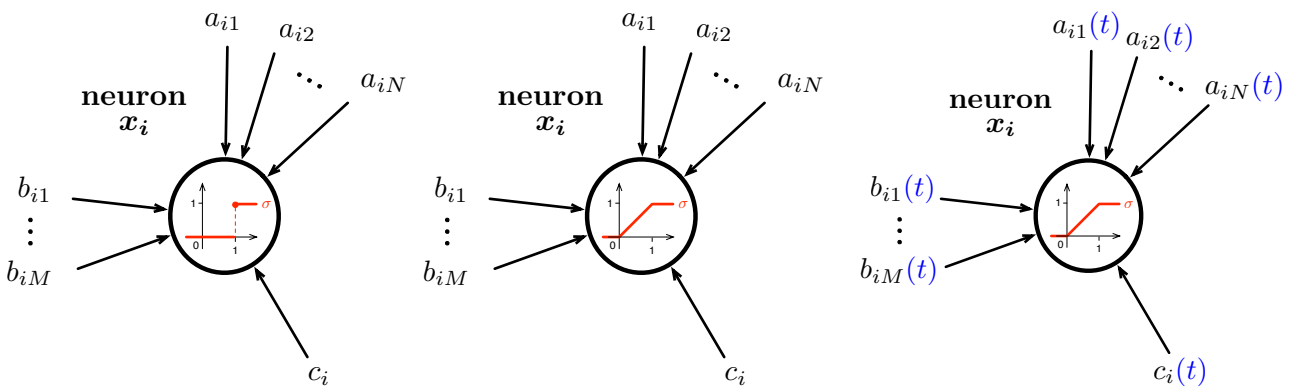

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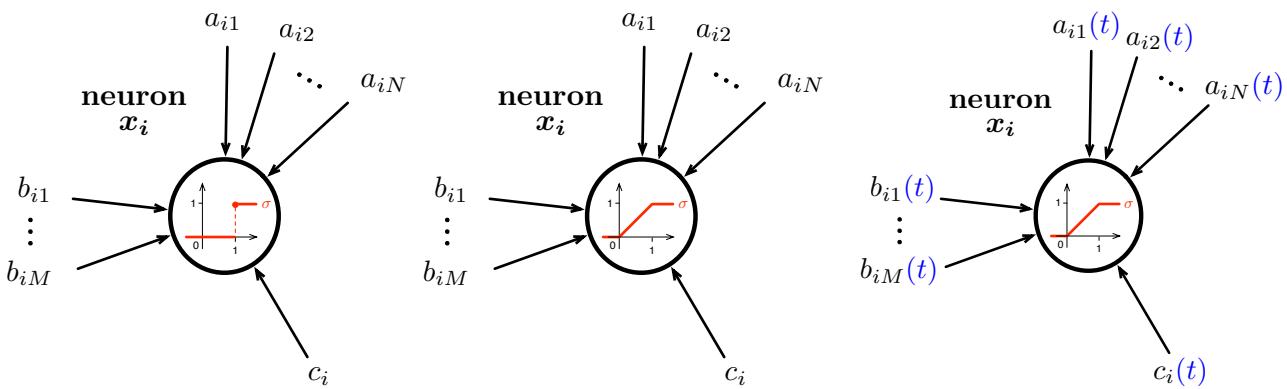

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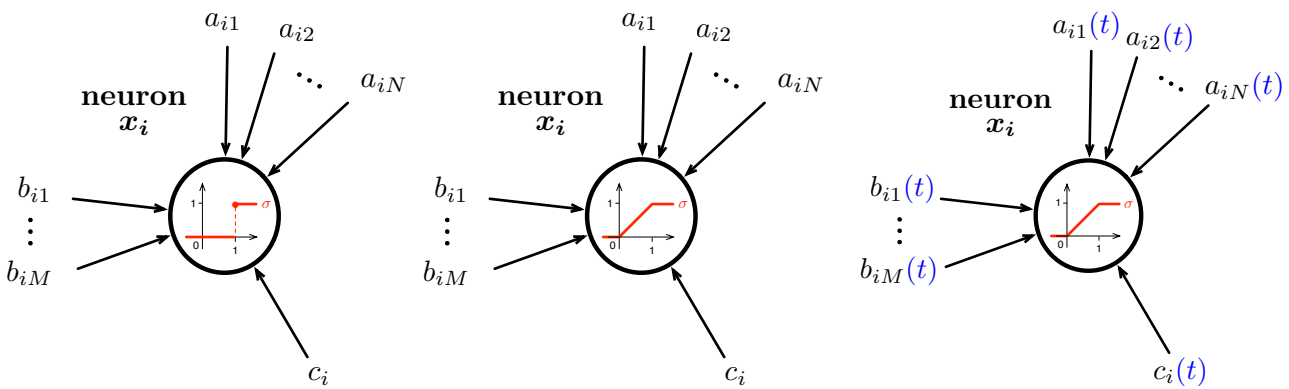

Results: classical computation

	Boolean	Sigmoid		
		Static	Bi-valued Evolving	Evolving
$\mathbb{Q}$	FSA	TM	TM/poly(A)	TM/poly(A)
	REG	**P**	**P/poly**	**P/poly**
	KI 56, Mi 67	Si & So 95	Ca & Si 11,14	Ca & Si 11,14
$\mathbb{R}$	FSA	TM/poly(A)	TM/poly(A)	TM/poly(A)
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	REG	**P/poly**	**P/poly**	**P/poly**
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	Kl 56, Mi 67	Si & So 95	Ca & Si 11,14	Ca & Si 11,14
$\mathbb{R}$	FSA	TM/poly(A)	TM/poly(A)	TM/poly(A)
	REG	**P/poly**	**P/poly**	**P/poly**
	Kl 56, Mi 67	Si & So 94	Ca & Si 11,14	Ca & Si 11,14

Results: classical computation

	Boolean	Sigmoid		
		Static	Bi-valued Evolving	Evolving
$\mathbb{Q}$	FSA **REG** Kl 56, Mi 67	TM **P** Si & So 95	TM/poly(A) **P/poly** Ca & Si 11,14	TM/poly(A) **P/poly** Ca & Si 11,14
$\mathbb{R}$	FSA **REG** Kl 56, Mi 67	TM/poly(A) **P/poly** Si & So 94	TM/poly(A) **P/poly** Ca & Si 11,14	TM/poly(A) **P/poly** Ca & Si 11,14

Results: classical computation

	Boolean	Sigmoid		
		Static	Bi-valued Evolving	Evolving
$\mathbb{Q}$	FSA **REG** Kl 56, Mi 67	TM **P** Si & So 95	TM/poly(A) **P/poly** Ca & Si 11,14	TM/poly(A) **P/poly** Ca & Si 11,14
$\mathbb{R}$	FSA **REG** Kl 56, Mi 67	TM/poly(A) **P/poly** Si & So 94	TM/poly(A) **P/poly** Ca & Si 11,14	TM/poly(A) **P/poly** Ca & Si 11,14

Results: infinite computation / det.

	Boolean	Sigmoid		
		Static	Bi-valued Evolving	Evolving
$\mathbb{Q}$	Muller FSA $\in BC(\Pi_2^0)$ Ca & Vi 10,14	Muller TM $= BC(\Pi_2^0)$ Ca & Vi 15,16	super-Turing $= BC(\mathbf{\Pi_2^0})$ Ca & Vi 15,16	super-Turing $= BC(\mathbf{\Pi_2^0})$ Ca & Vi 15,16
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Results: infinite computation / nondet.

	Boolean	Sigmoid		
		Static	Bi-valued Evolving	Evolving
$\mathbb{Q}$	Muller FSA $\in \Sigma_1^1$ Ca & Vi 10,14	Muller TM $= \Sigma_1^1$ Ca & Vi 15,16	super-Turing $= \mathbf{\Sigma_1^1}$ Ca & Vi 15,16	super-Turing $= \mathbf{\Sigma_1^1}$ Ca & Vi 15,16
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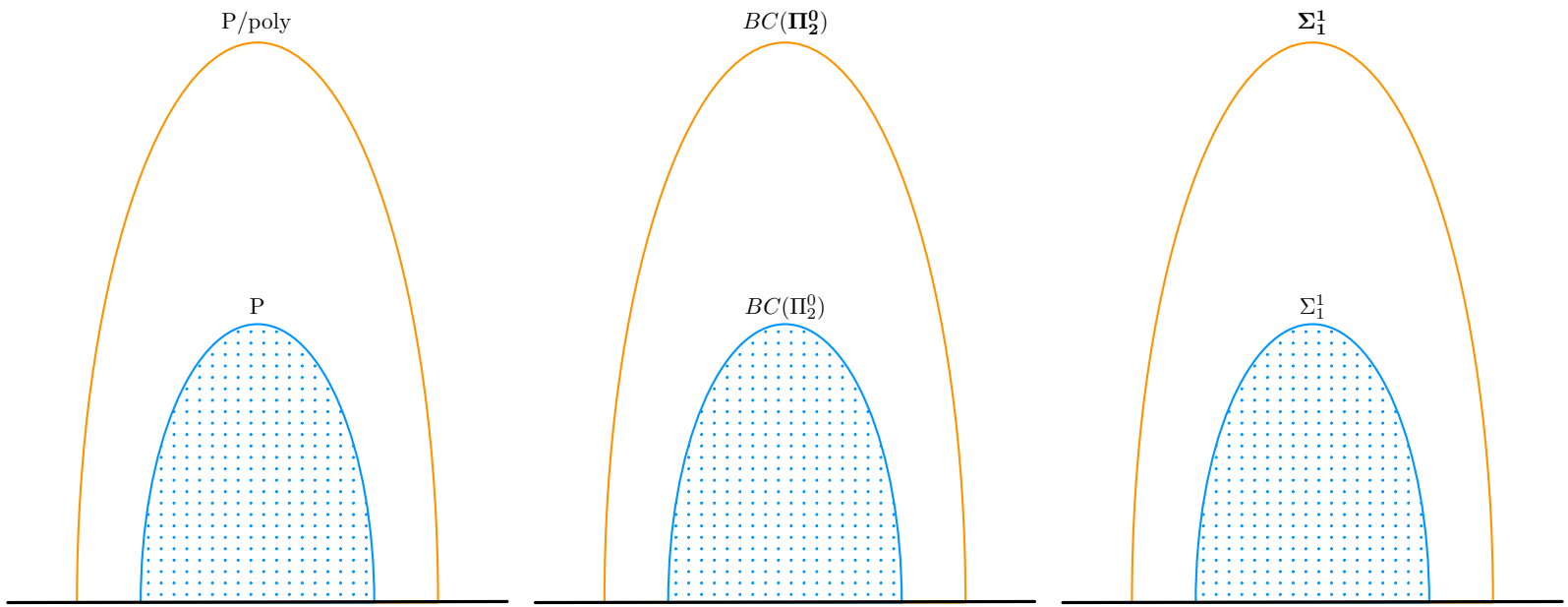
Hierarchy Theorems: stratification of the super-Turing "world"

- In both finite and infinite computational contexts, we can define infinitely many complexity classes between the Turing and super-Turing levels.



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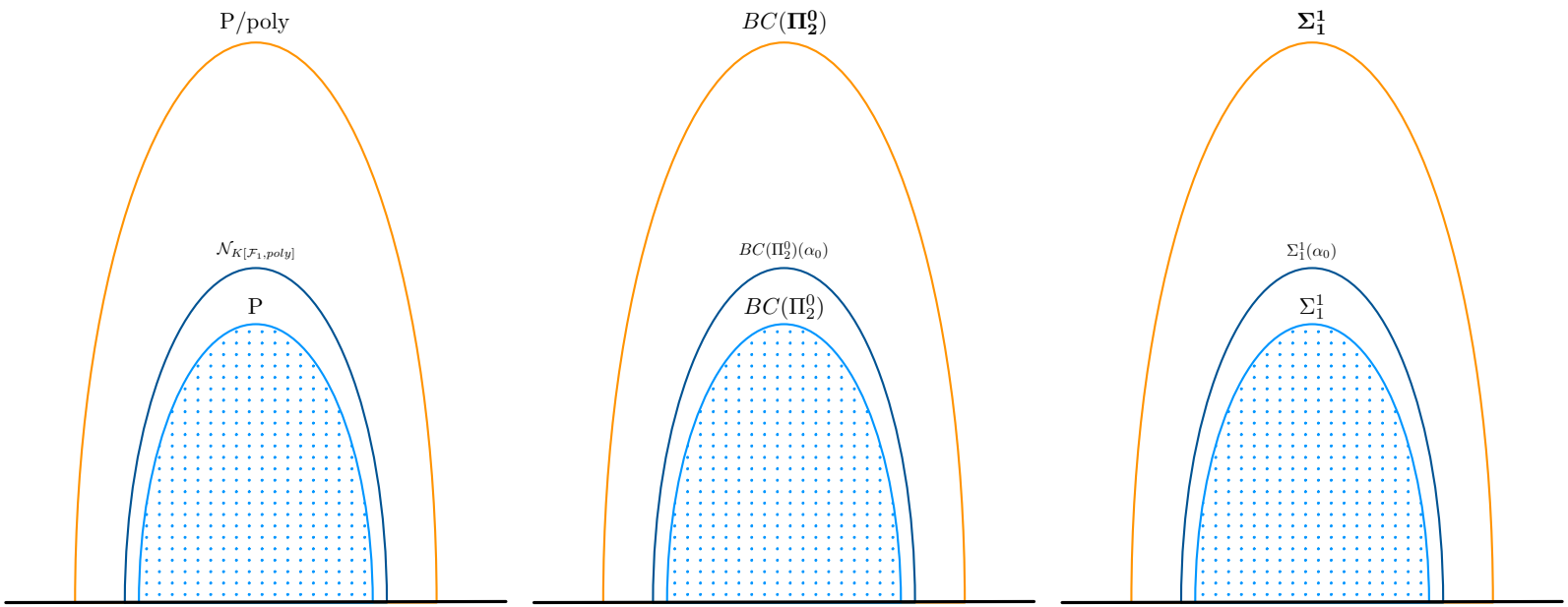

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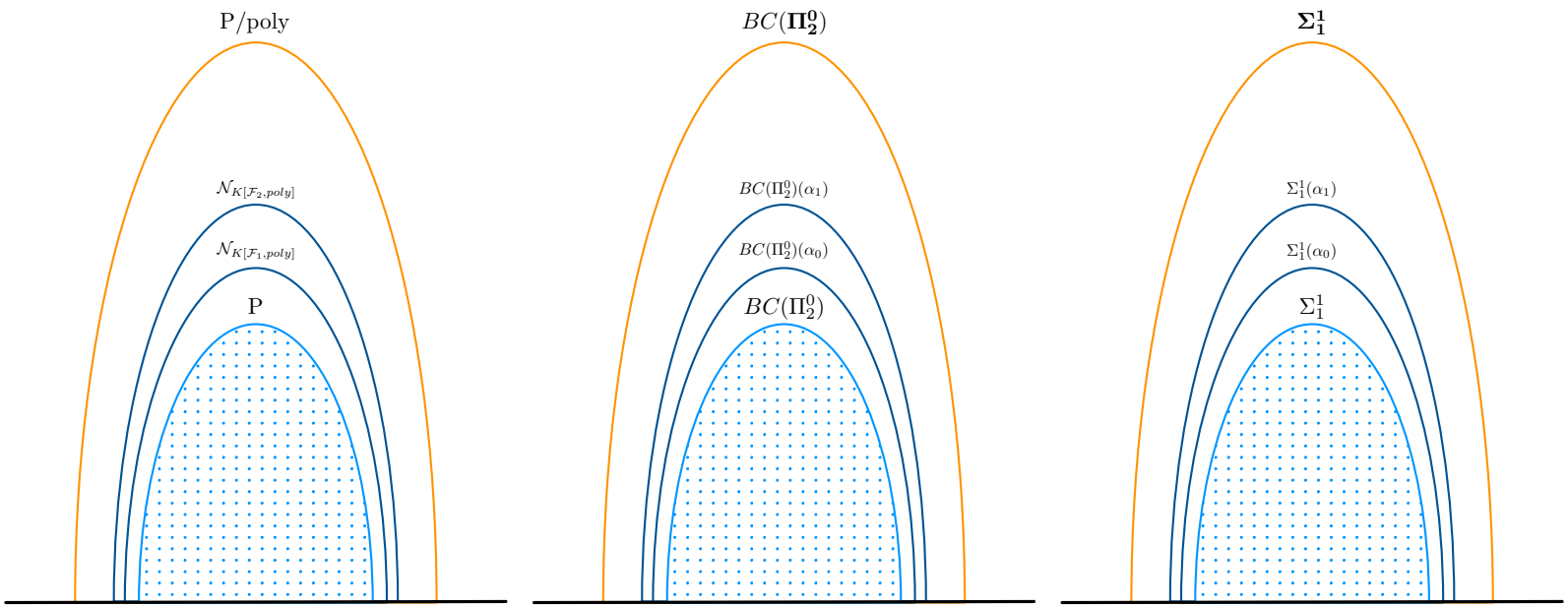

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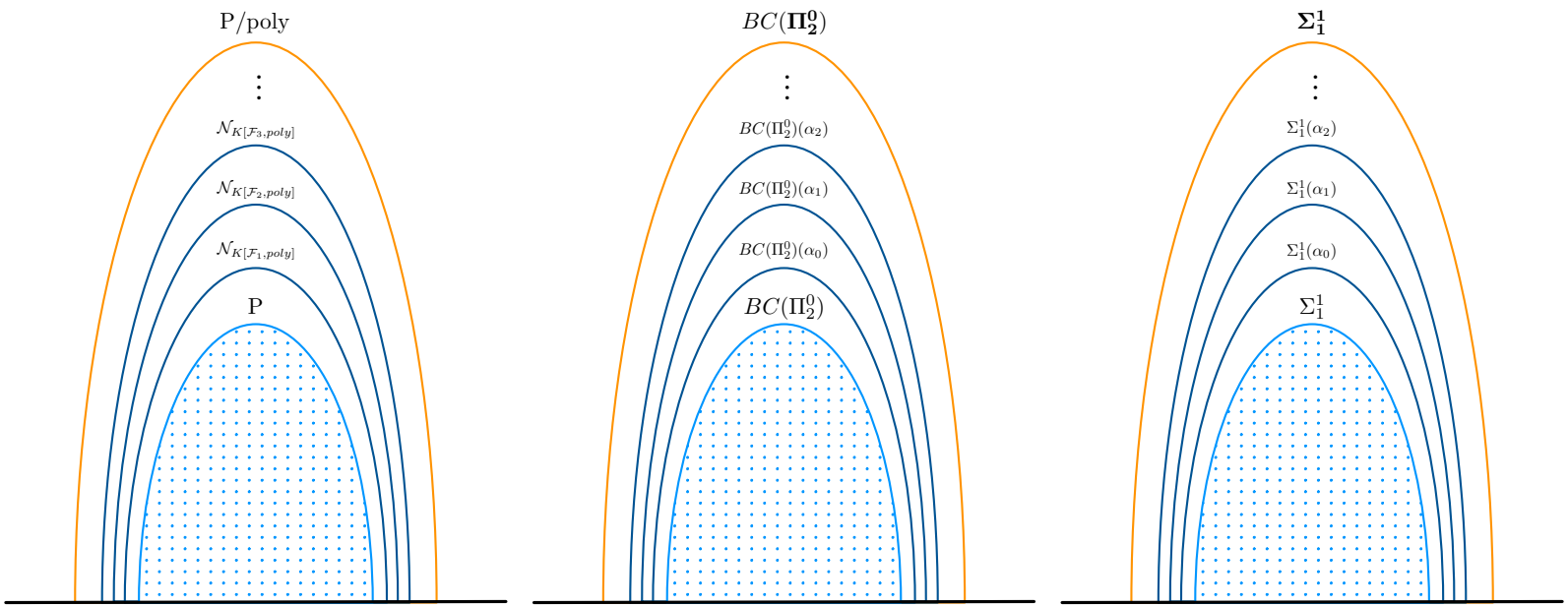

Intermediate Conclusions

- Recurrent neural networks is a natural model for *oracle-based (super-Turing) computation*.
- In all these results, the simulation of finite state machines by recurrent neural networks is not "biologically plausible".
- We propose a novel paradigm for *abstract neural computation* that takes into account important biological features.

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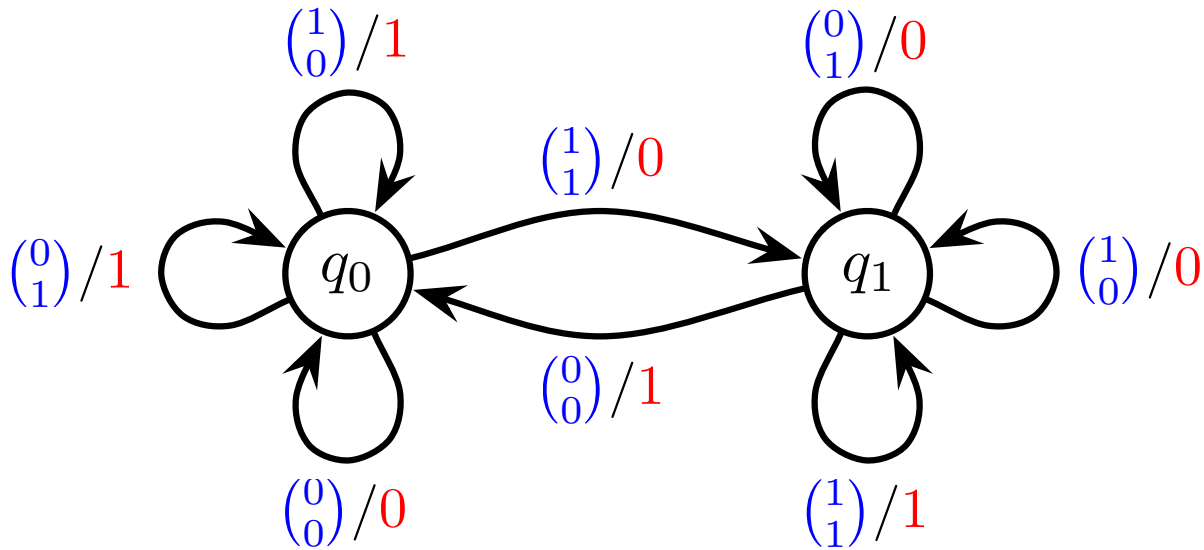
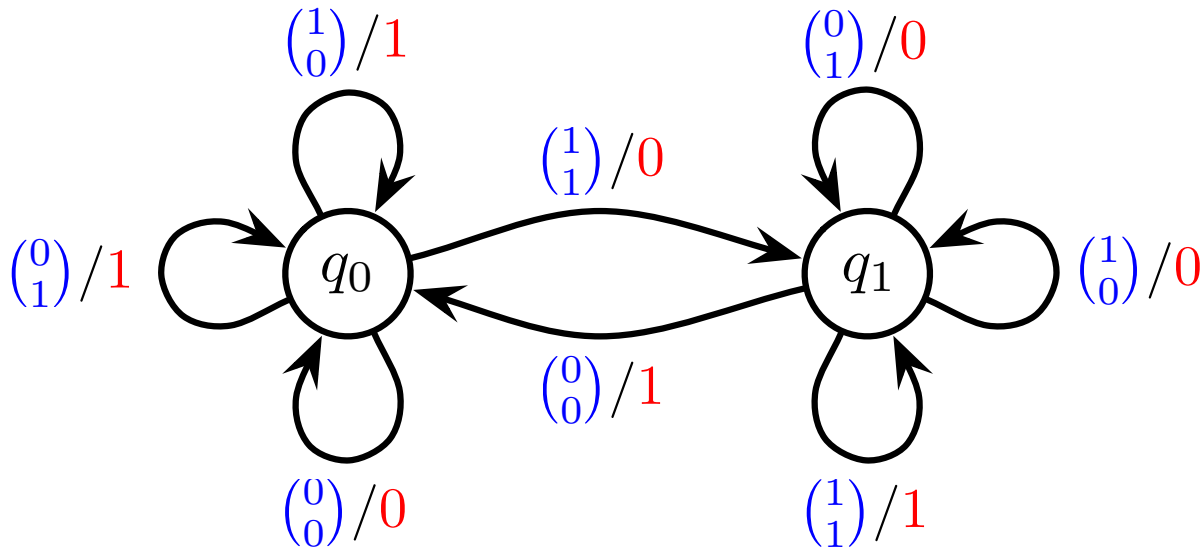
Binary Adder Automaton



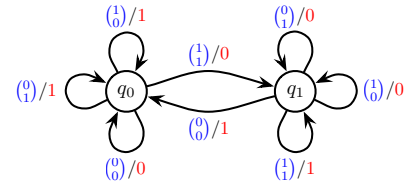
$$\begin{array}{cccccccc} & & 1 & 1^{1} & 0^{1} & 1 & 1 & 0^{1} & 1 \\ + & & & 1 & 1 & 1 & 0 & 0 & 1 \\ \hline & & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{array}$$

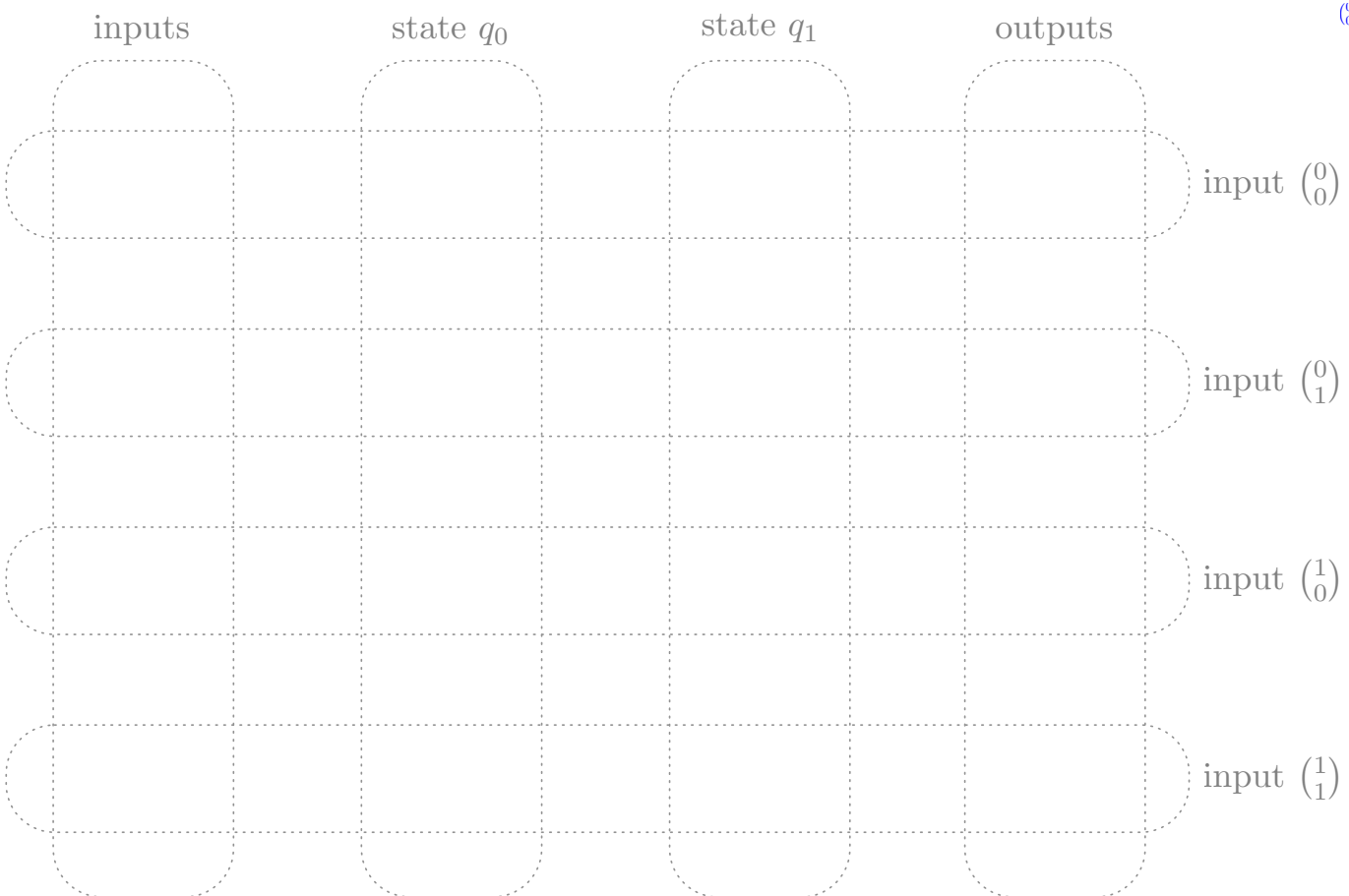
Binary Adder Automaton



$$
\begin{array}{cccccccc}
 & {\scriptstyle 1} & 1^{\scriptstyle 1} & 0^{\scriptstyle 1} & 1 & 1 & 0^{\scriptstyle 1} & 1 \\
+ & & 1 & 1 & 1 & 0 & 0 & 1 \\
\hline
 & 1 & 1 & 0 & 0 & 1 & 1 & 0
\end{array}
$$

Binary Adder Boolean Neural Network





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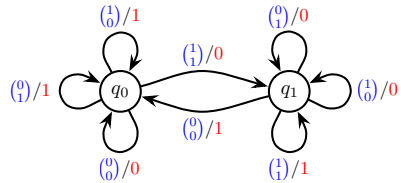

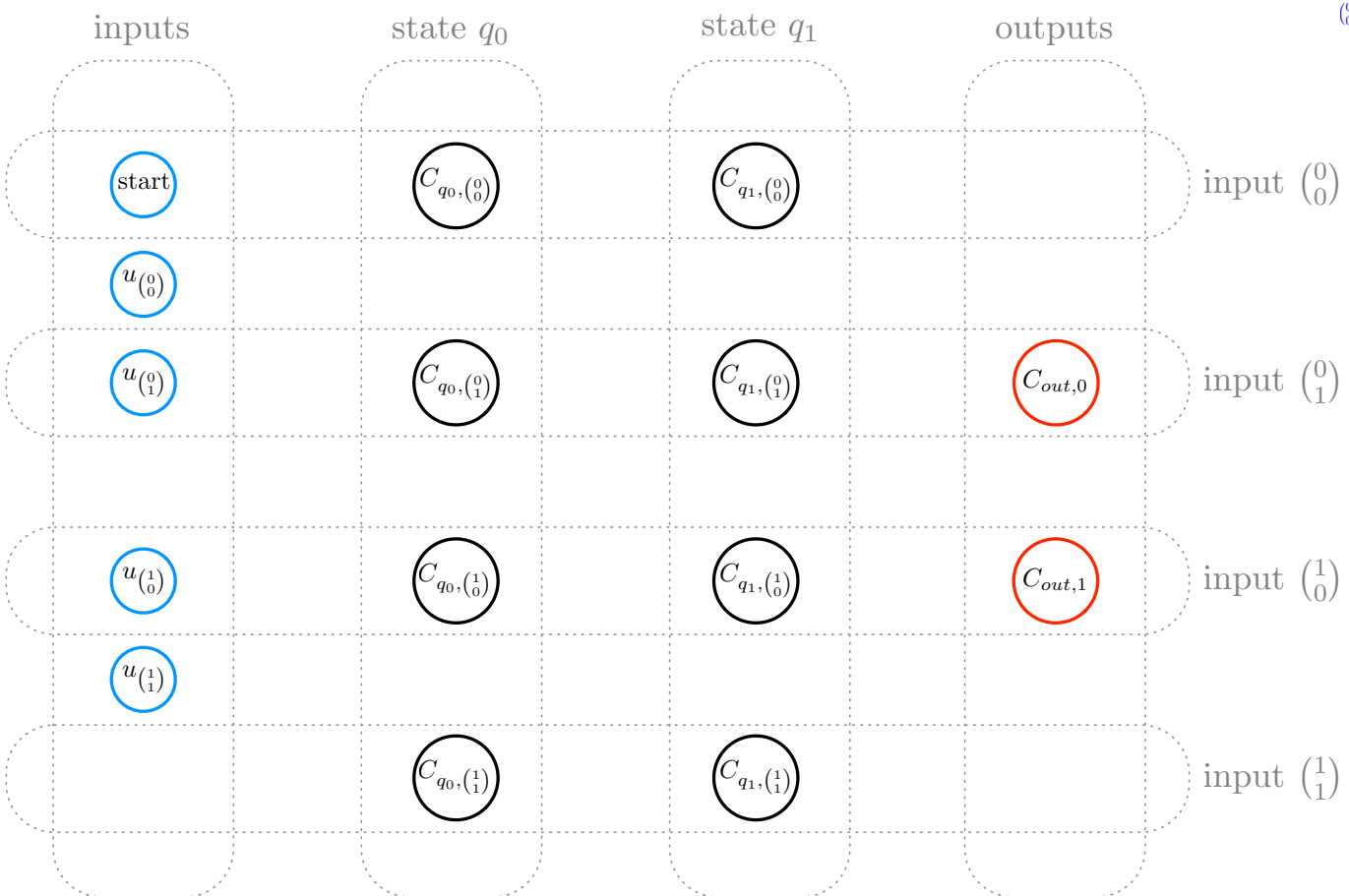

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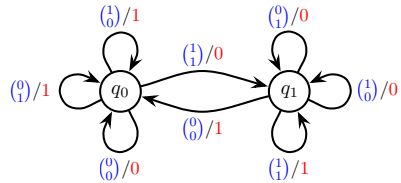

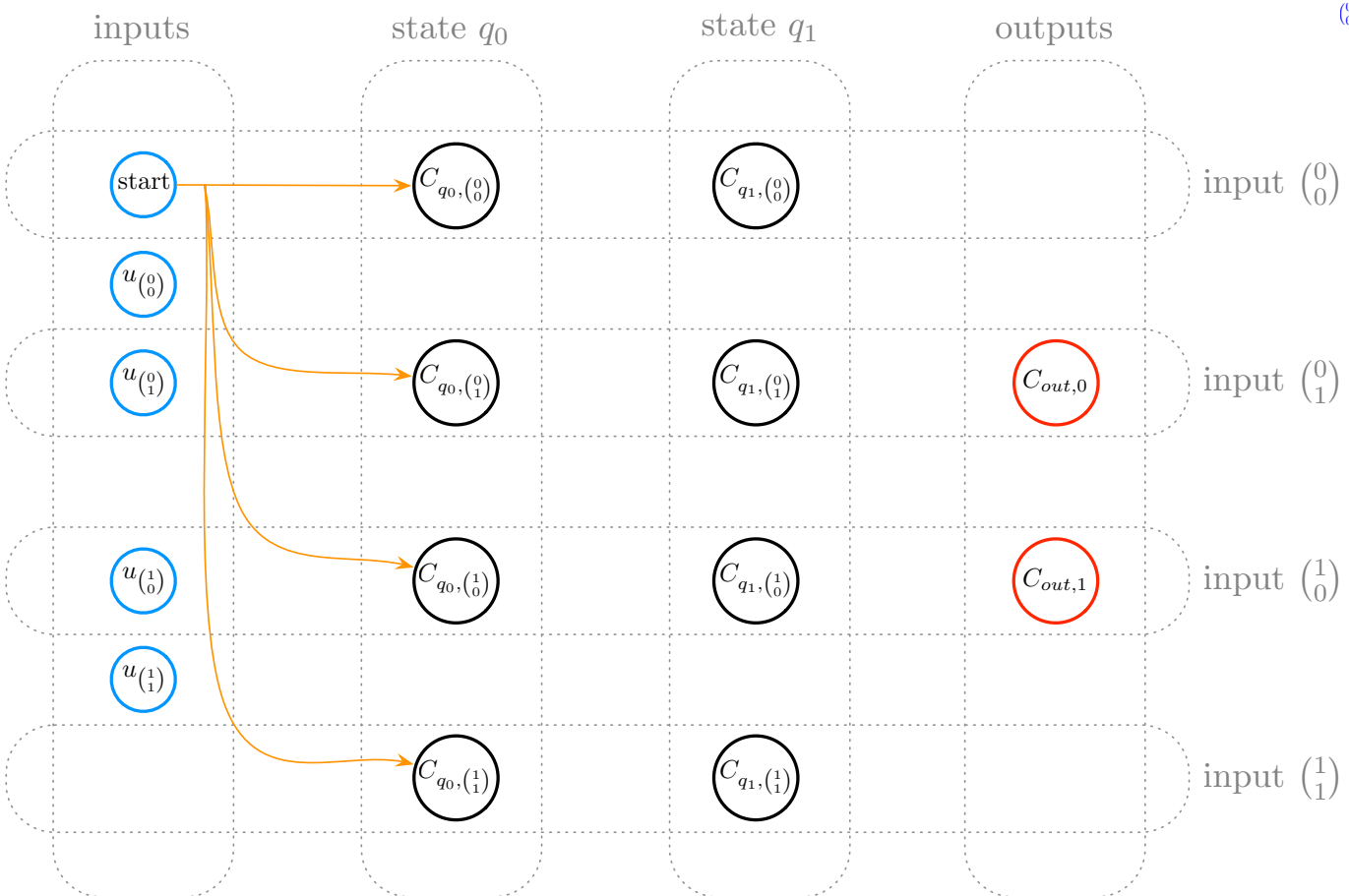

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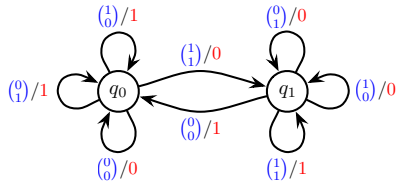

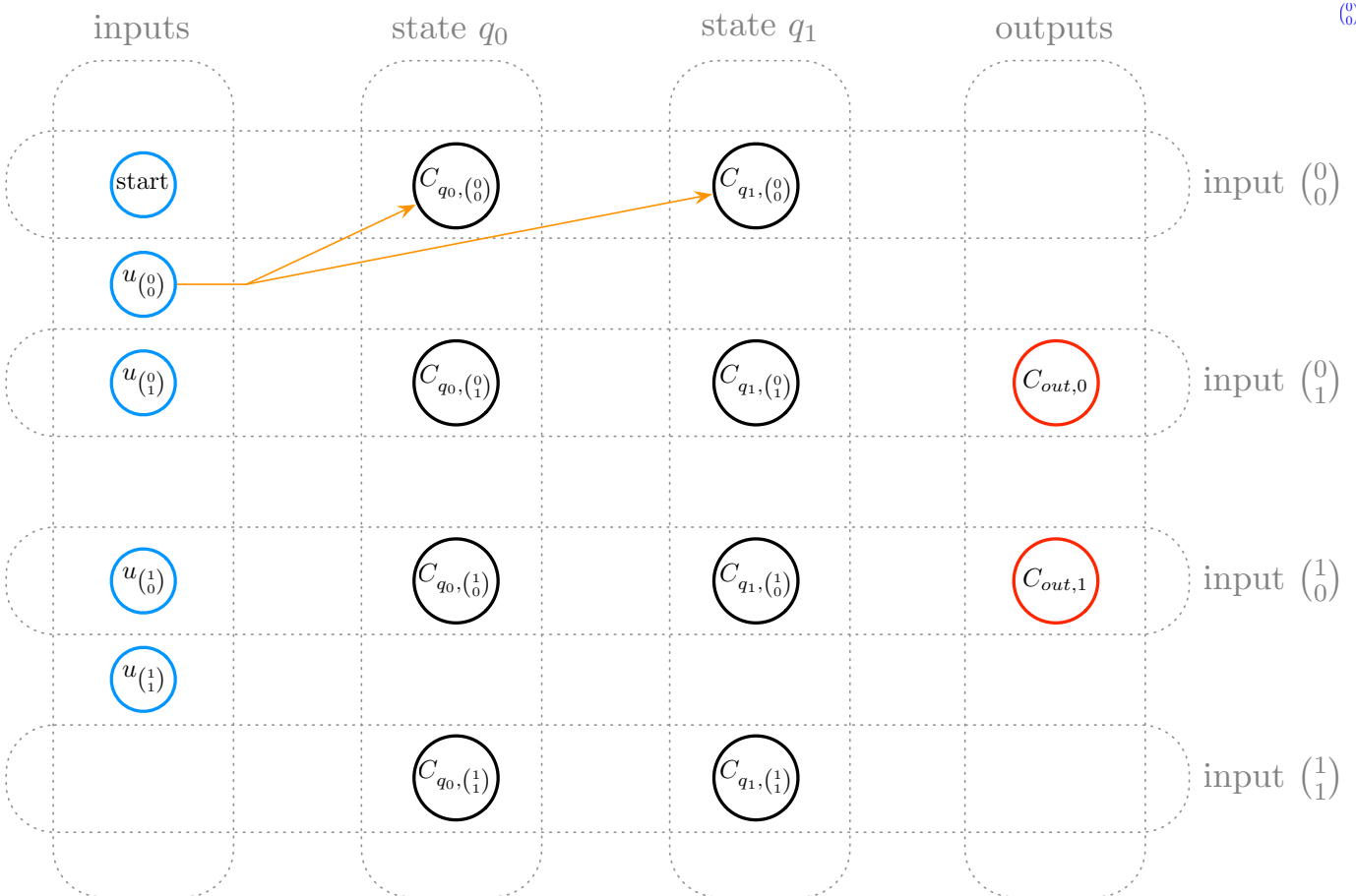

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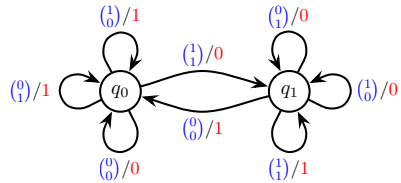

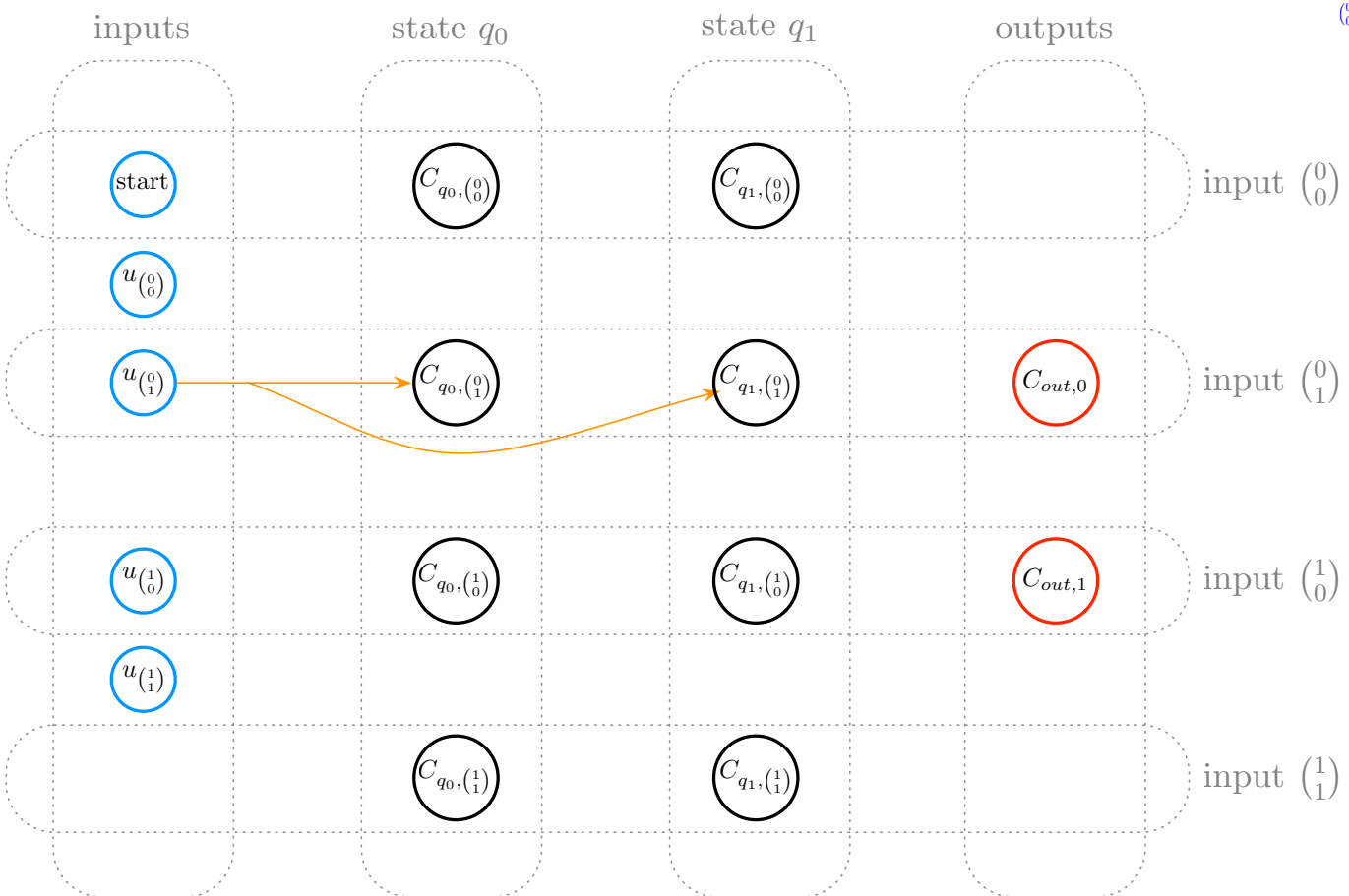

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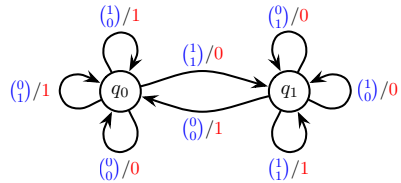

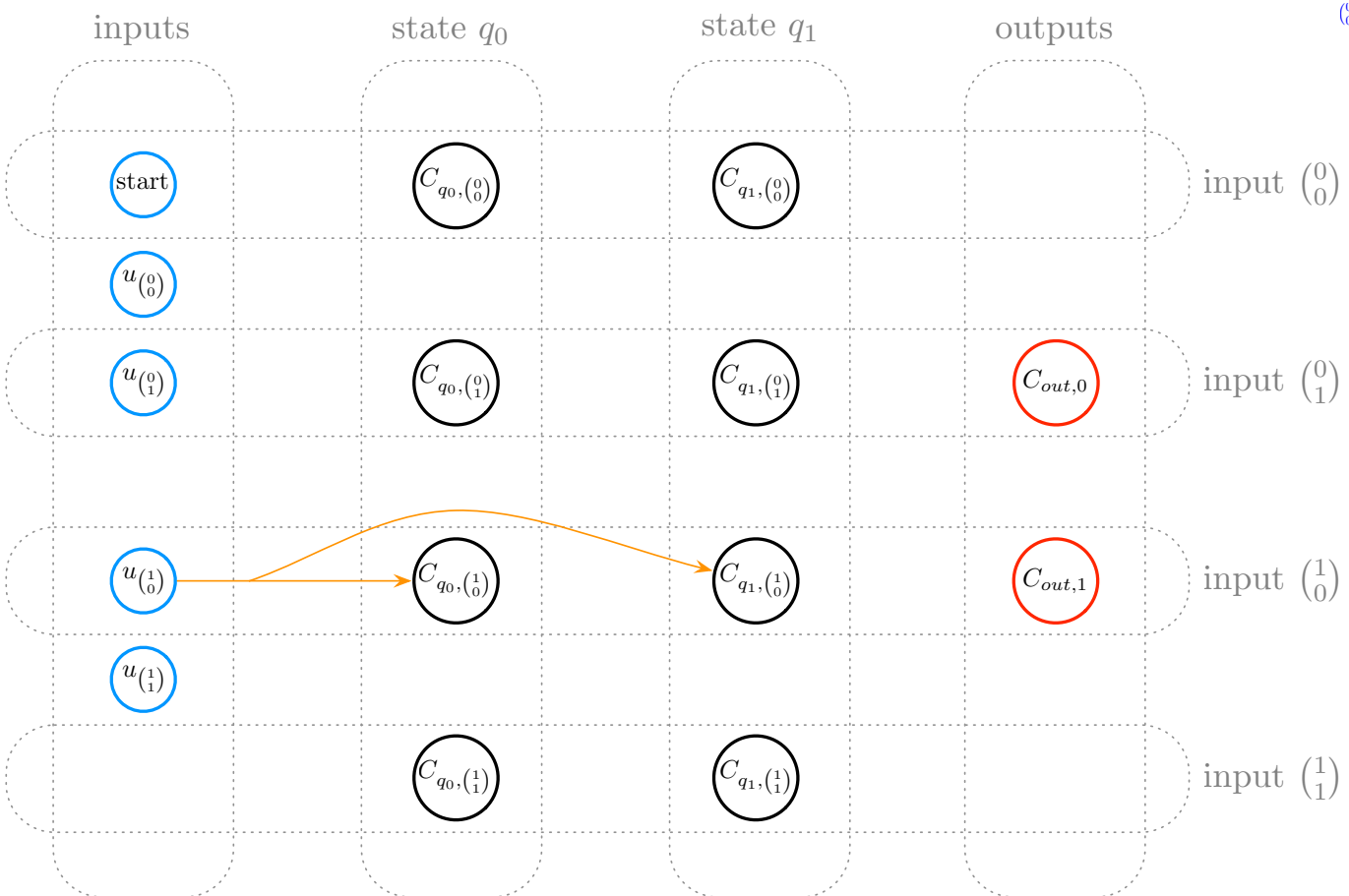

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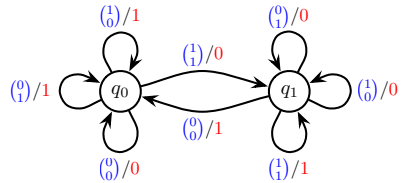

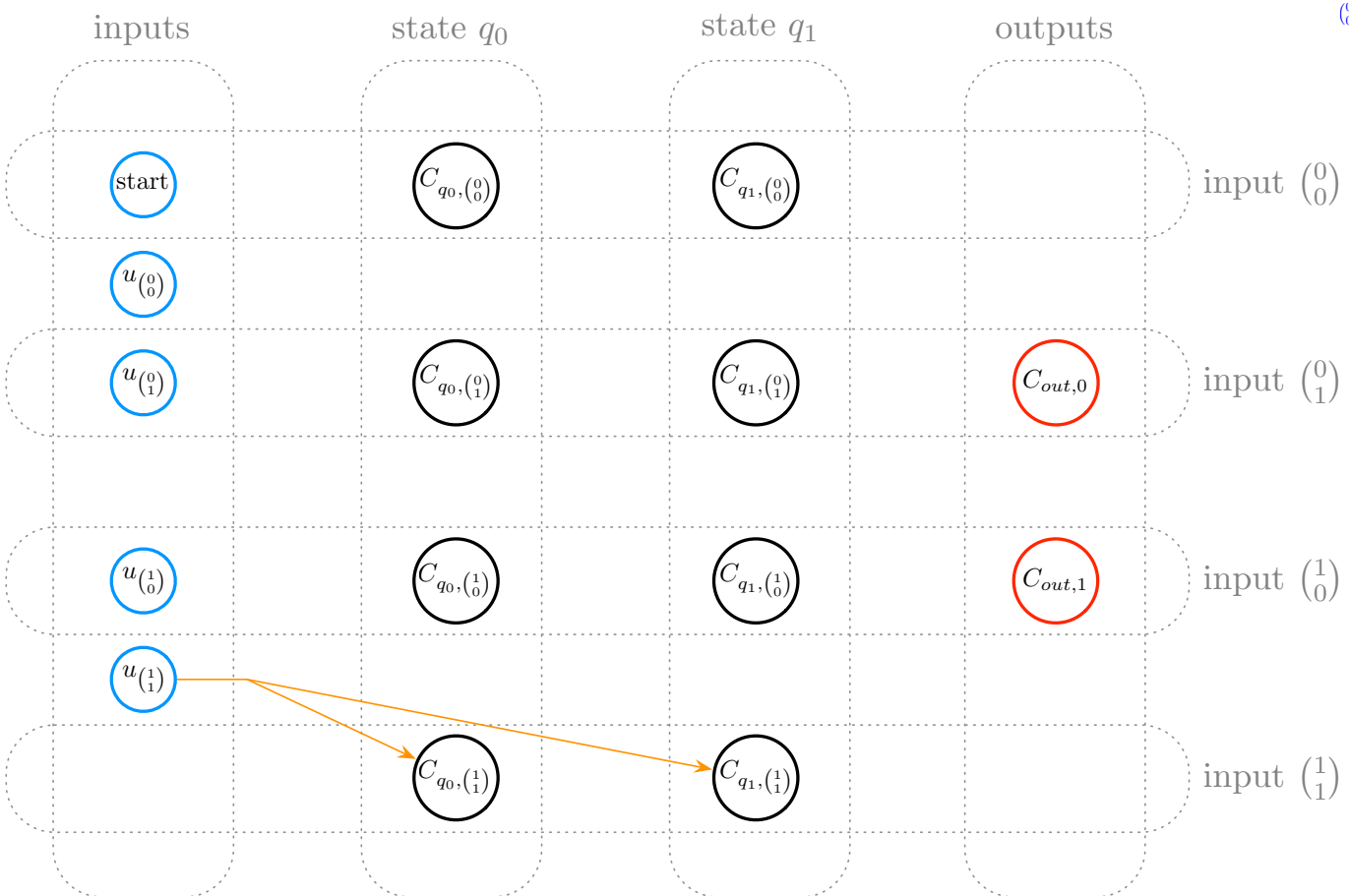

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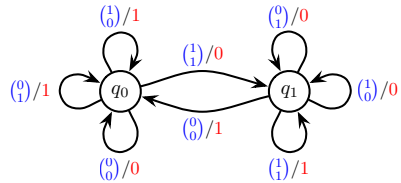

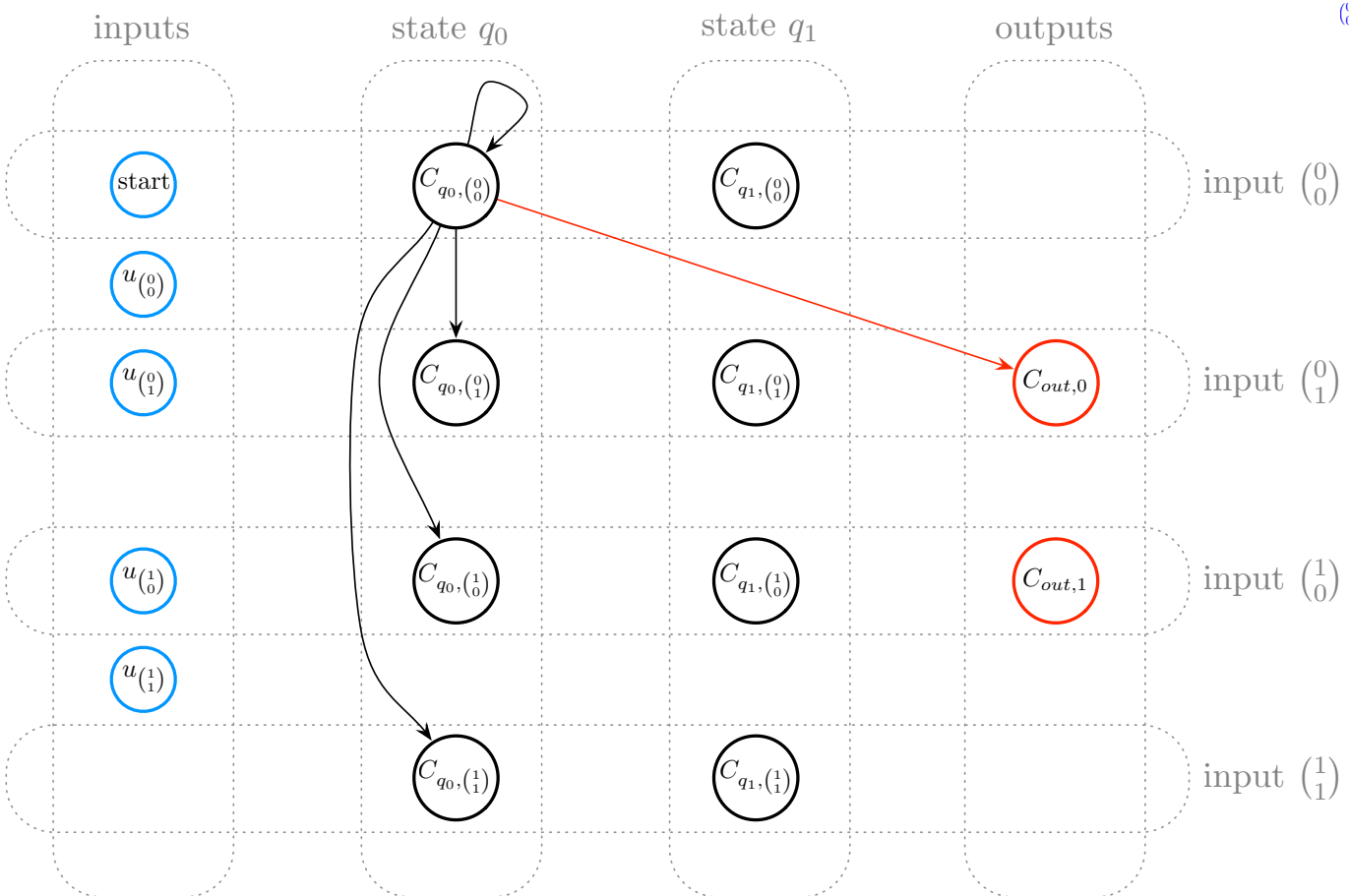

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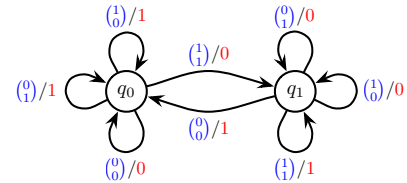

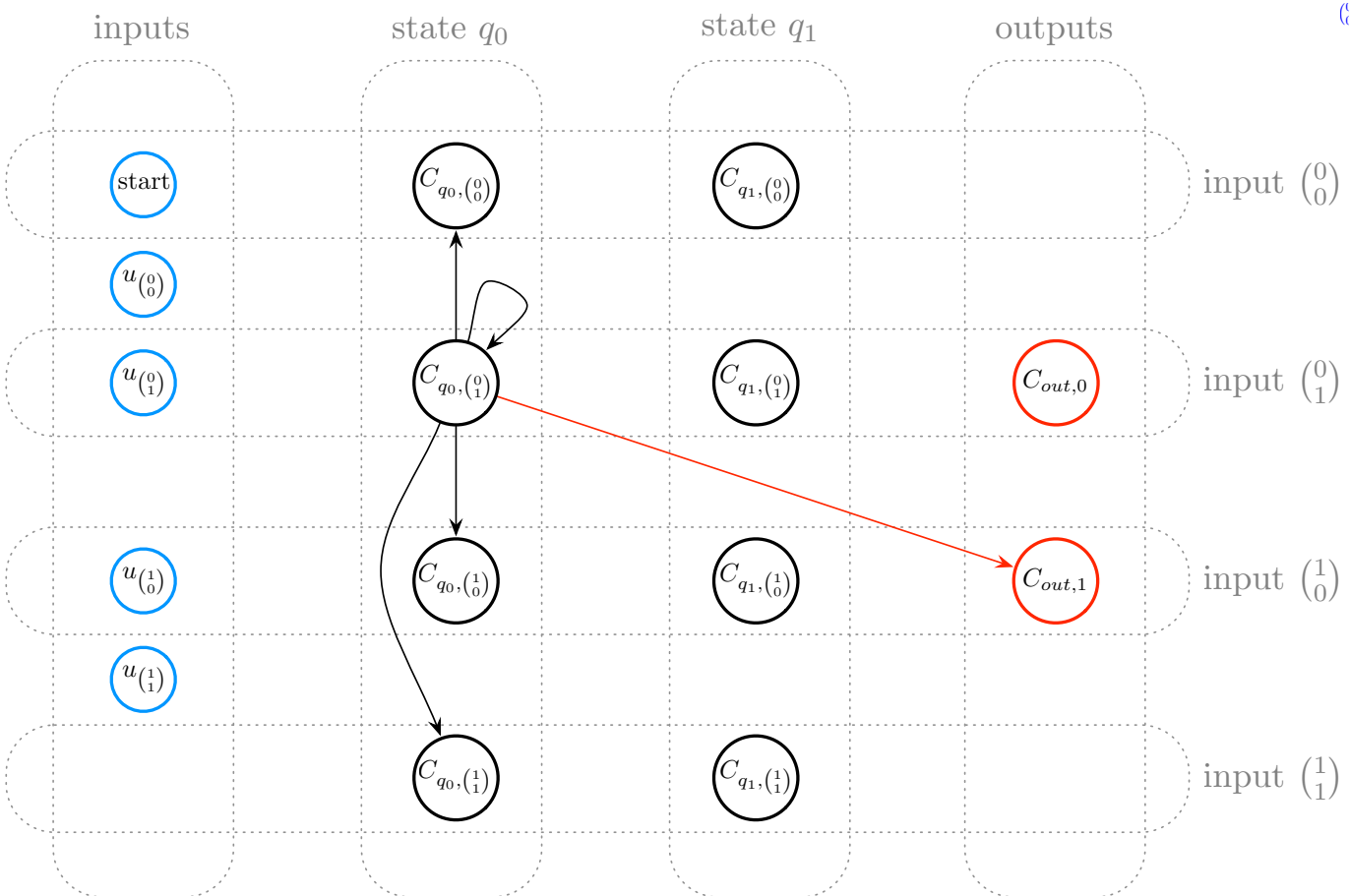

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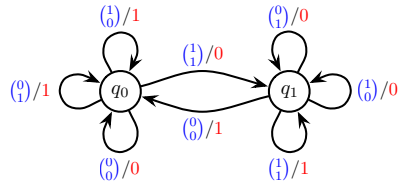

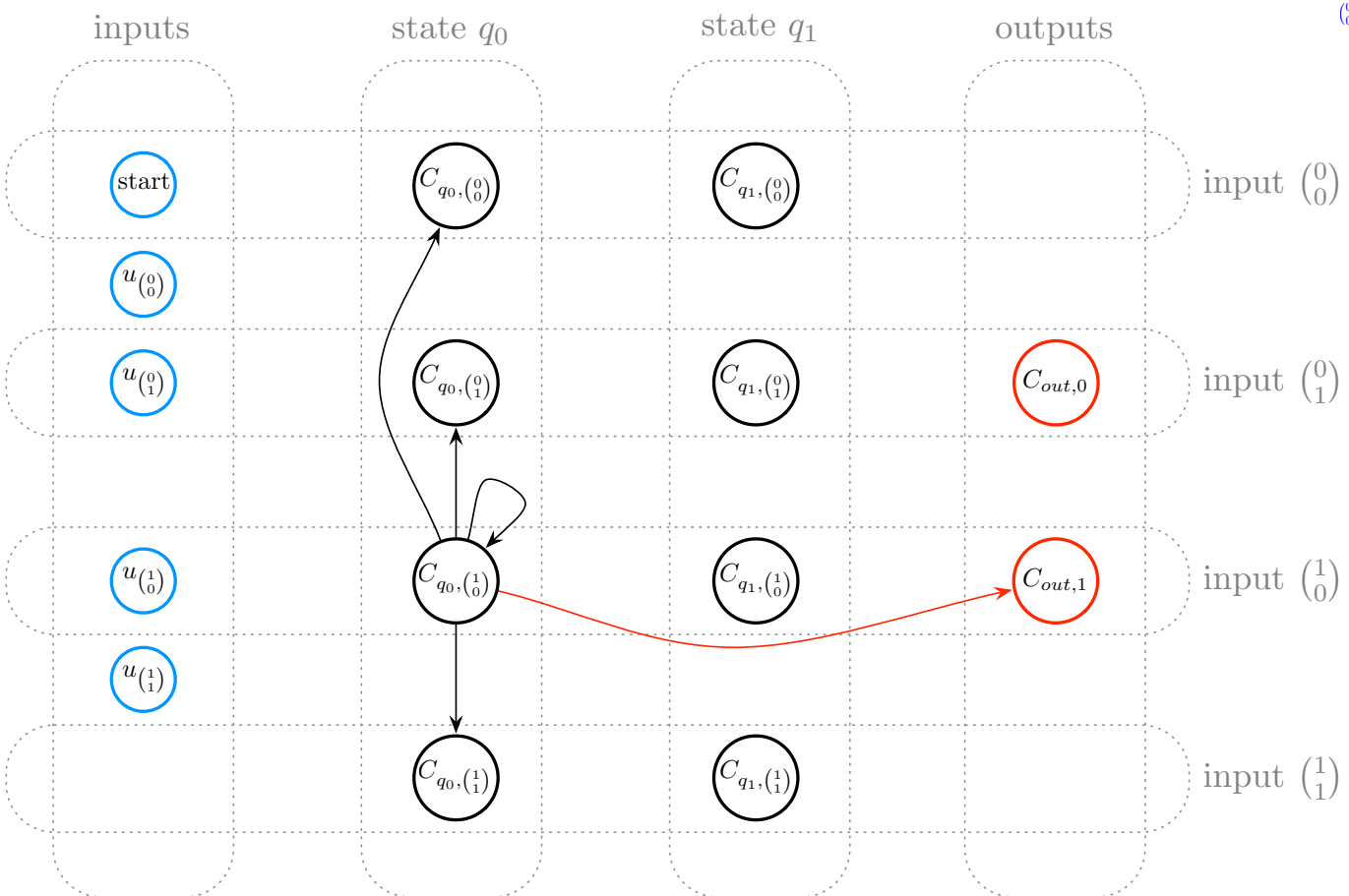

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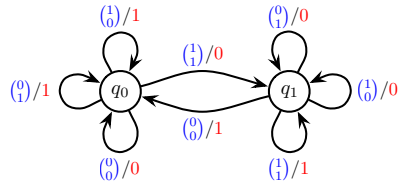

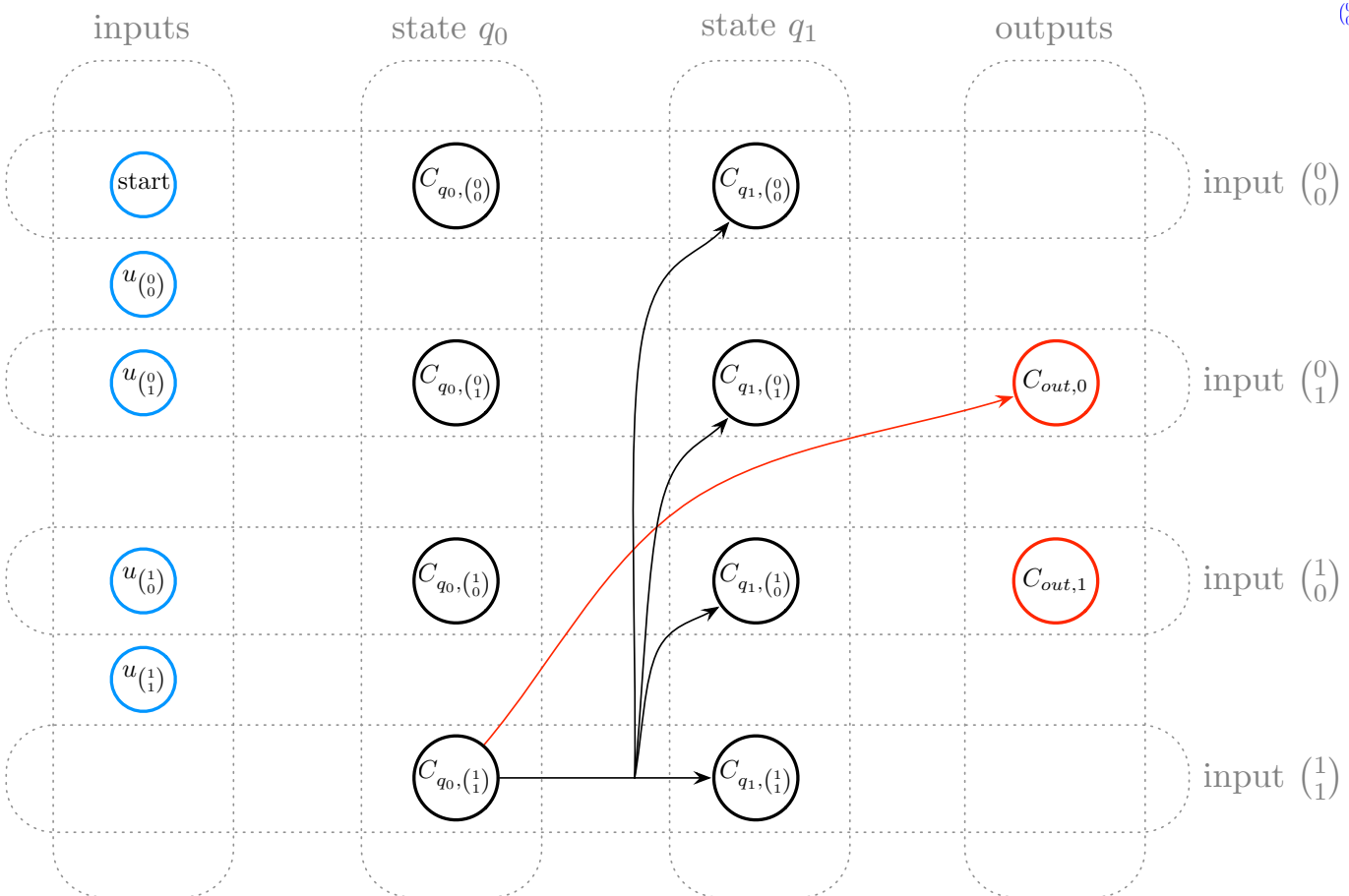

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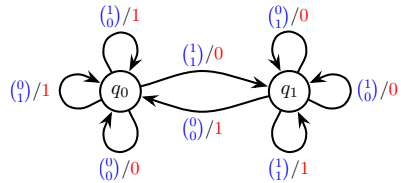

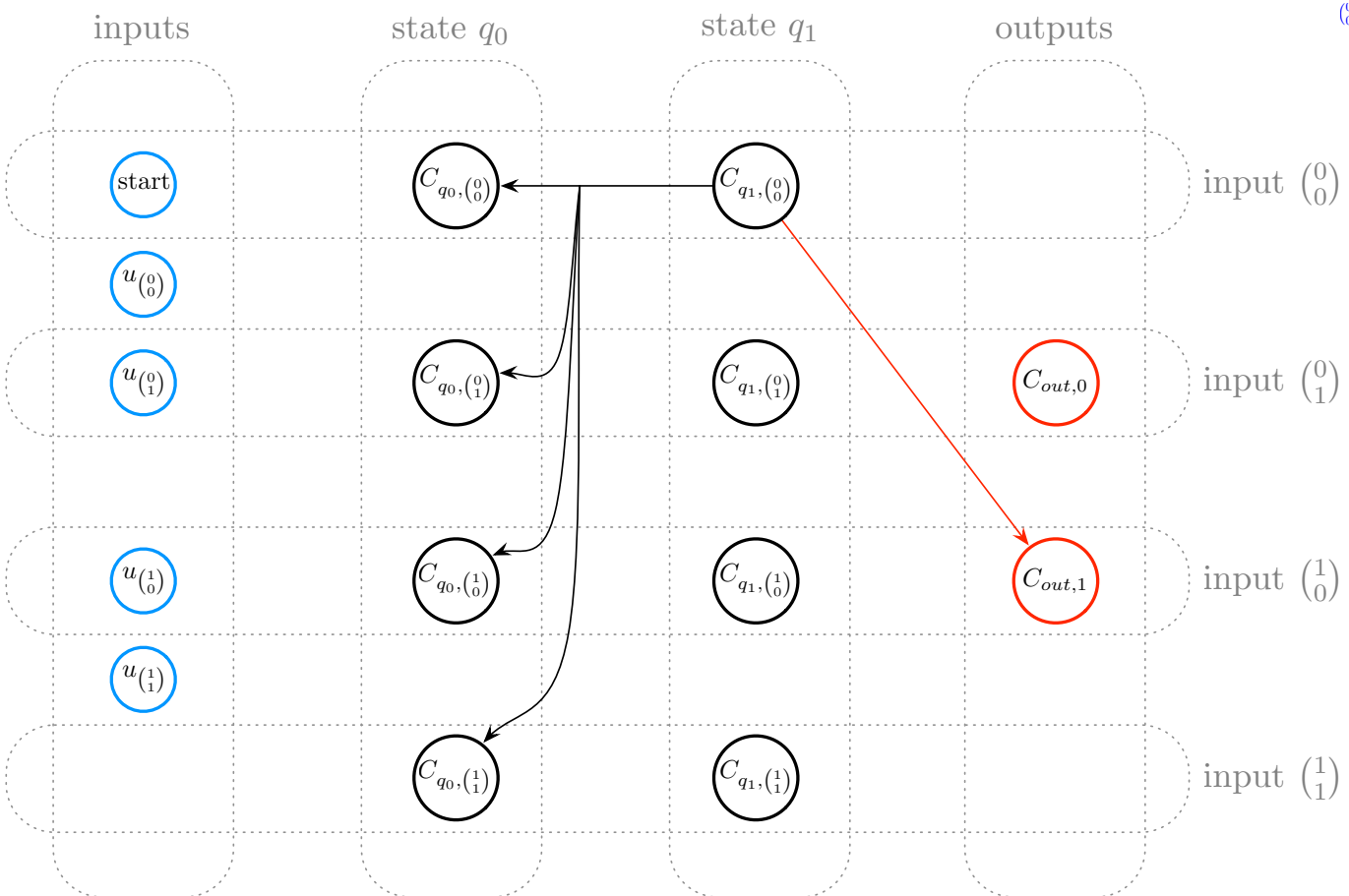

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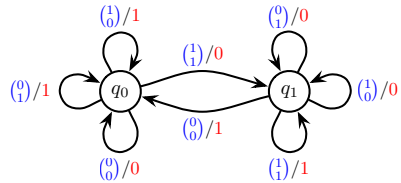

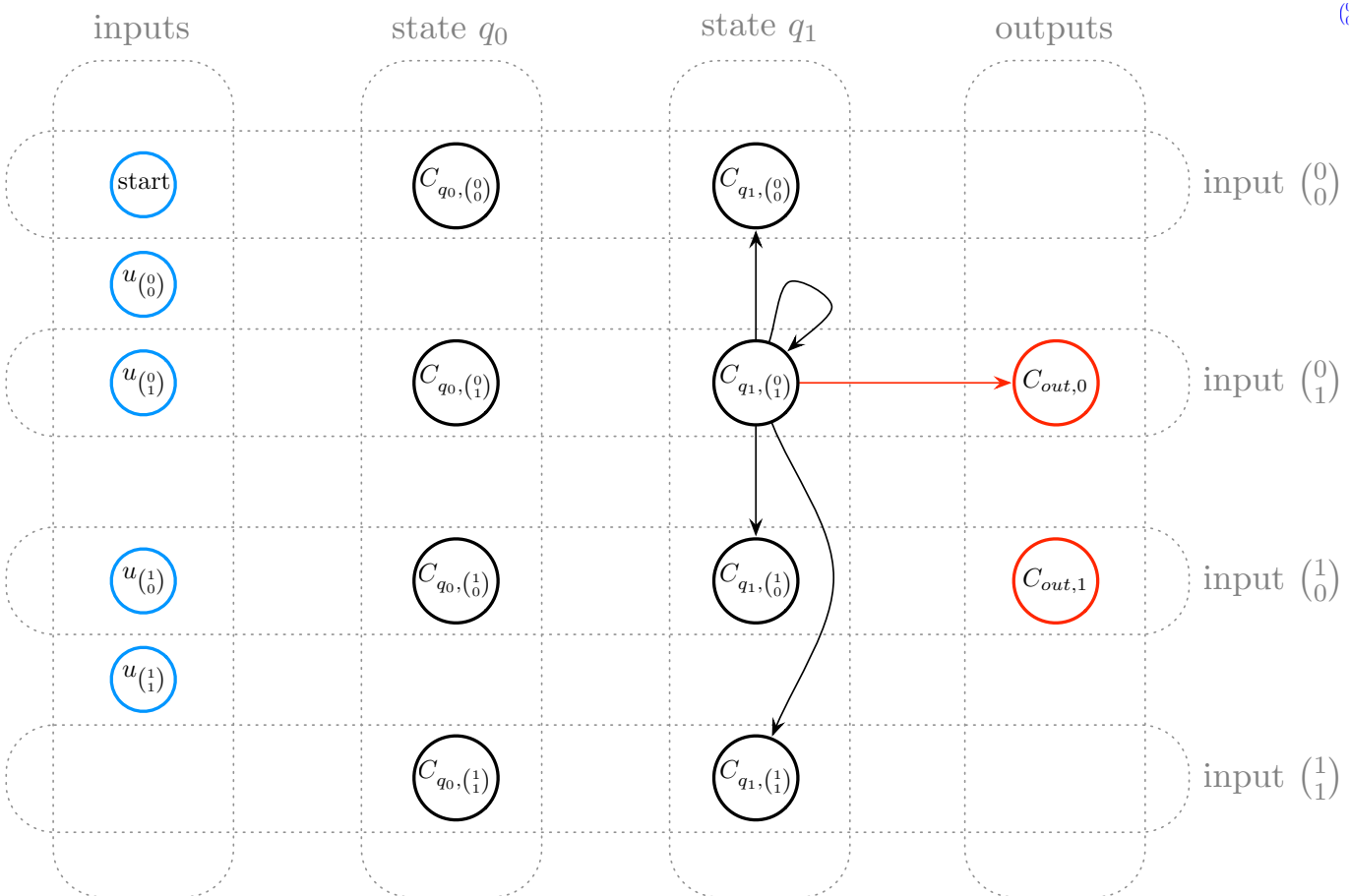

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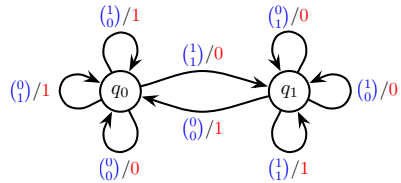

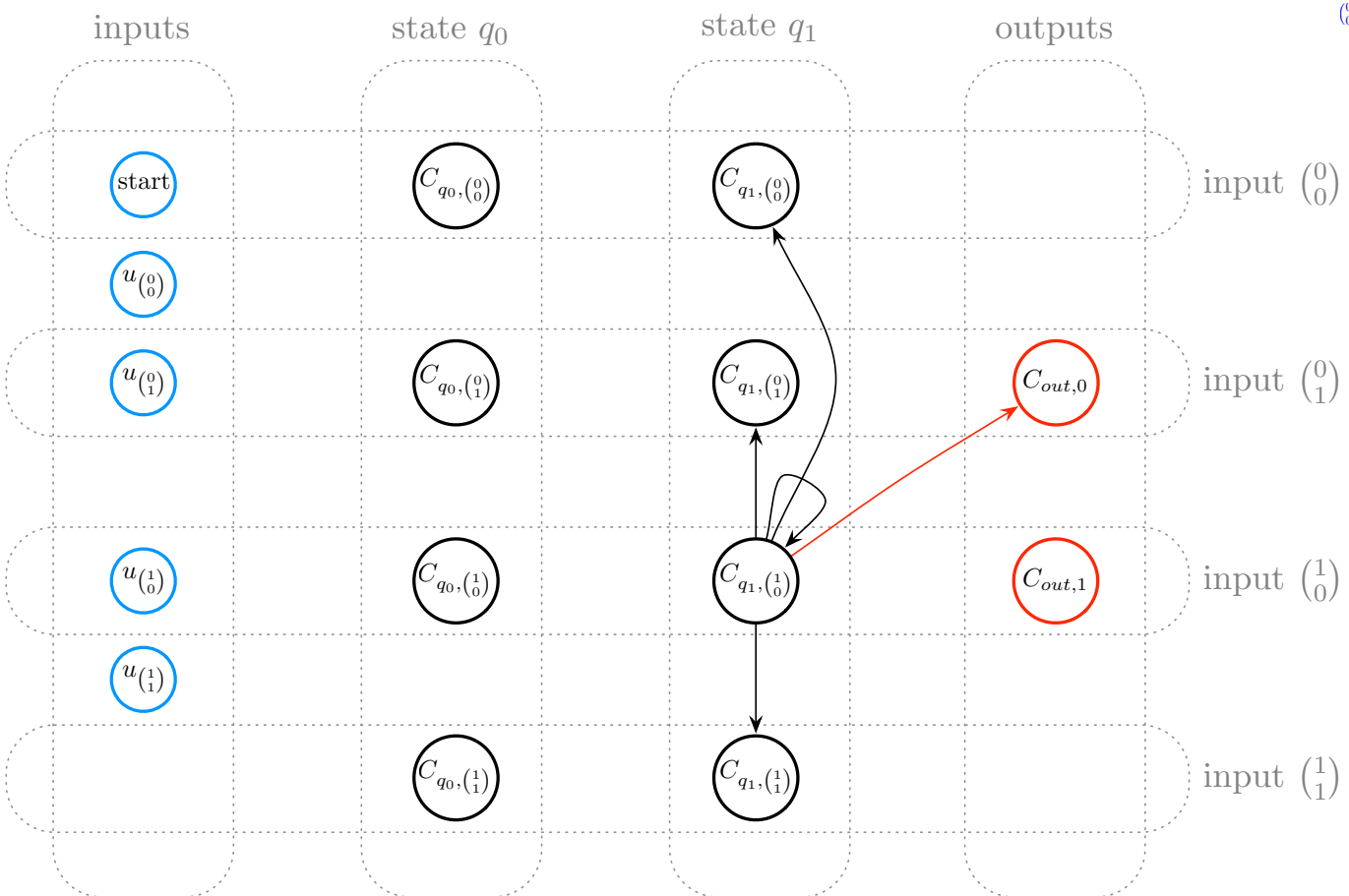

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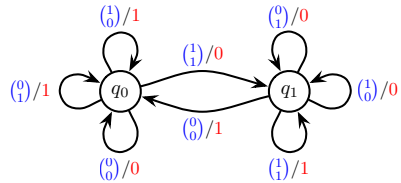

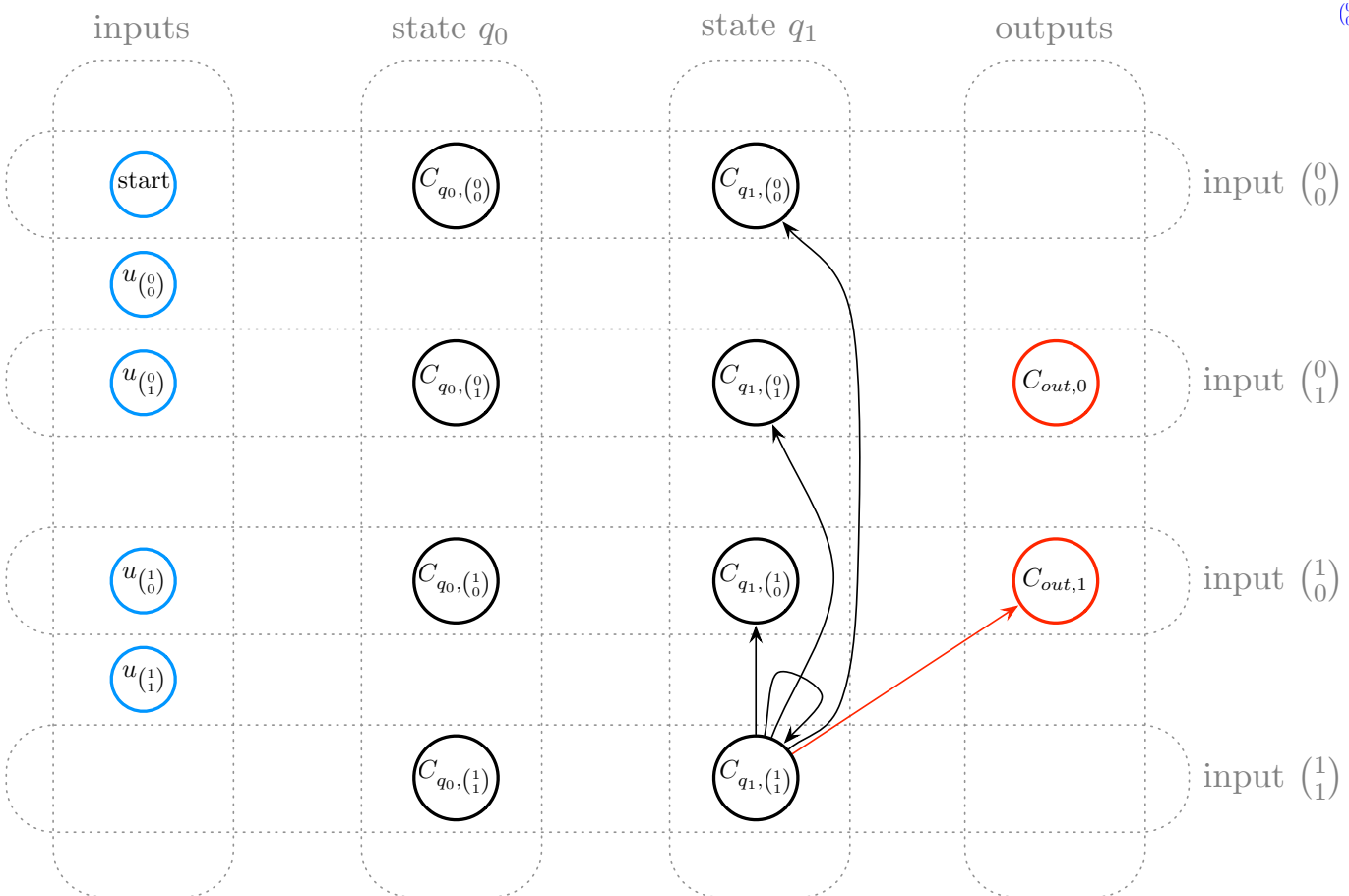

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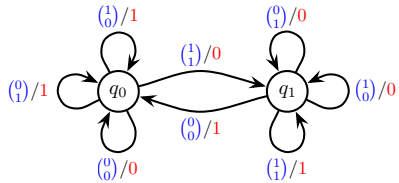
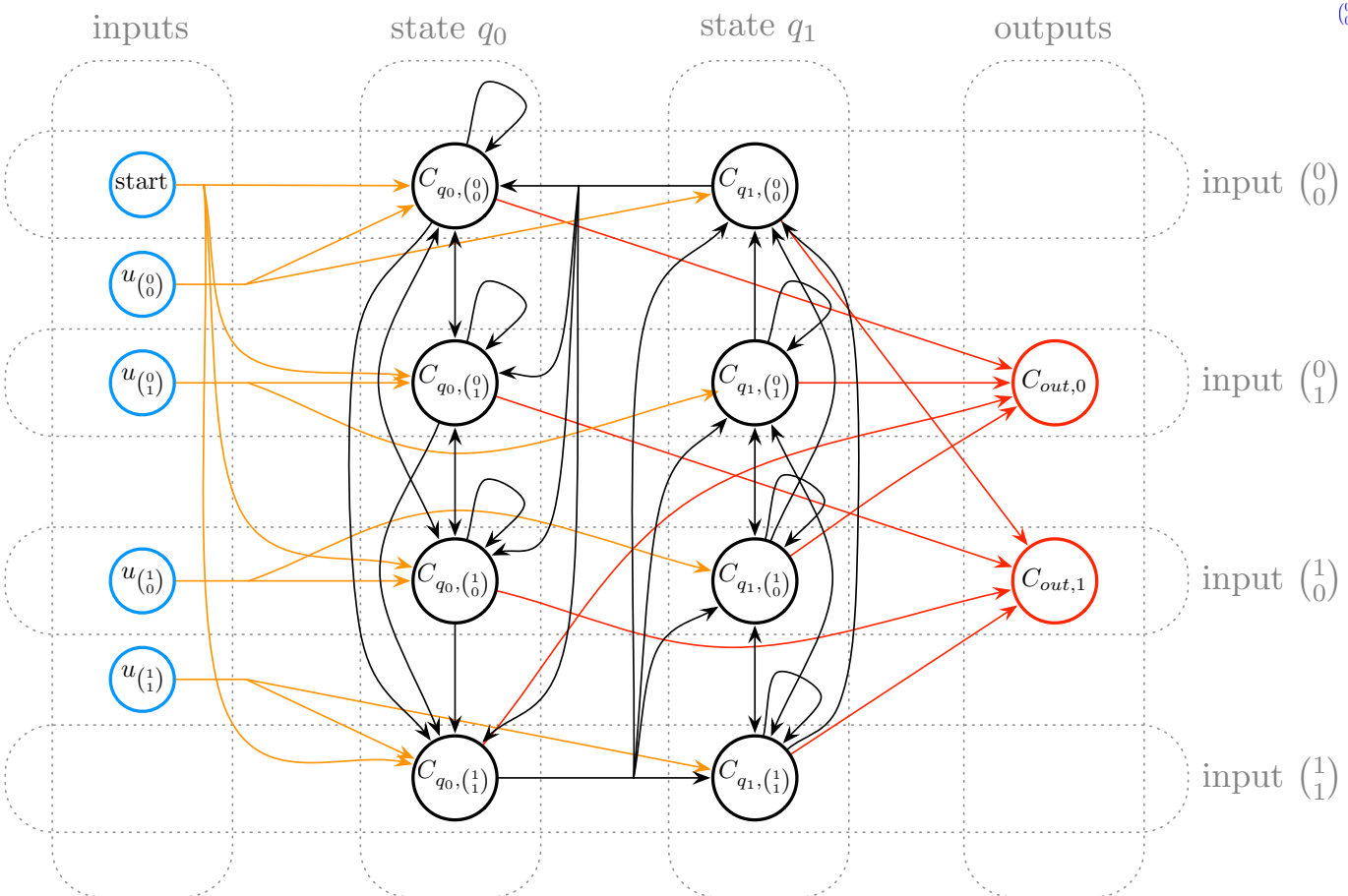
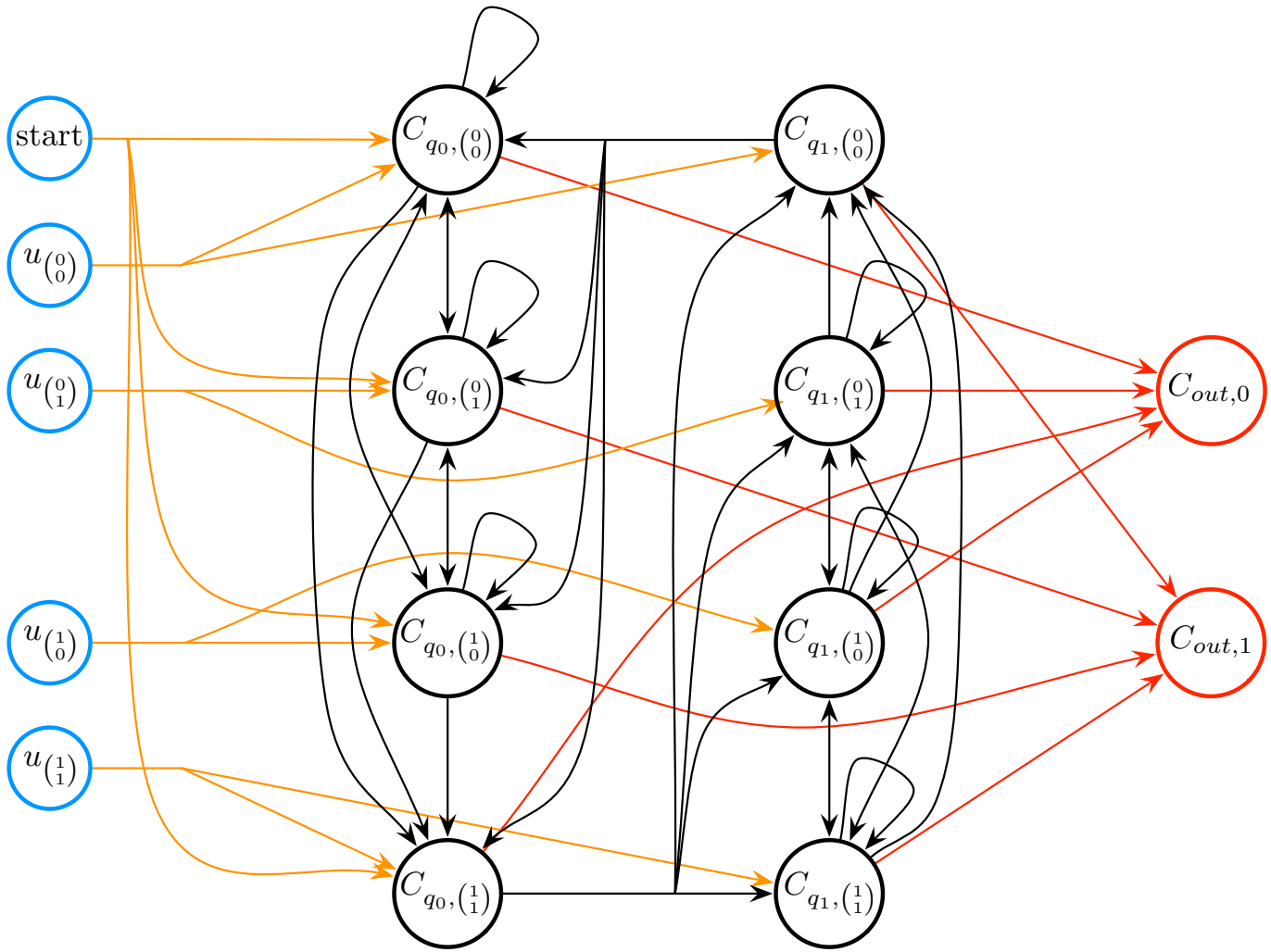




Simulation


start
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$C_{q_1,\binom{0}{1}}$
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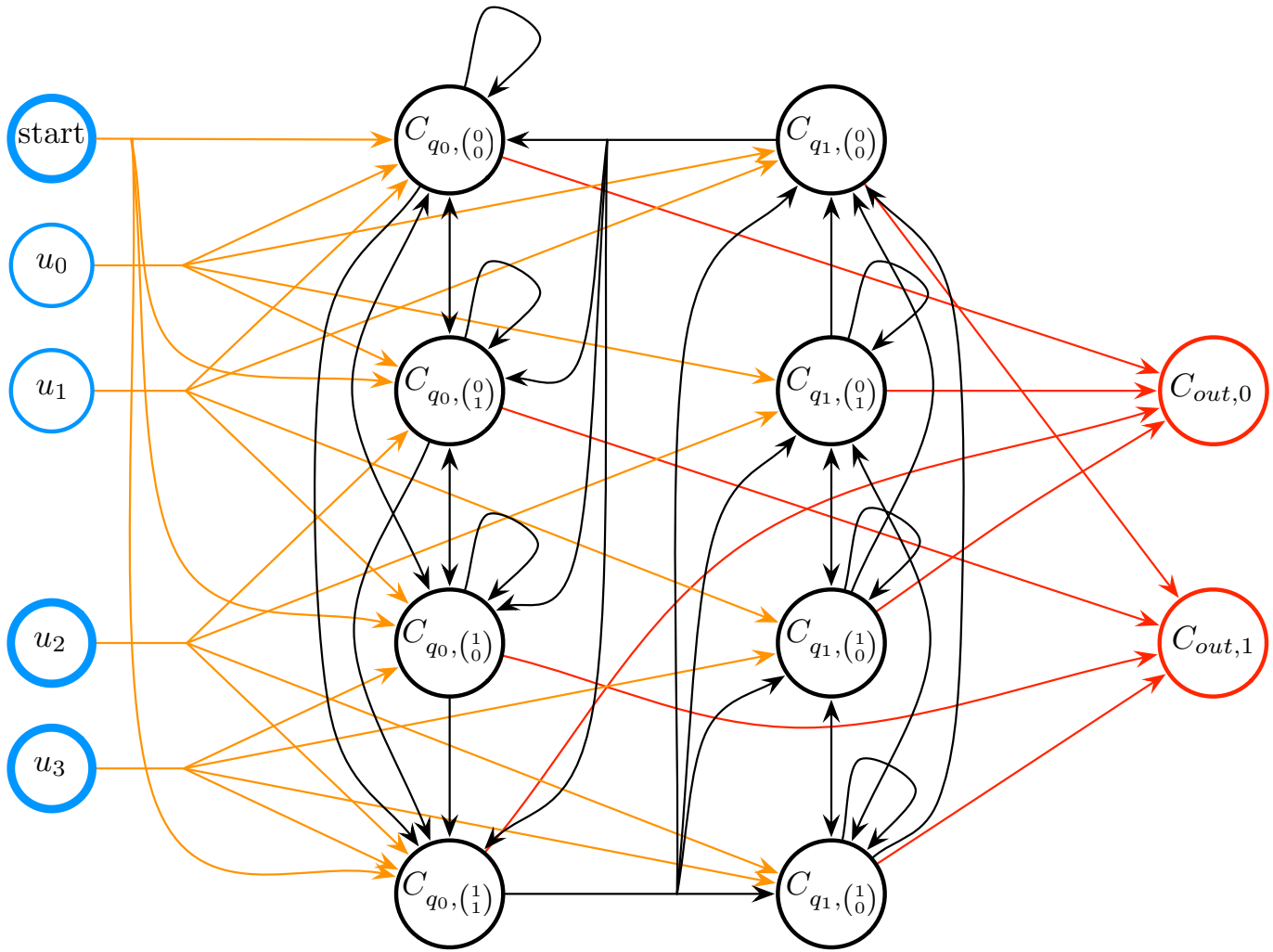
$C_{q_1,\binom{1}{1}}$
$C_{out,0}$
$C_{out,1}$

Simulation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$C_{q_0,\binom{0}{0}}$
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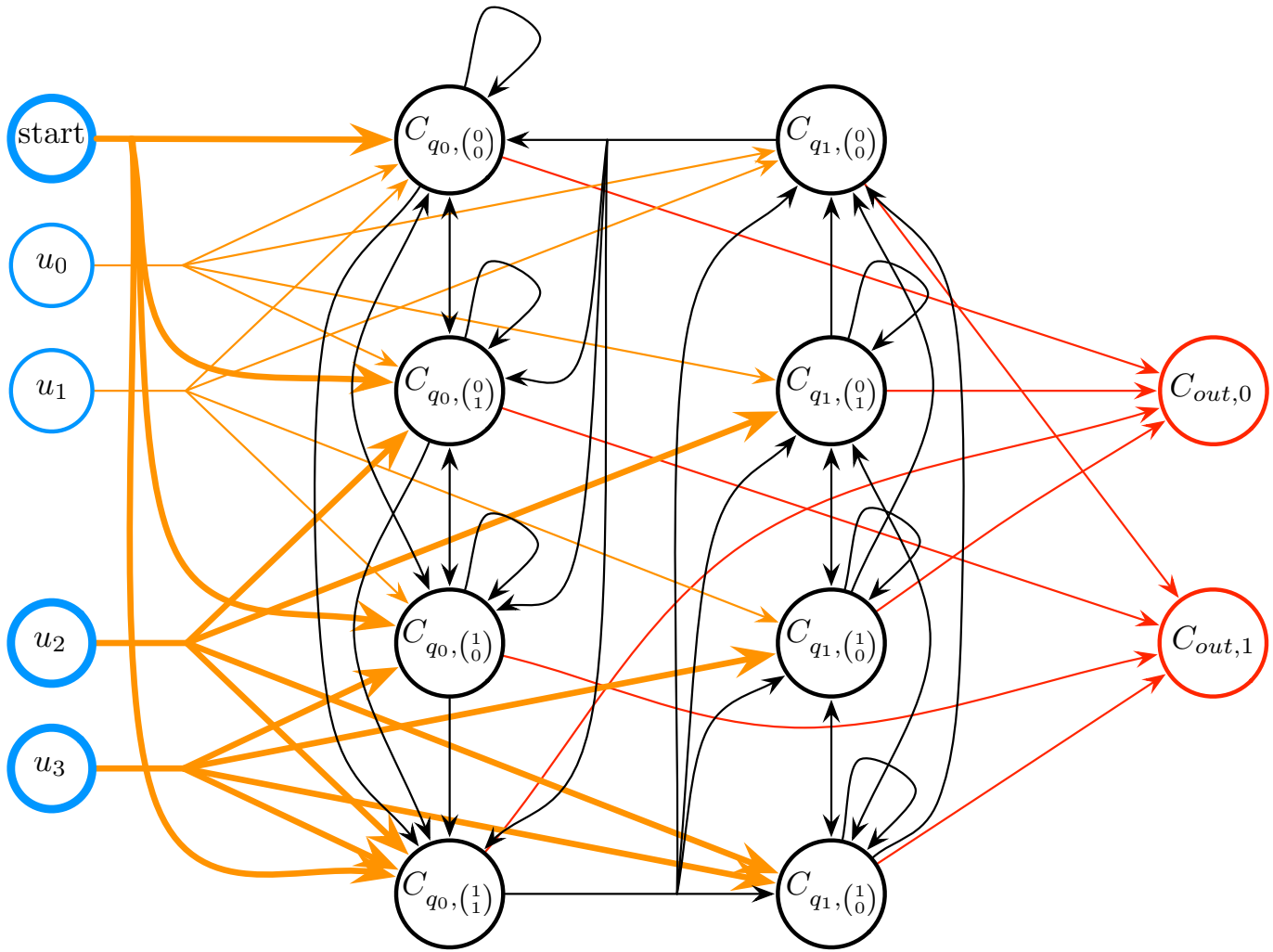
$C_{out,0}$
$C_{out,1}$

Simulation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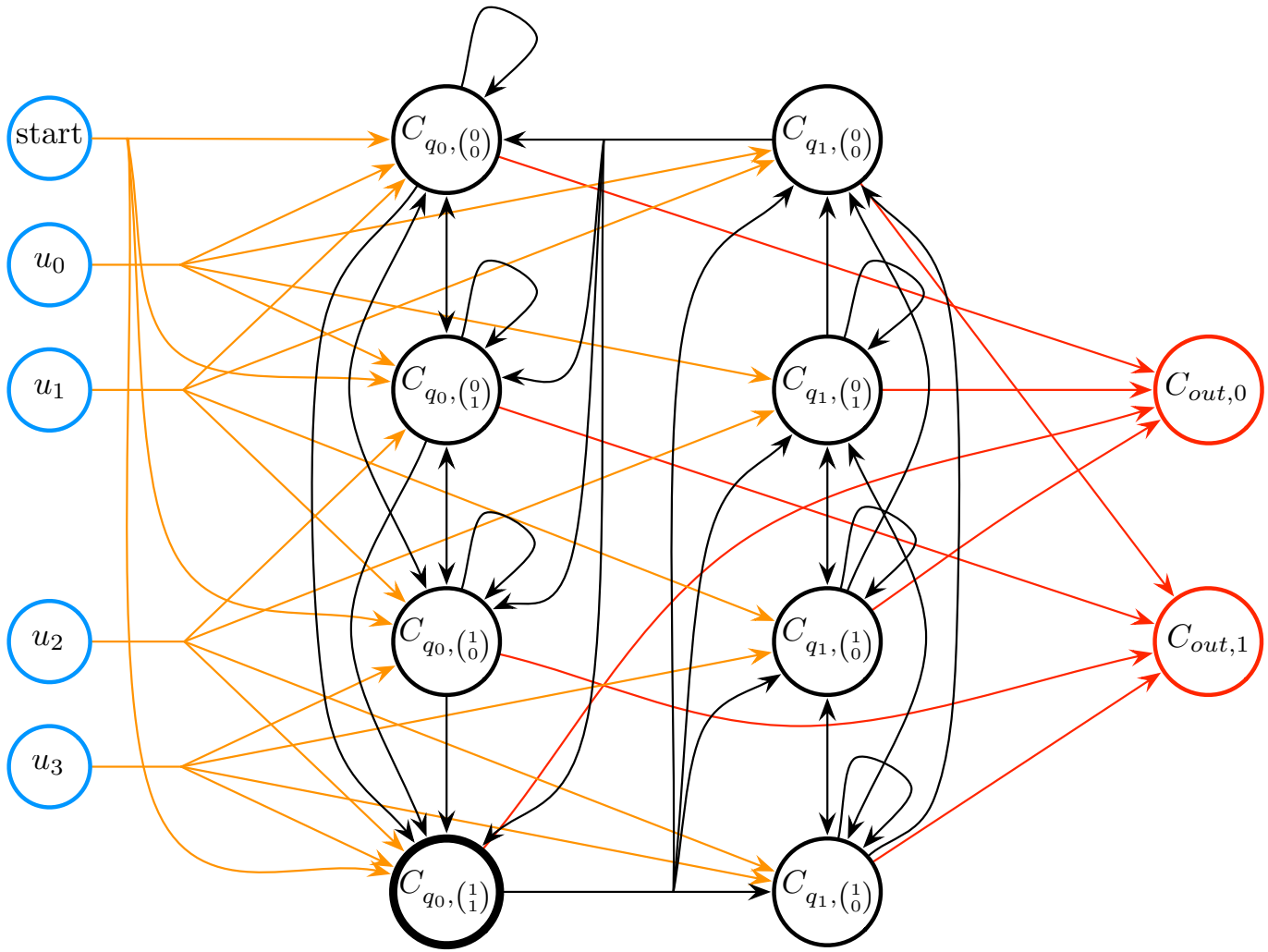
$C_{out,1}$

Simulation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$C_{q_1,\binom{0}{0}}$
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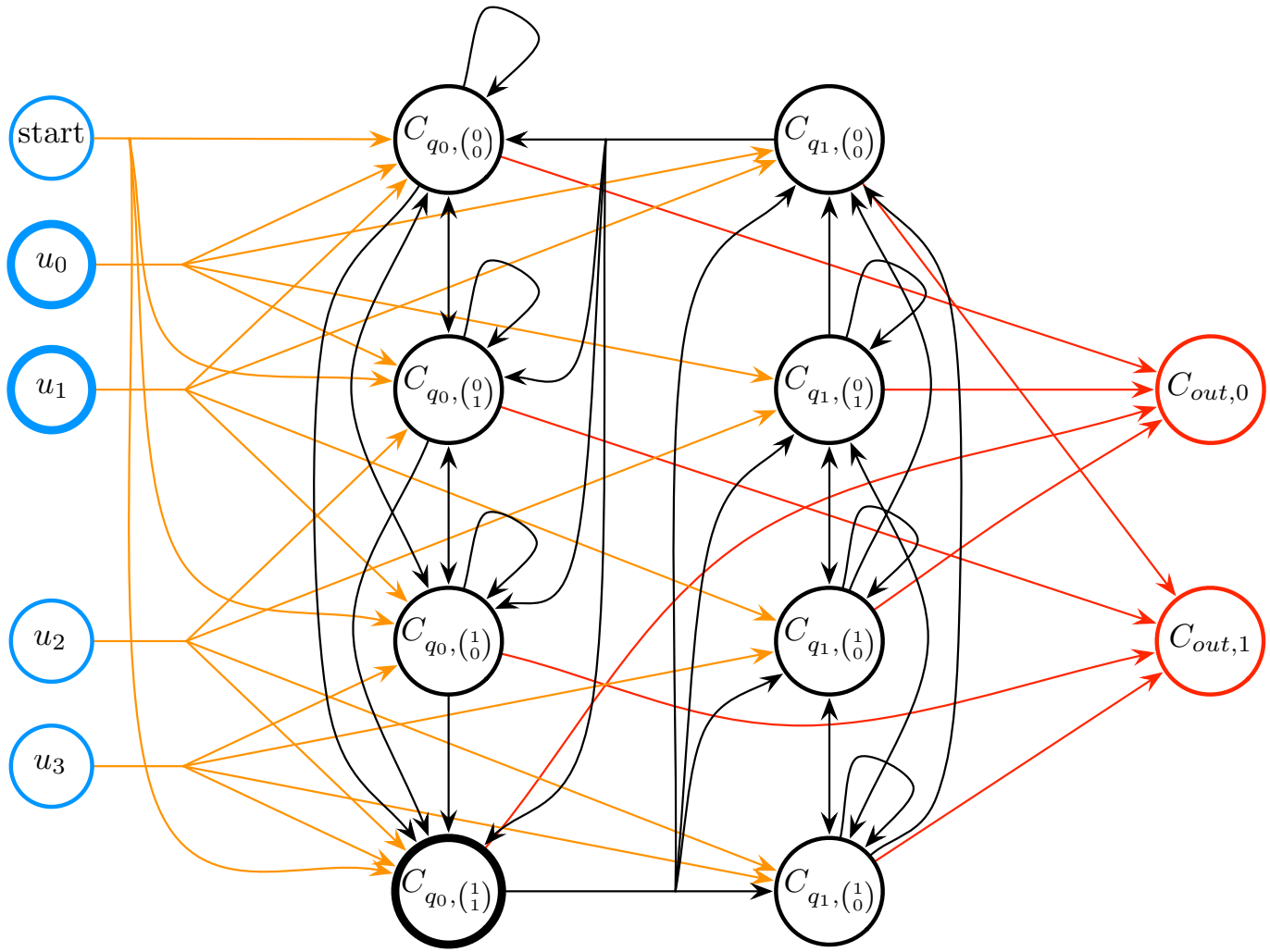
$C_{out,0}$
$C_{out,1}$

Simulation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$C_{q_1,\binom{0}{0}}$
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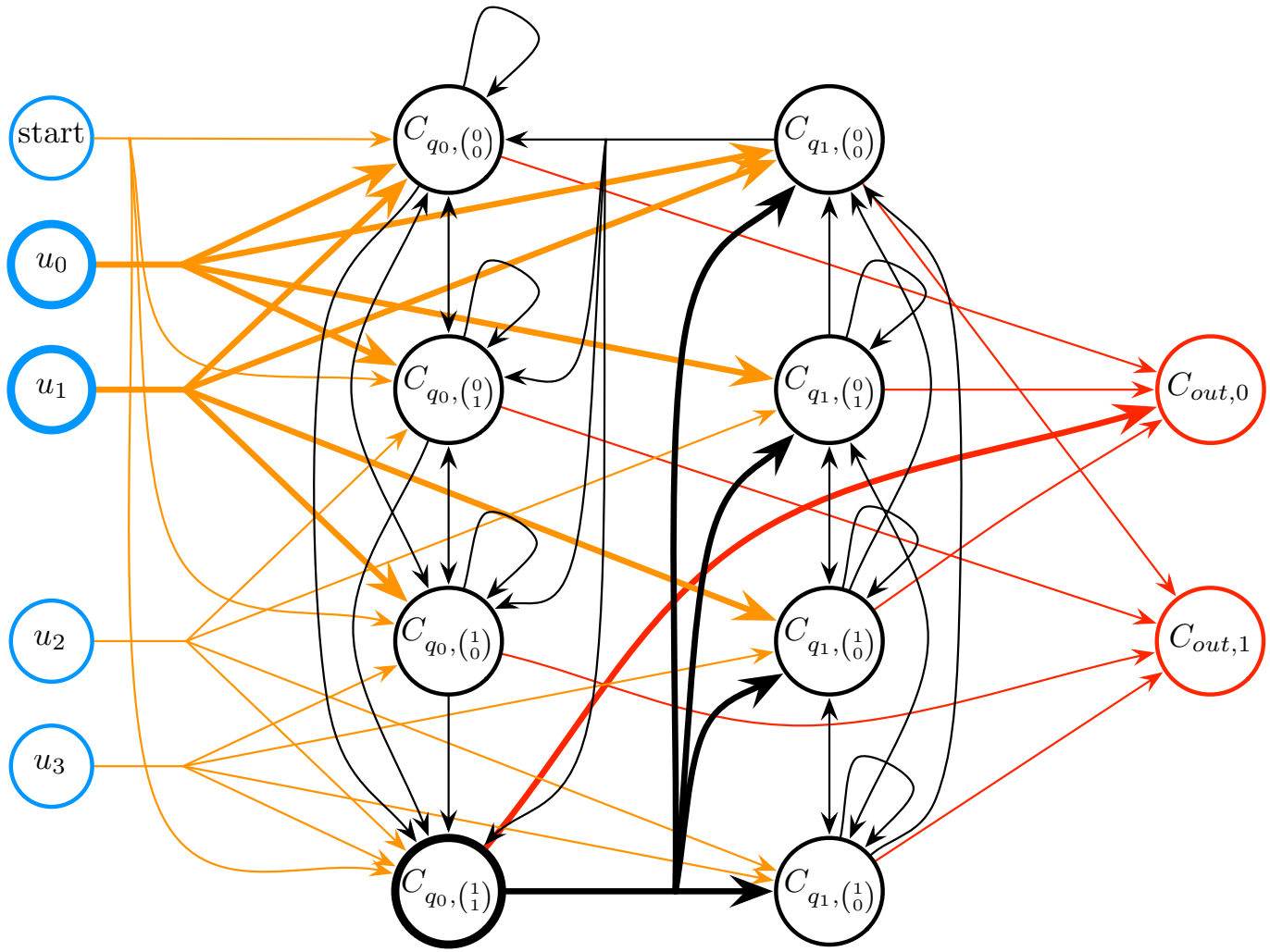
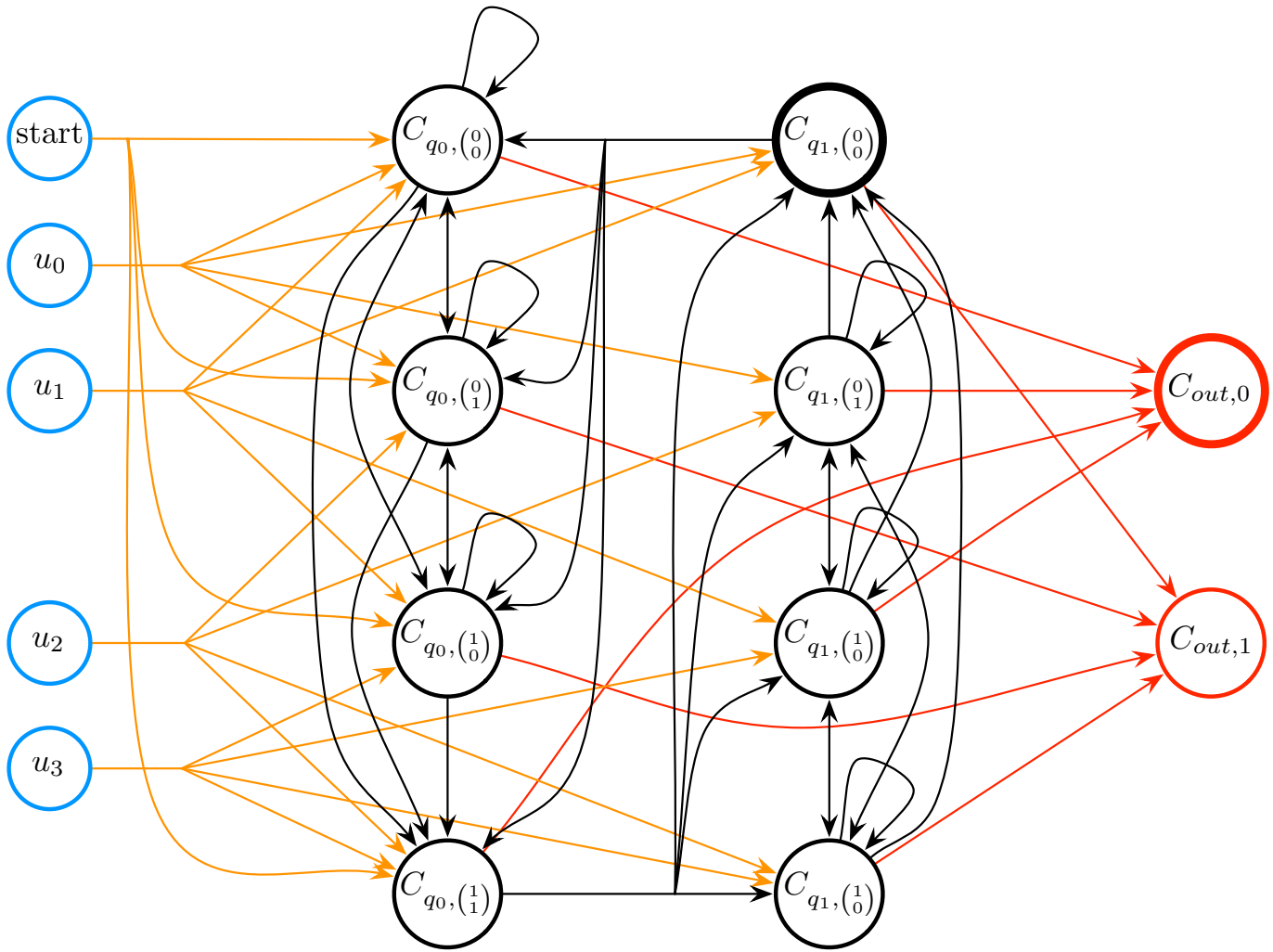
$C_{out,1}$

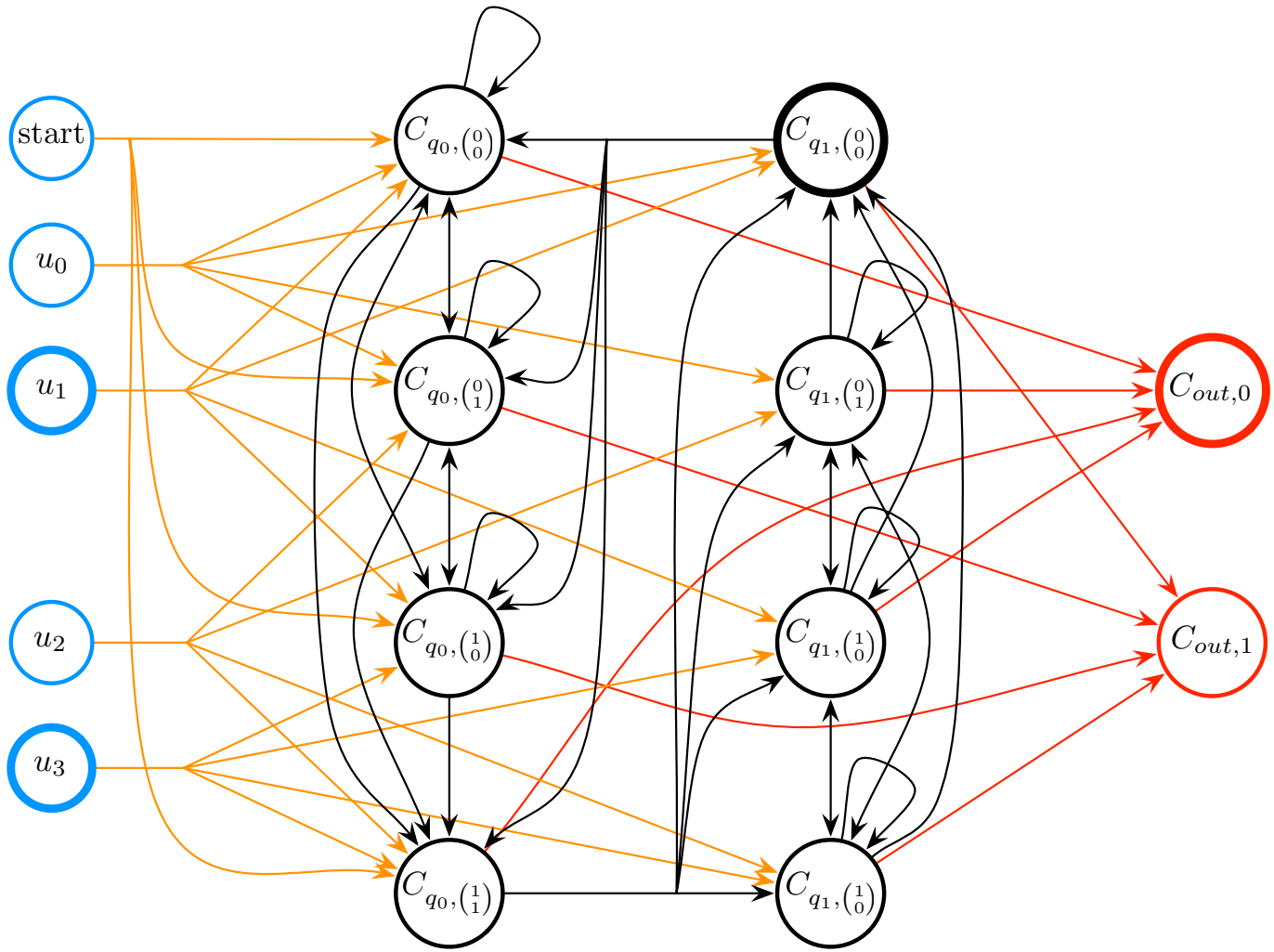
Simulation


start
u_0
u_1
u_2
u_3
$C_{q_0,\binom{0}{0}}$
$C_{q_0,\binom{0}{1}}$
$C_{q_0,\binom{1}{0}}$
$C_{q_0,\binom{1}{1}}$
$C_{q_1,\binom{0}{0}}$
$C_{q_1,\binom{0}{1}}$
$C_{q_1,\binom{1}{0}}$
$C_{q_1,\binom{1}{0}}$
$C_{out,0}$
$C_{out,1}$

Simulation


start
u_0
u_1
u_2
u_3
$C_{q_0,\binom{0}{0}}$
$C_{q_0,\binom{0}{1}}$
$C_{q_0,\binom{1}{0}}$
$C_{q_0,\binom{1}{1}}$
$C_{q_1,\binom{0}{0}}$
$C_{q_1,\binom{0}{1}}$
$C_{q_1,\binom{1}{0}}$
$C_{q_1,\binom{1}{0}}$
$C_{out,0}$
$C_{out,1}$

Simulation


start
u_0
u_1
u_2
u_3
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$C_{q_0,\binom{0}{1}}$
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$C_{out,0}$
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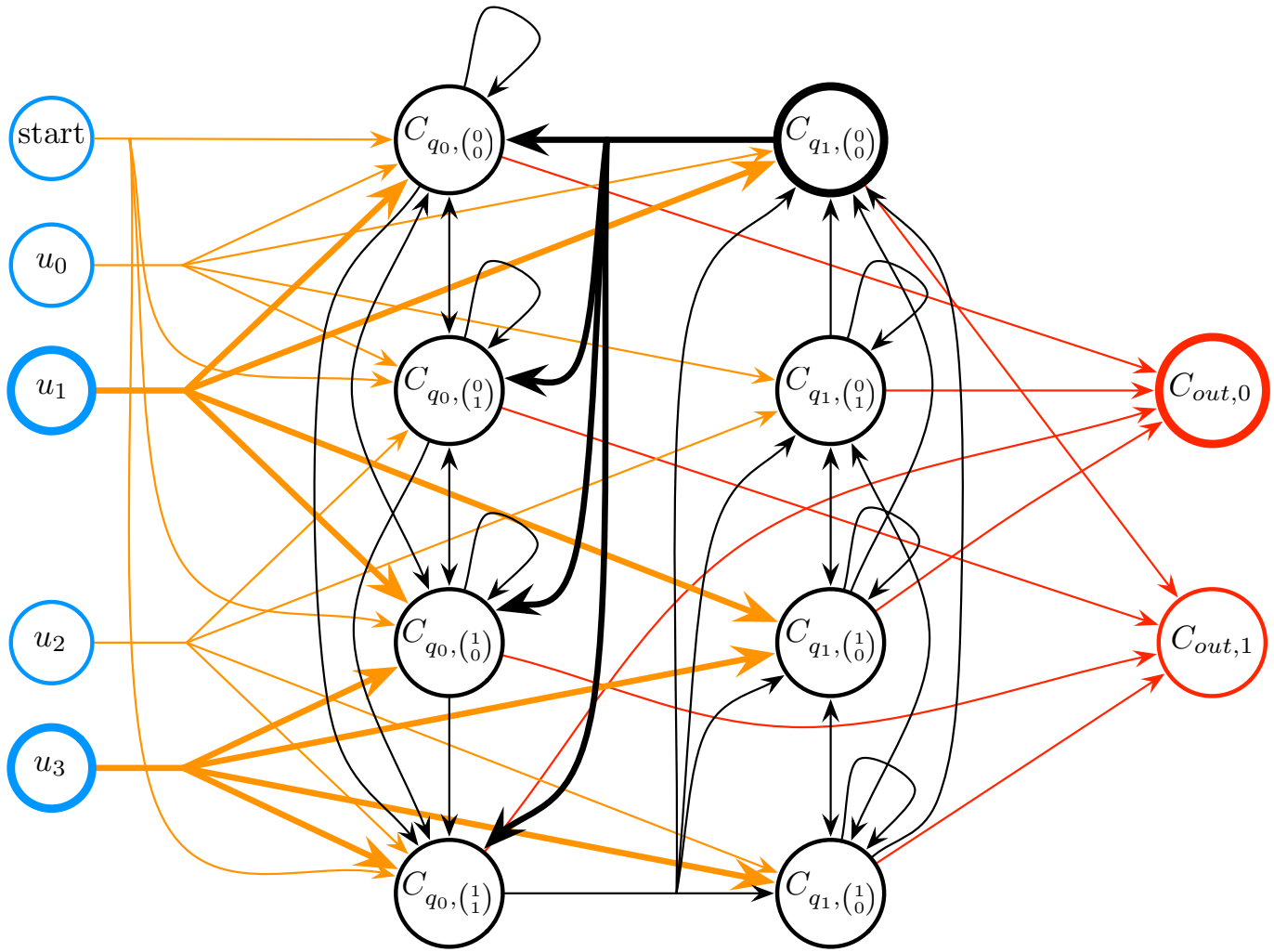
start
u_0
u_1
u_2
u_3
$C_{q_0,\binom{0}{0}}$
$C_{q_0,\binom{0}{1}}$
$C_{q_0,\binom{1}{0}}$
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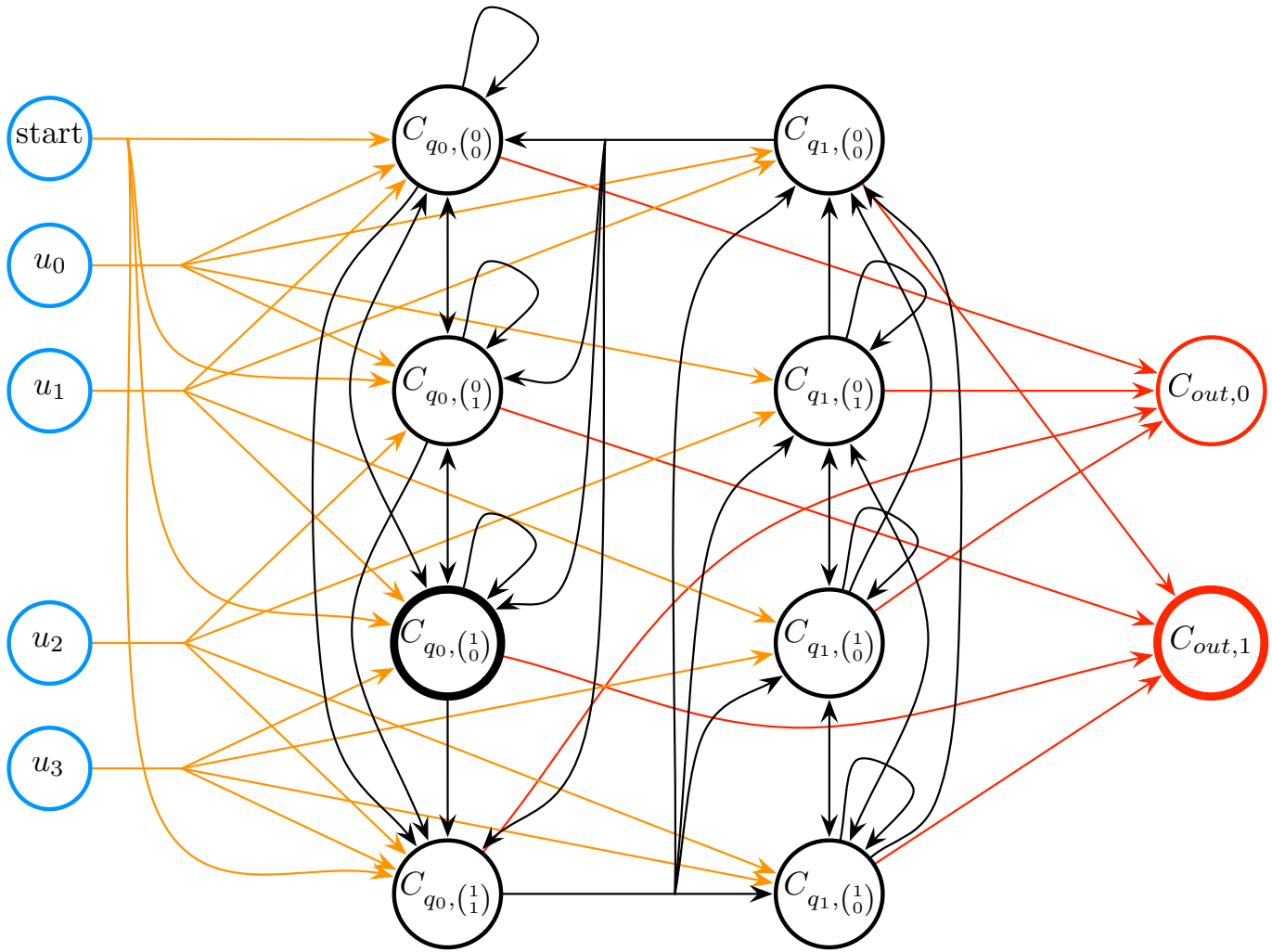
start
u_0
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u_3
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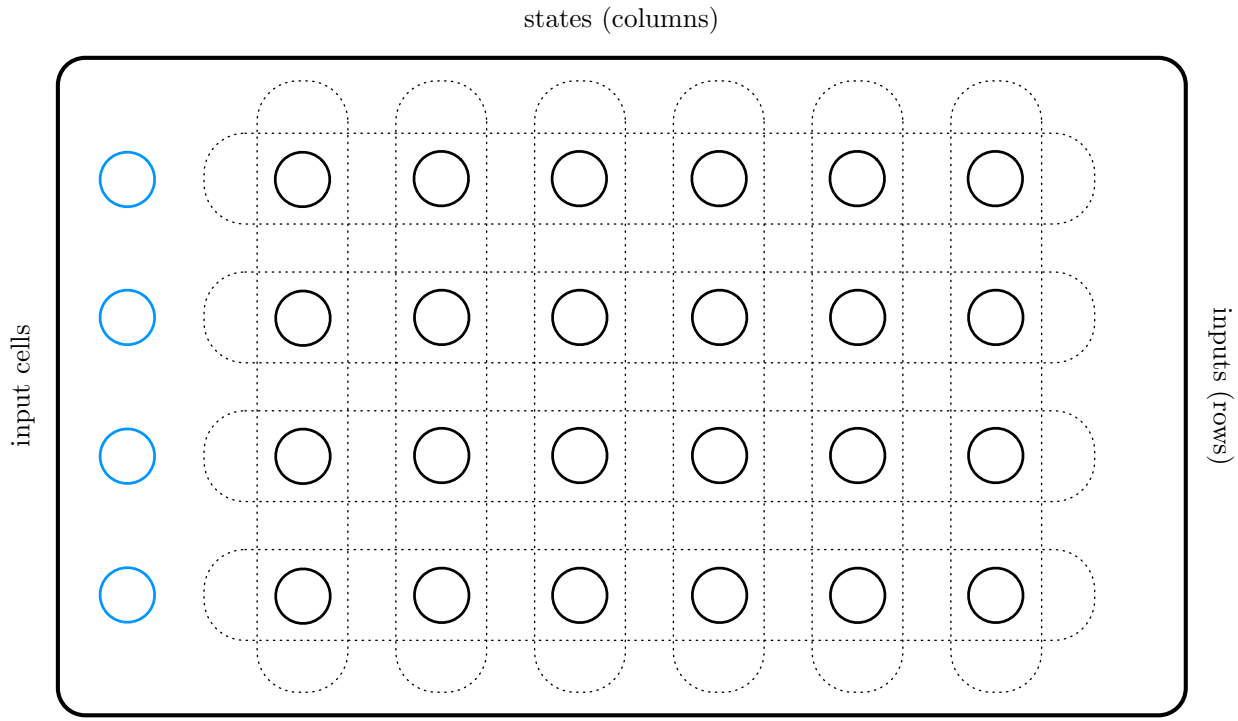
Simulation

time	0	1	2	3	4	5	6	7	8
states	q_0	q_1	q_0	q_0	q_1	q_1	q_1	q_0	–
inputs	$\binom{1}{1}$	$\binom{0}{0}$	$\binom{1}{0}$	$\binom{1}{1}$	$\binom{0}{1}$	$\binom{1}{1}$	$\binom{0}{0}$	–	–
outputs	0	1	1	0	0	1	1	–	–
$start$	1	0	0	0	0	0	0	0	0
u_0	0	1	0	0	1	0	1	0	0
u_1	0	1	1	0	0	0	1	0	0
u_2	1	0	0	1	1	1	0	0	0
u_3	1	0	1	1	0	1	0	0	0
$C_{s,i}$	–	$q_0, \binom{1}{1}$	$q_1, \binom{0}{0}$	$q_0, \binom{1}{0}$	$q_0, \binom{1}{1}$	$q_1, \binom{0}{1}$	$q_1, \binom{1}{1}$	$q_1, \binom{0}{0}$	–
$C_{out,0}$	0	0	1	0	0	1	1	0	0
$C_{out,1}$	0	0	0	1	1	0	0	1	1

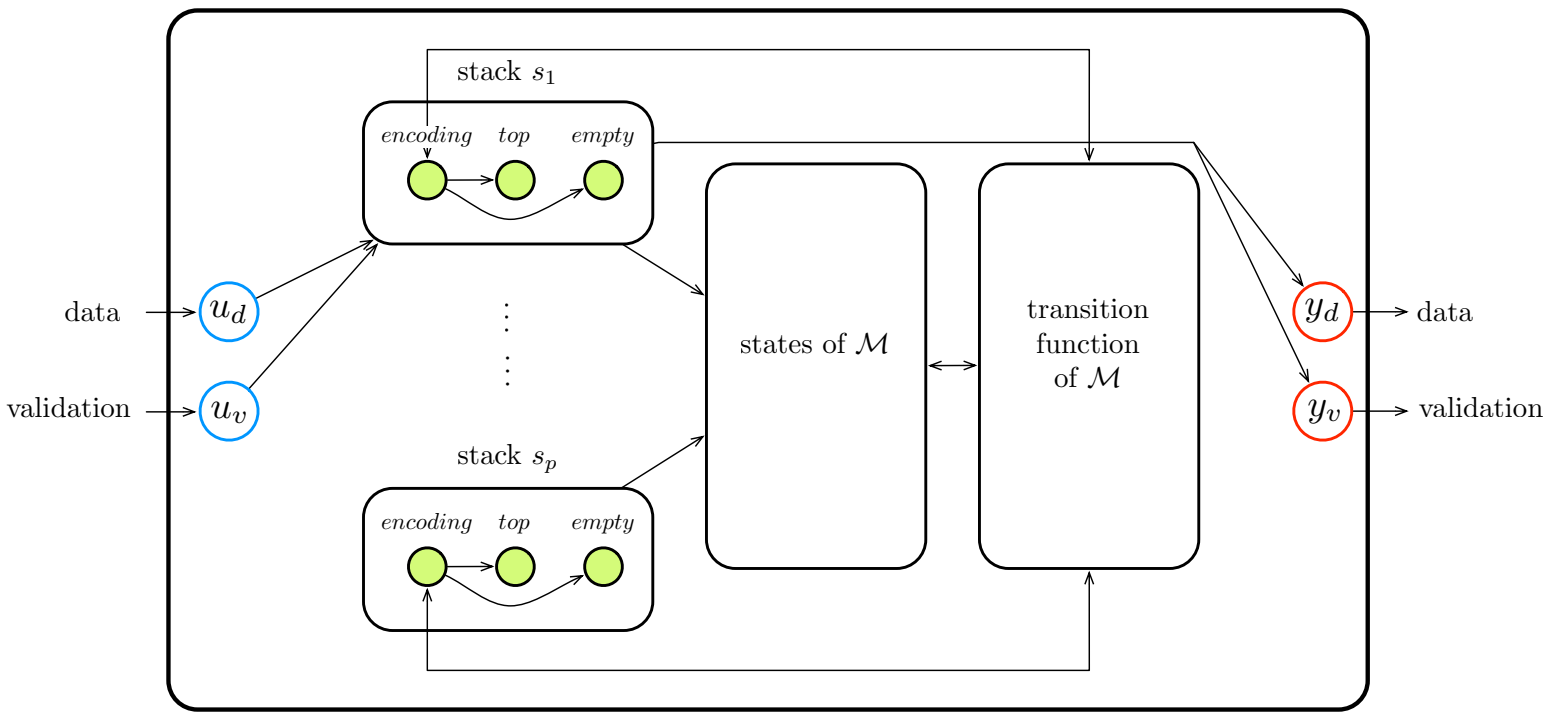
Simulating FSA with B-RNNs (summary)

- Proof idea: grid-like B-RNN (state $\times$ inputs)...



Simulating TMs with St-RNN[$\mathbb{Q}$]s (proof idea)

- Simulating a multistack machine.
- Stacks operations carried out by rational-weighted neural nets.



Drawbacks of the Constructions

- Computational states of the machines are represented as Boolean states, i.e., spiking configurations of the network.
- ⋆ Computational states should rather be represented by *sustained activities of neural assemblies*, e.g., by *cyclic attractors*.
- Network is not robust to cell death, synaptic plasticity, architectural plasticity in general.
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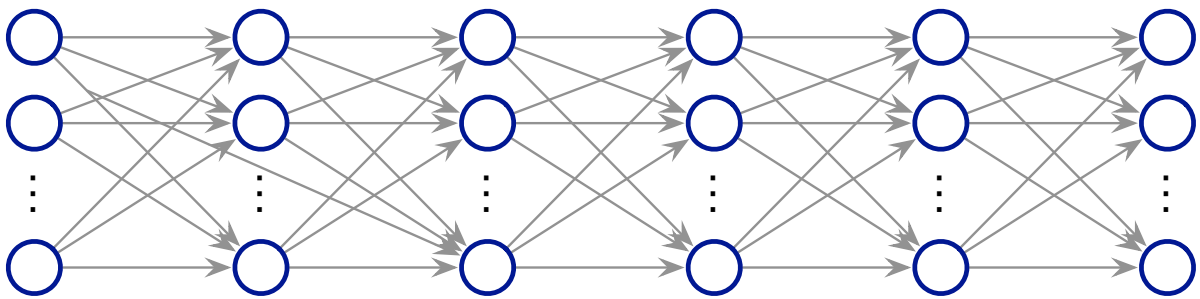
- *Synfire chains* allow for robust and highly precise transmission of information in neural networks.
- *Synfire chains* are likely to be crucially involved in the processing and coding of information in neural networks.
- *Synfire chains* have been theorized as fundamental neuronal structures (Abeles 82).



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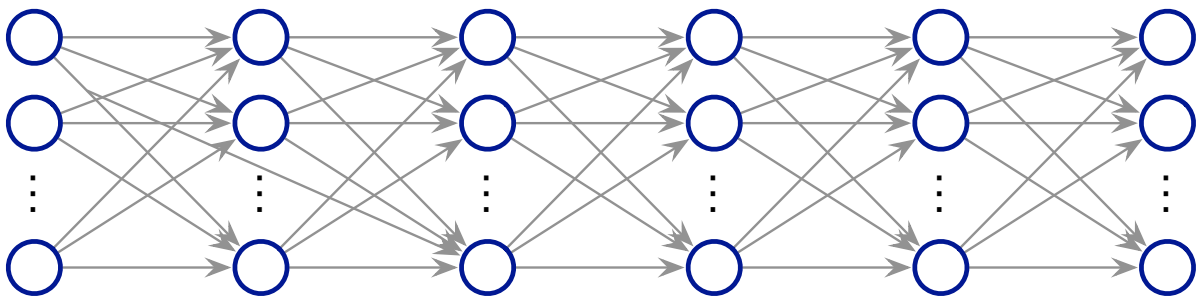

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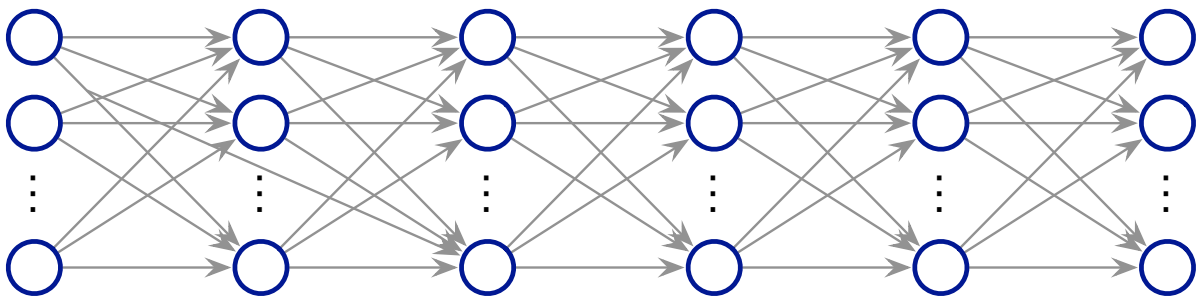

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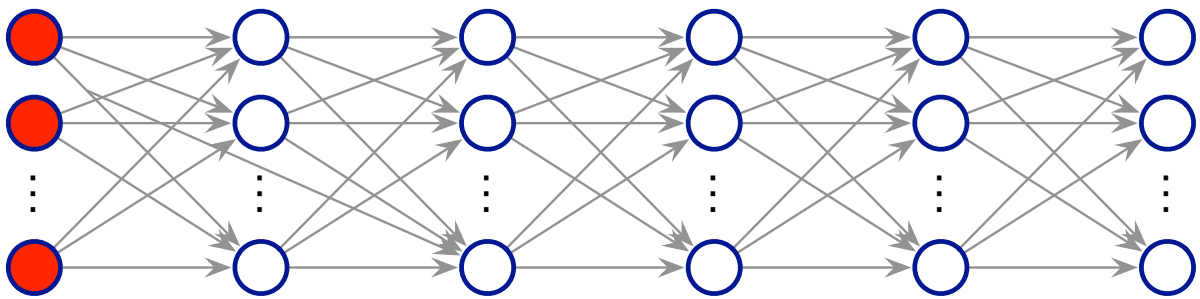

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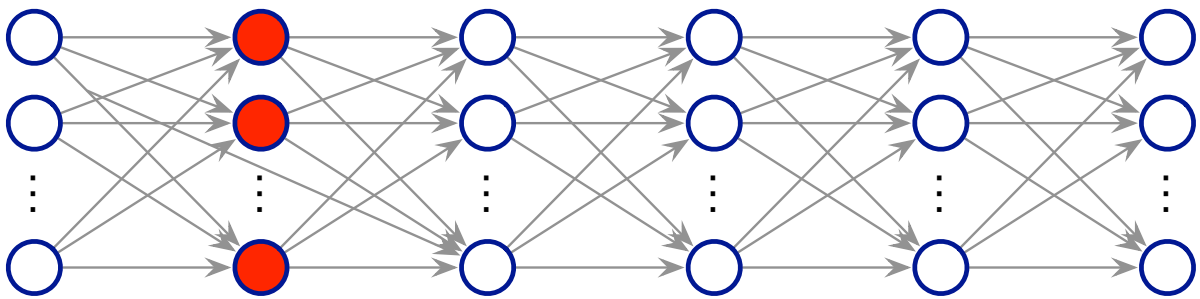

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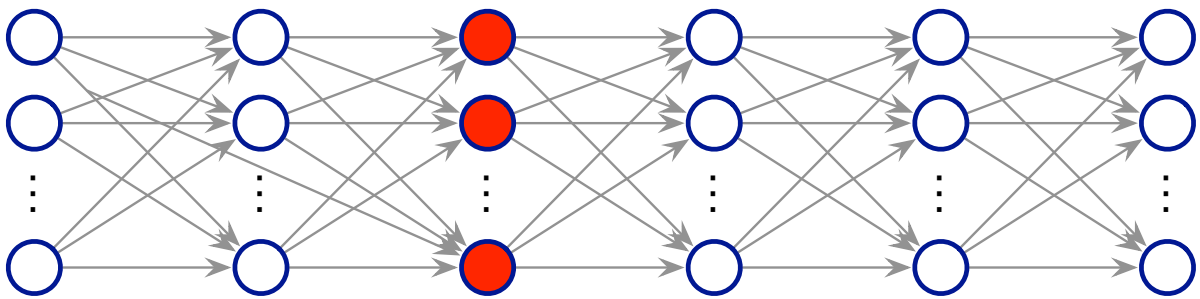

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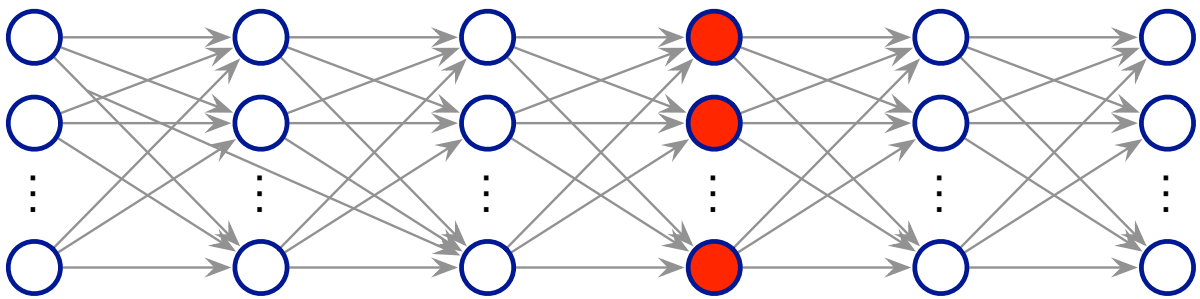

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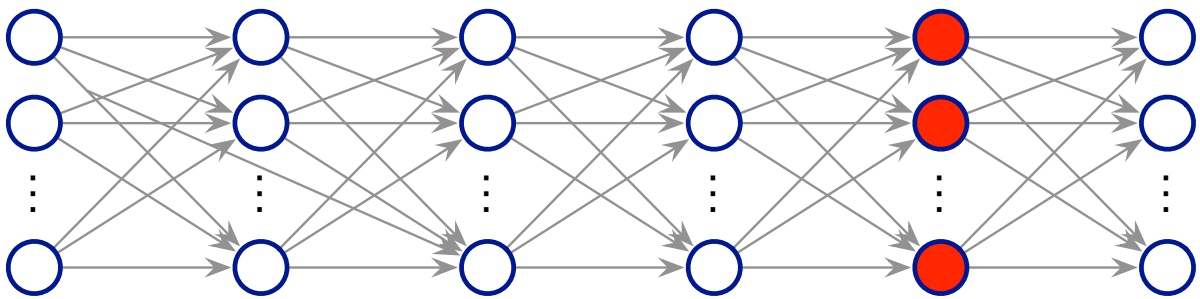

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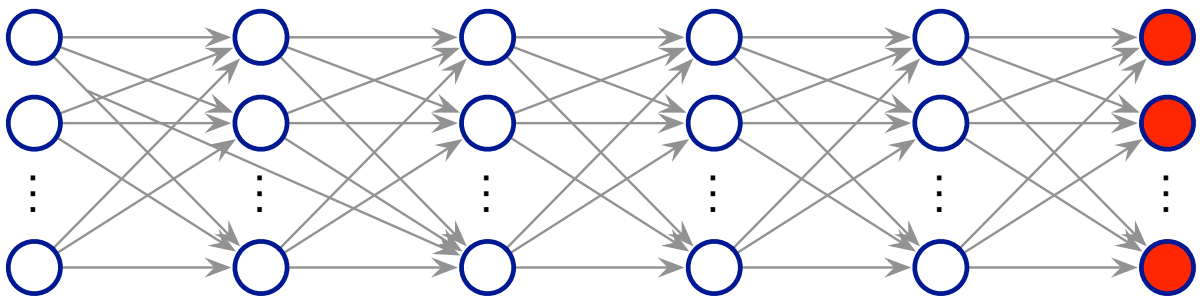

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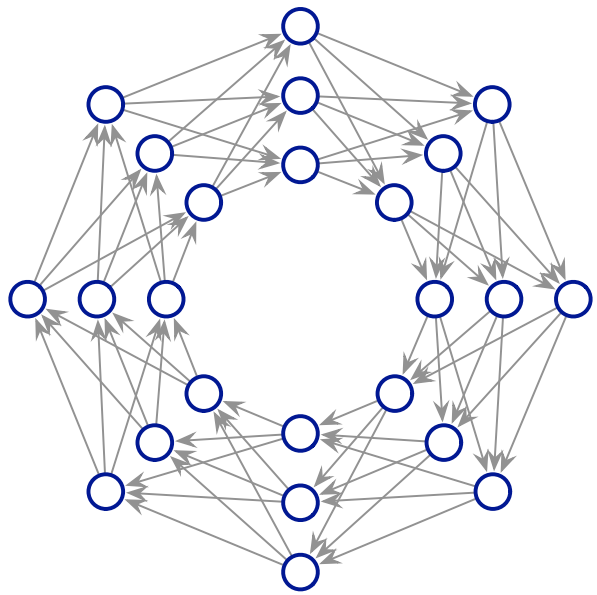

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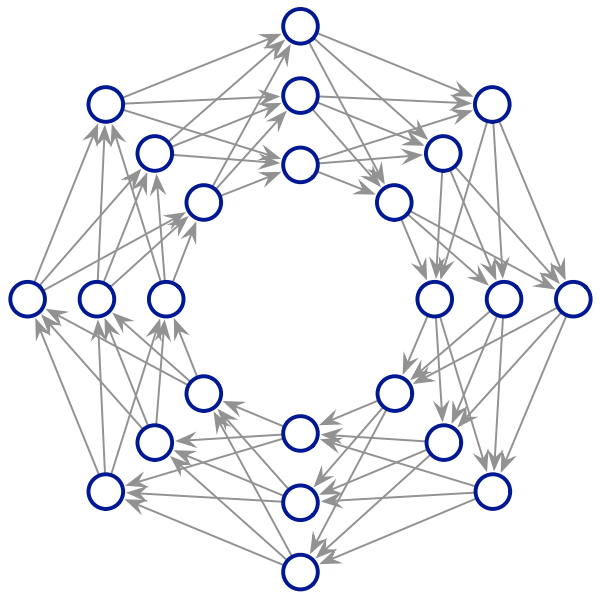

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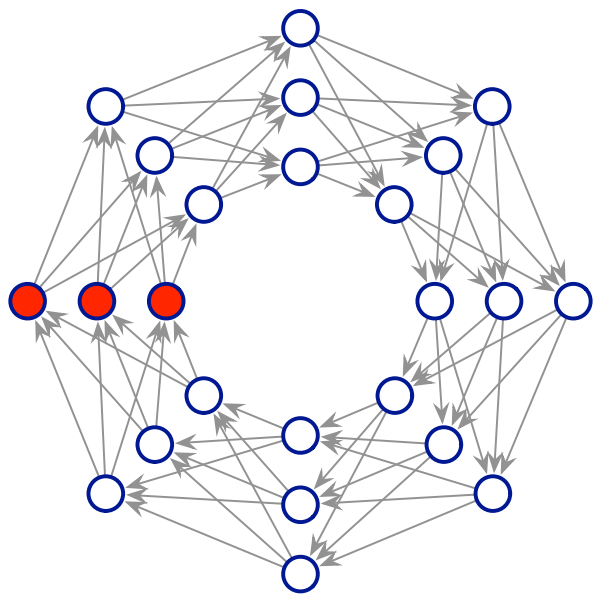

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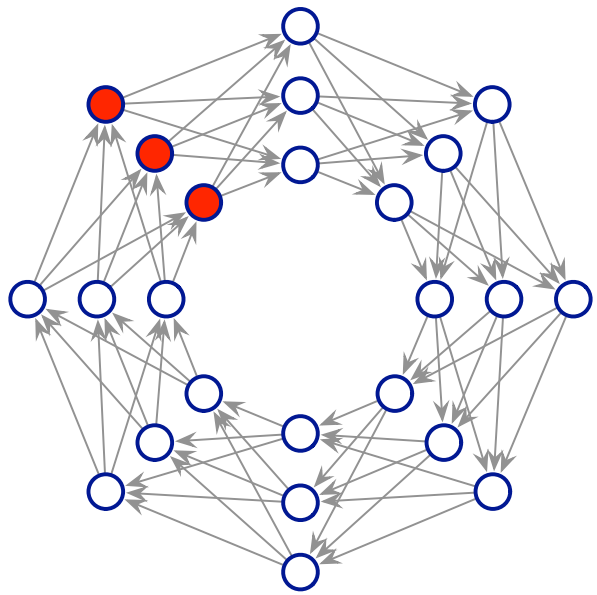

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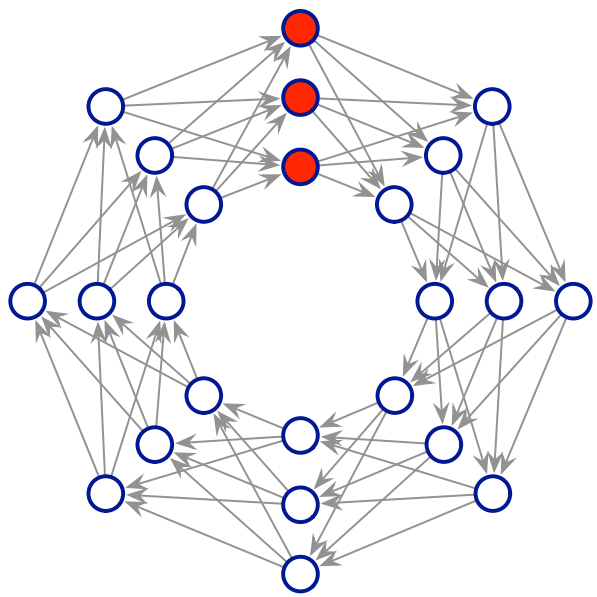

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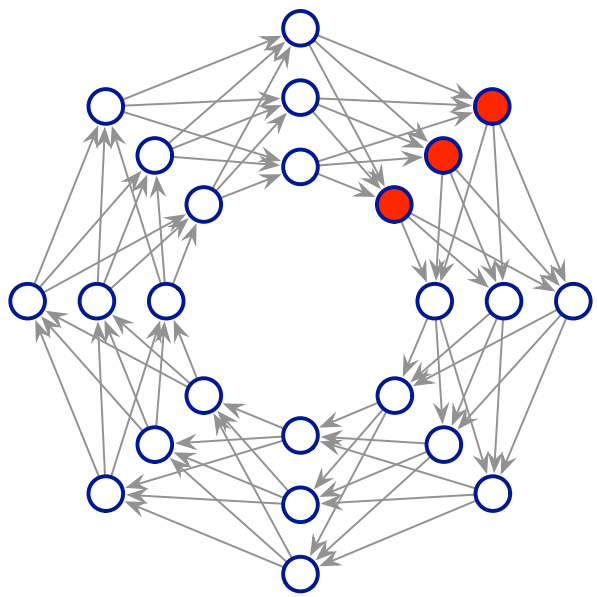

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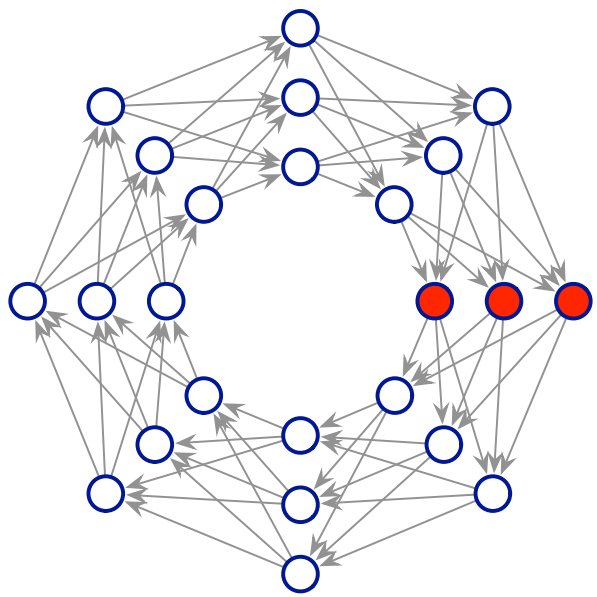

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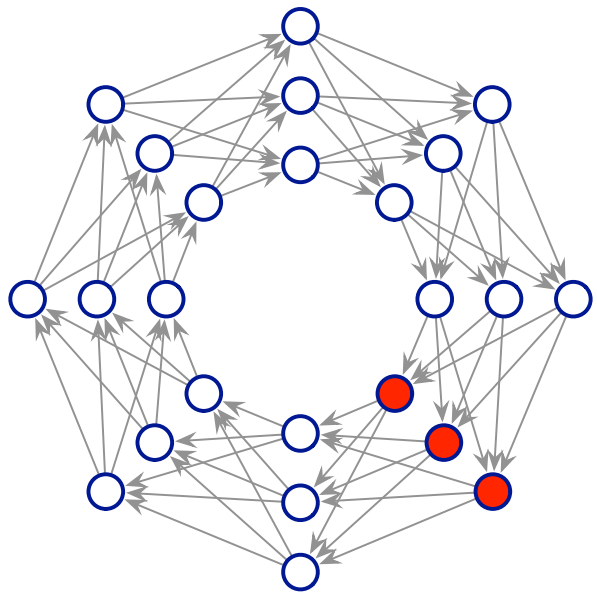

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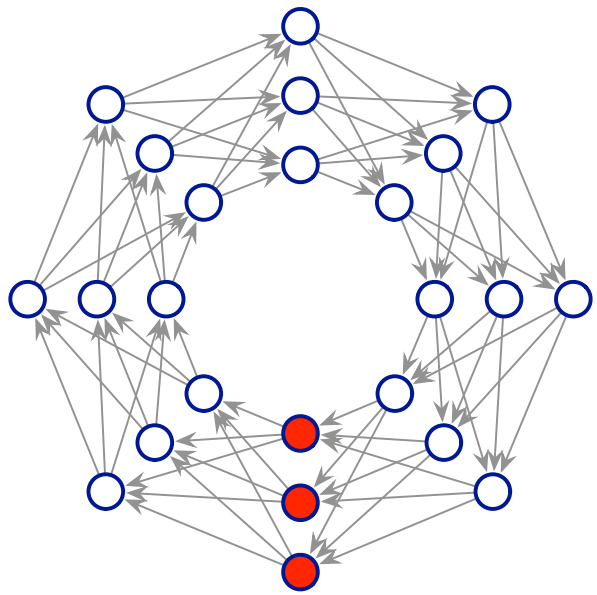

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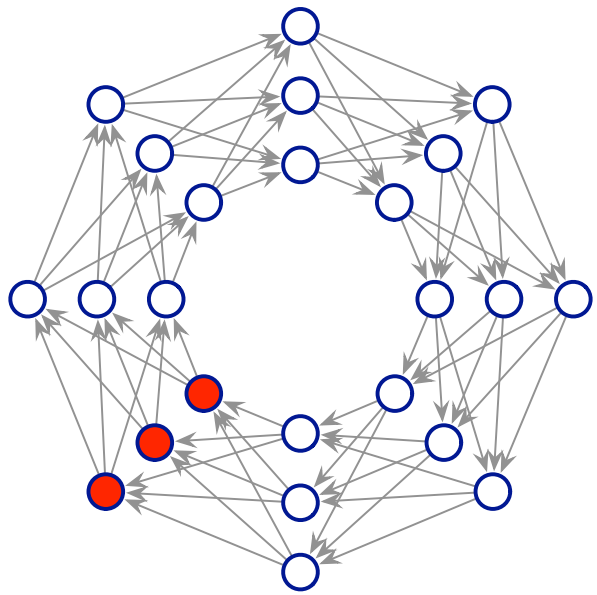

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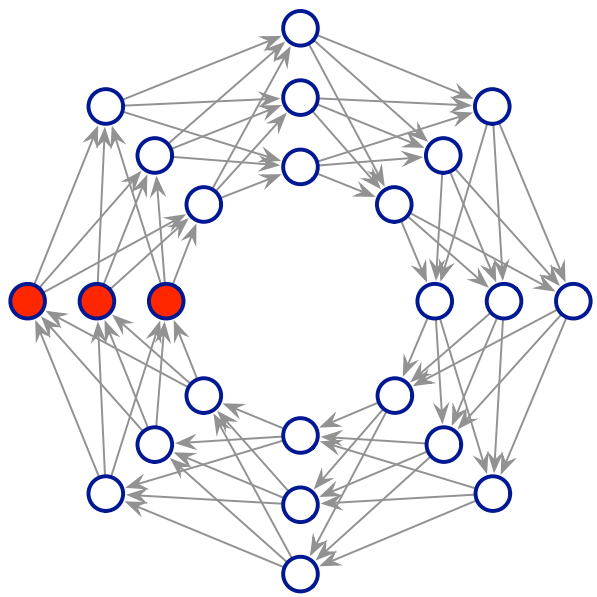

Neural Computation with Synfire Rings

- We introduce a paradigm of neural computation based on *synfire rings*.
- Computational states are represented by sustained activities within synfire rings.
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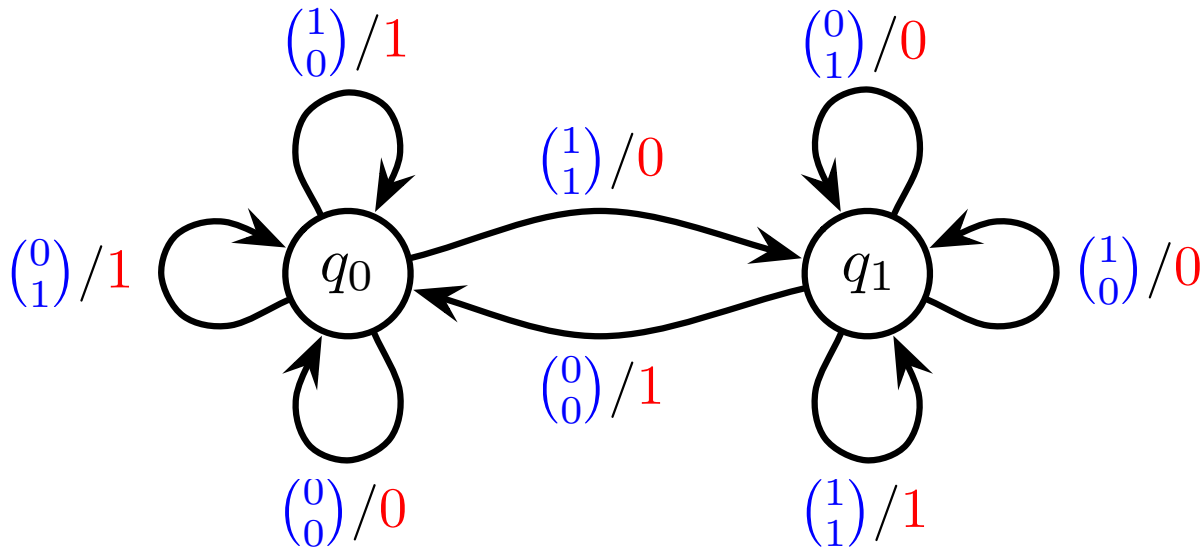
Boolean RNNs

- We simulate finite state automata with Boolean recurrent neural networks composed of synfire rings.

General Construction



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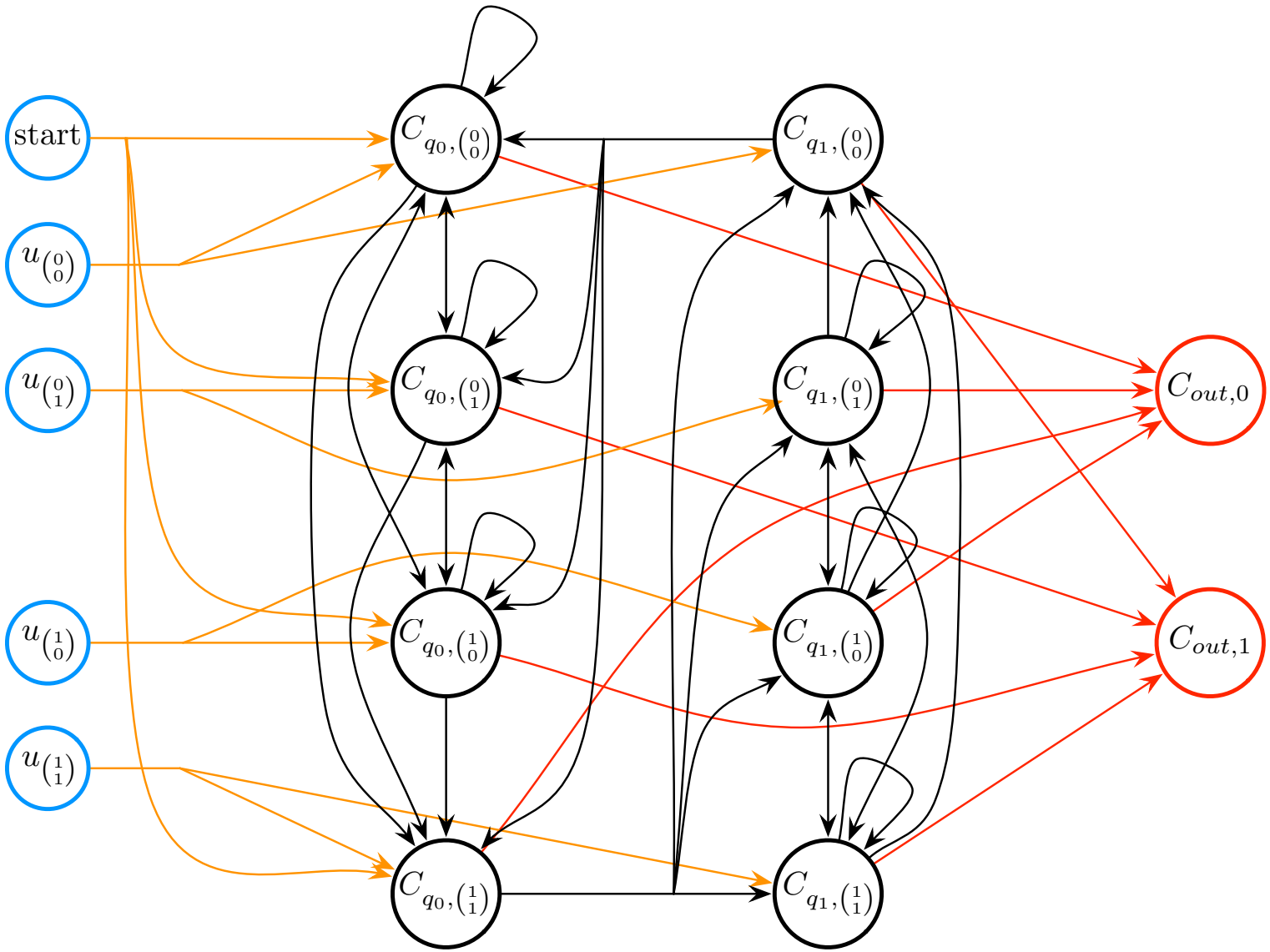

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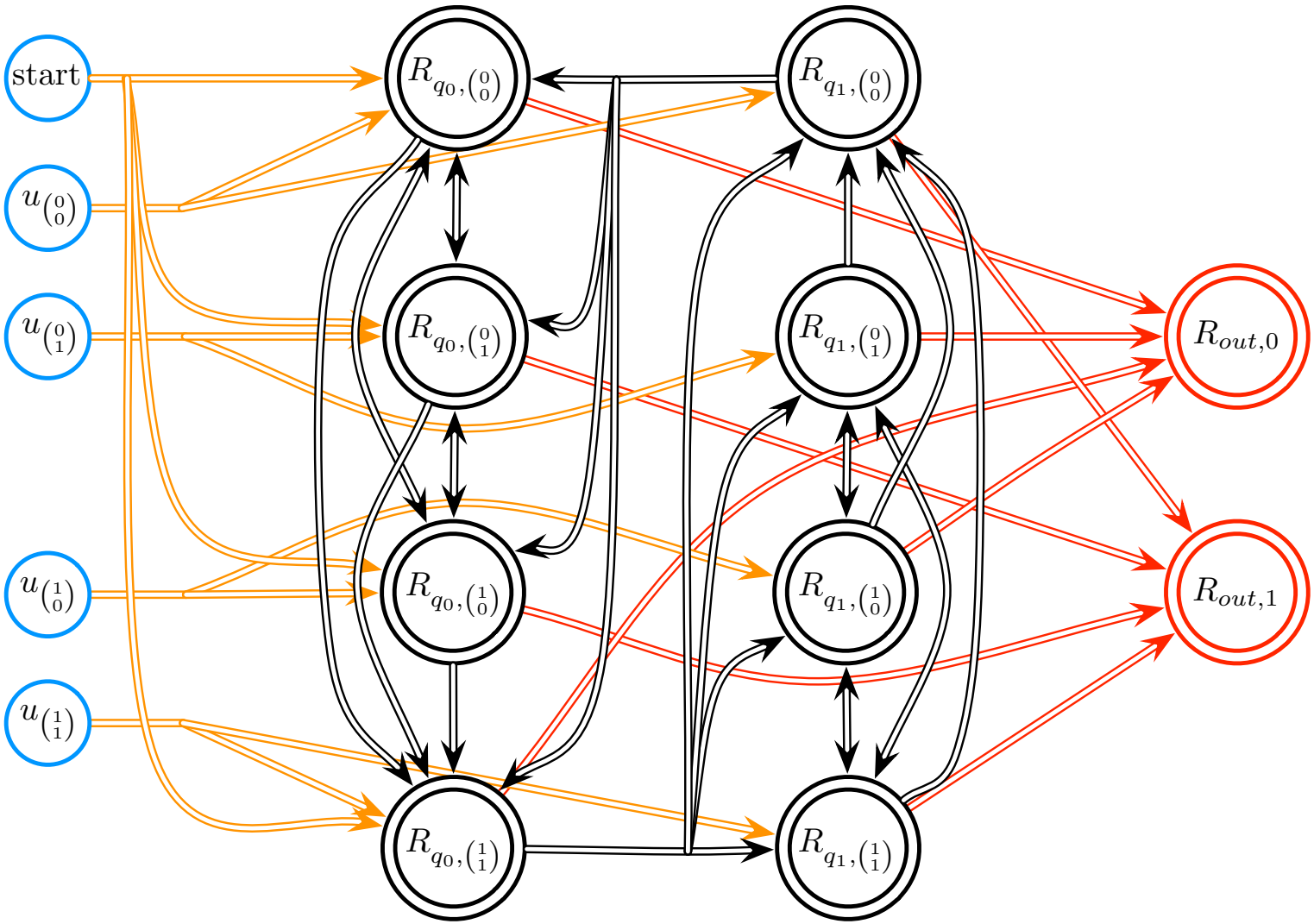
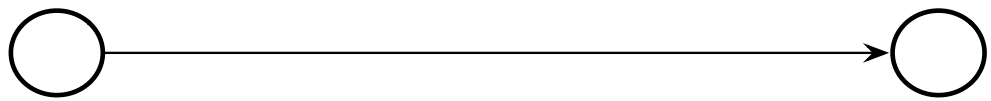


Fibres of connections & Inhibitory System



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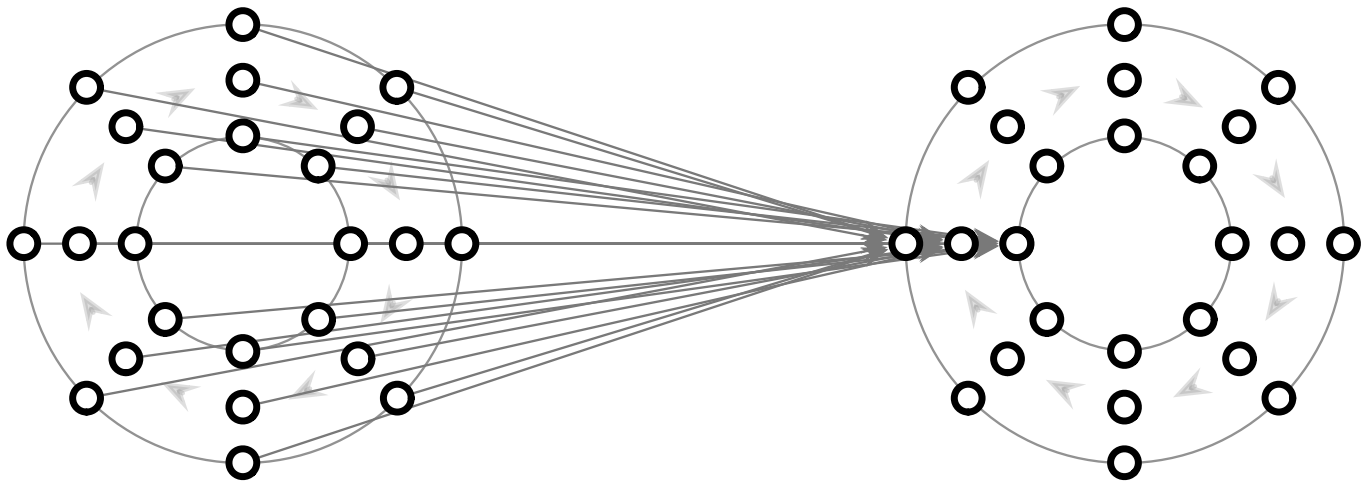

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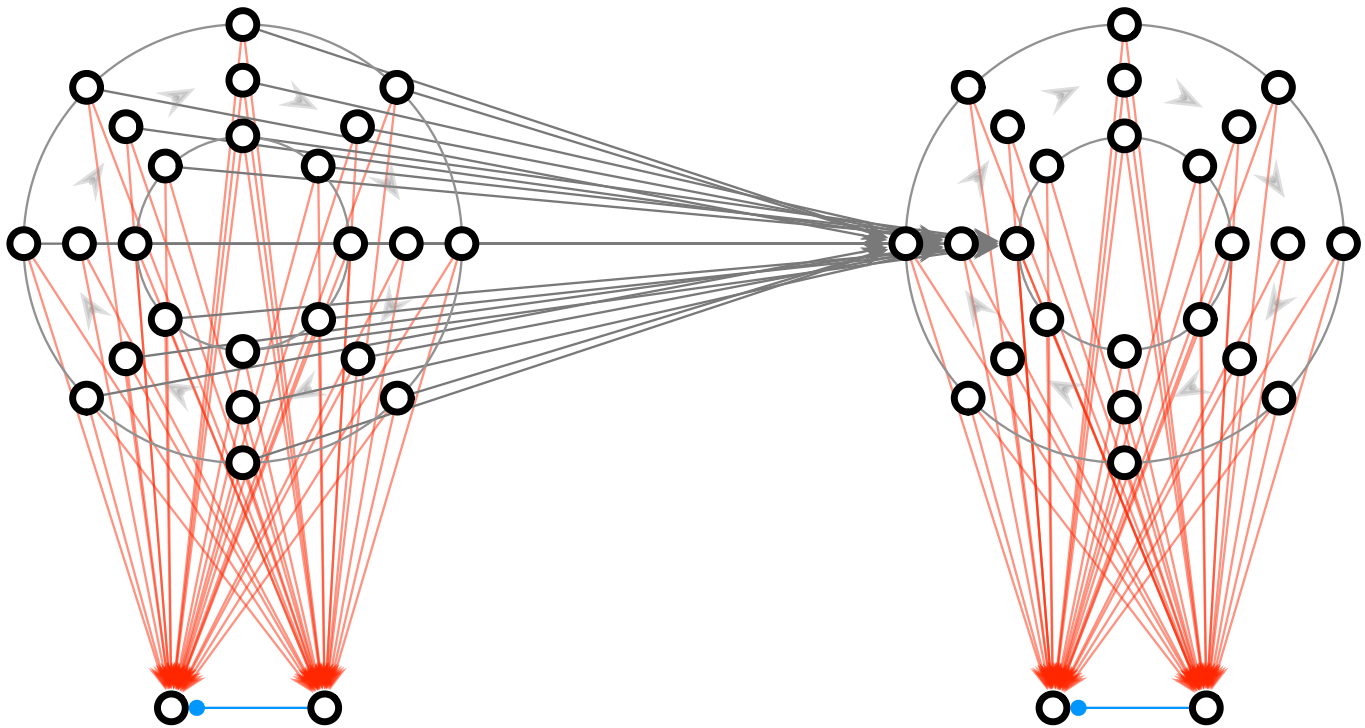

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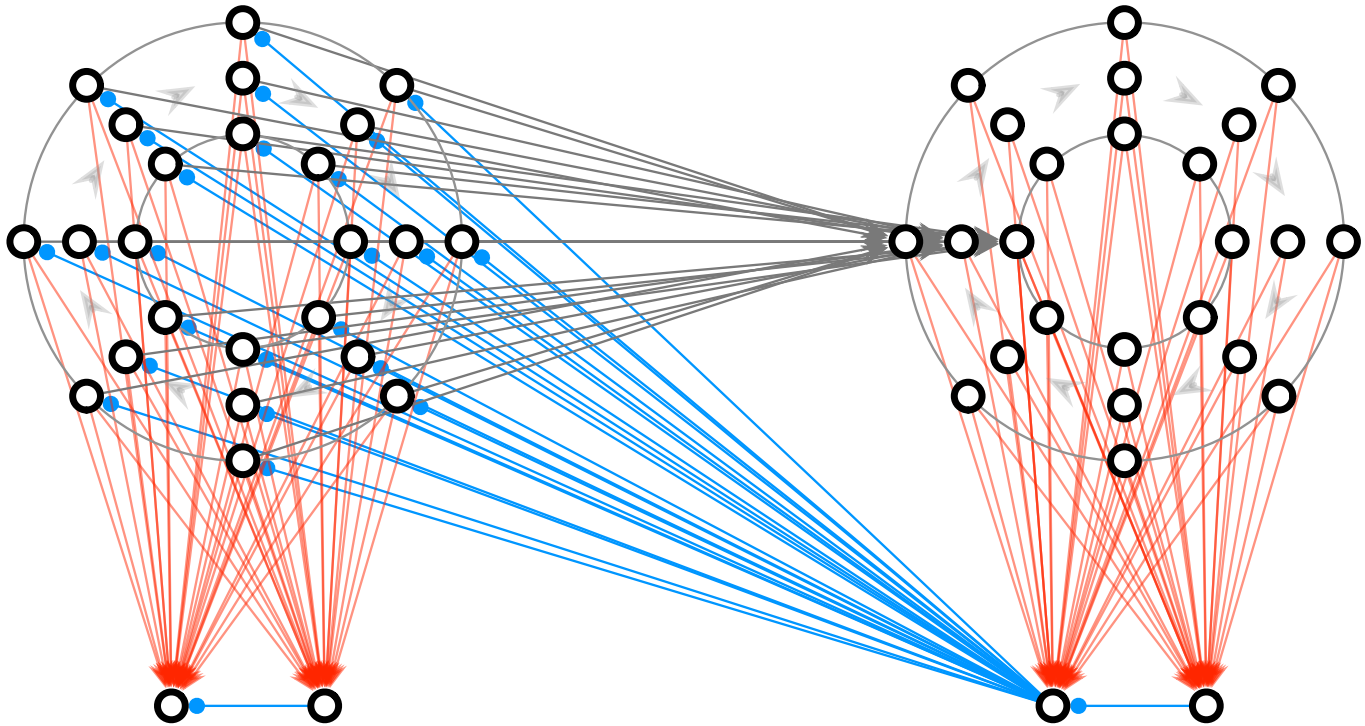

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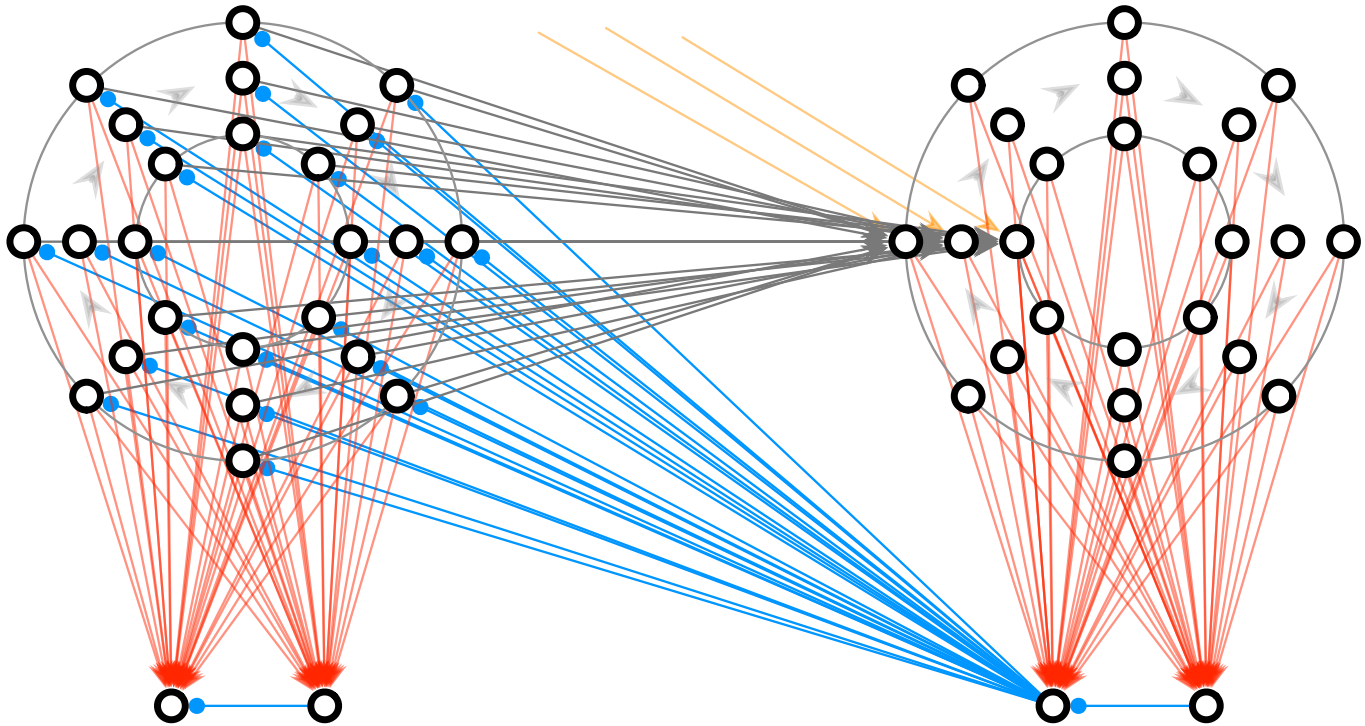

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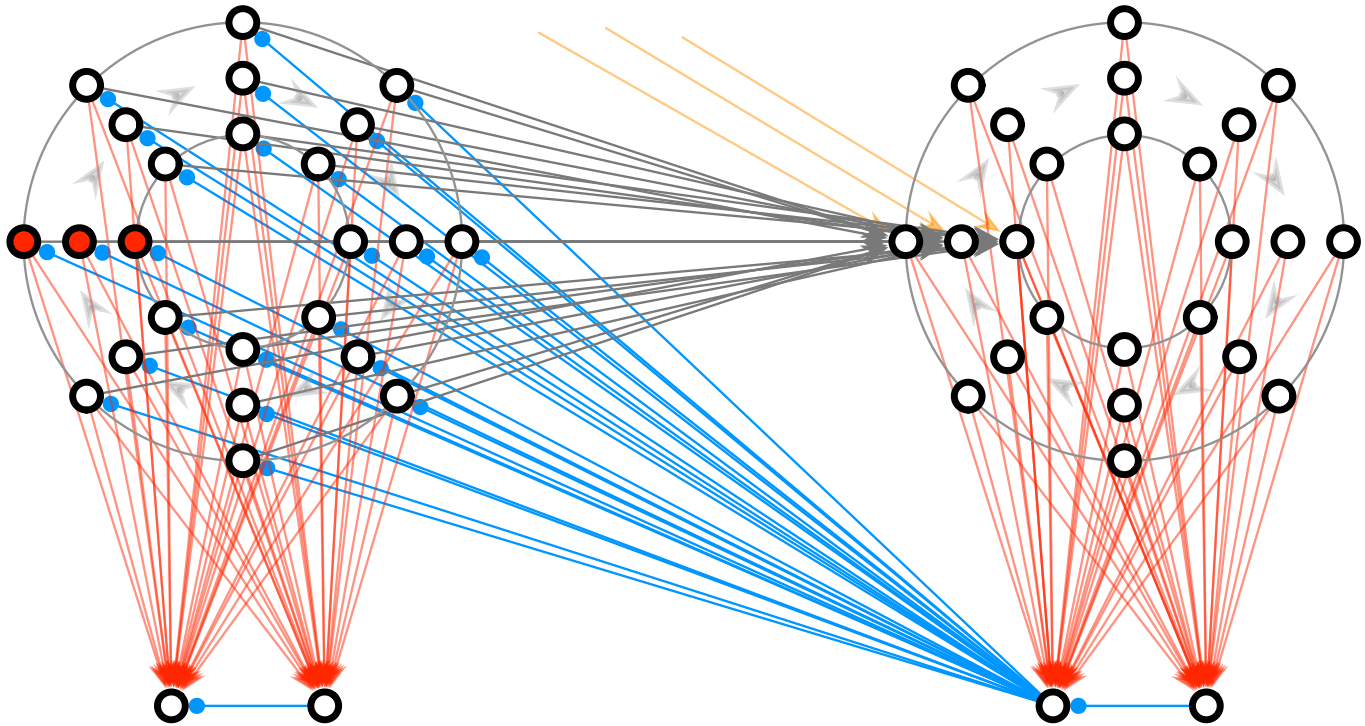
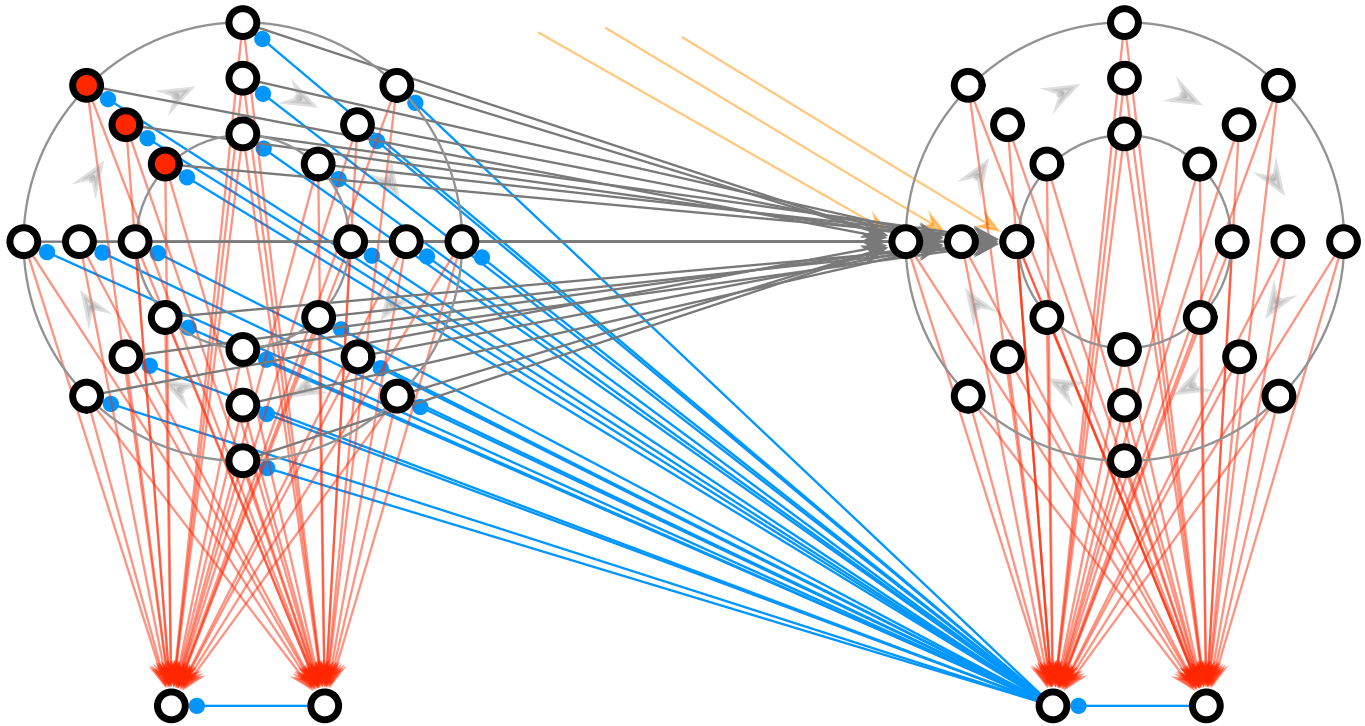
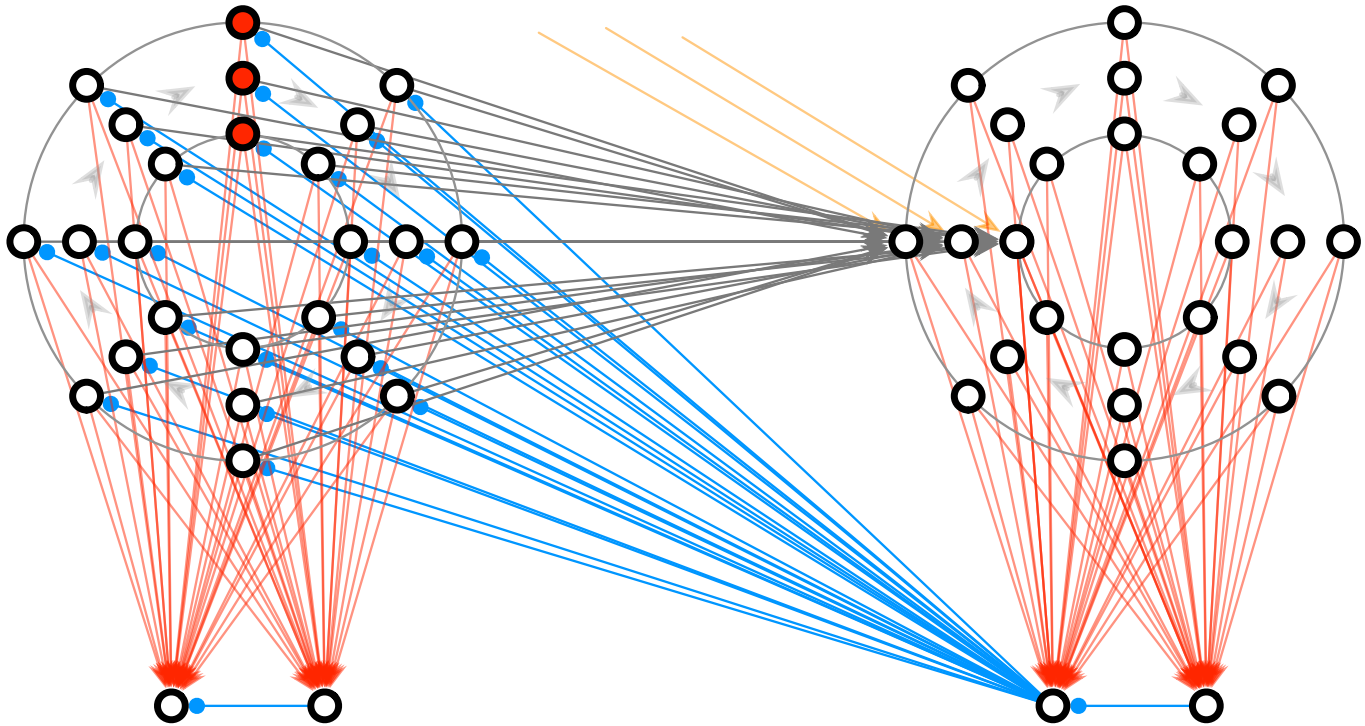


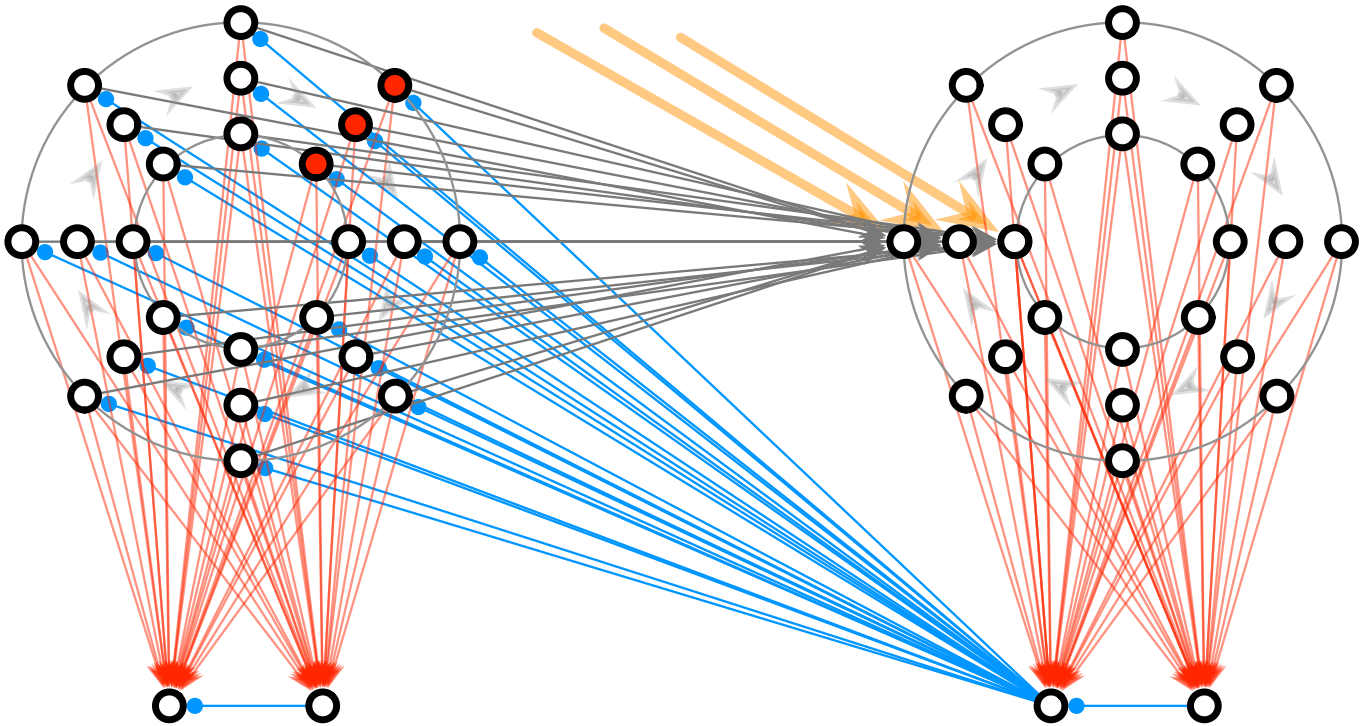
Fibres of connections & Inhibitory System



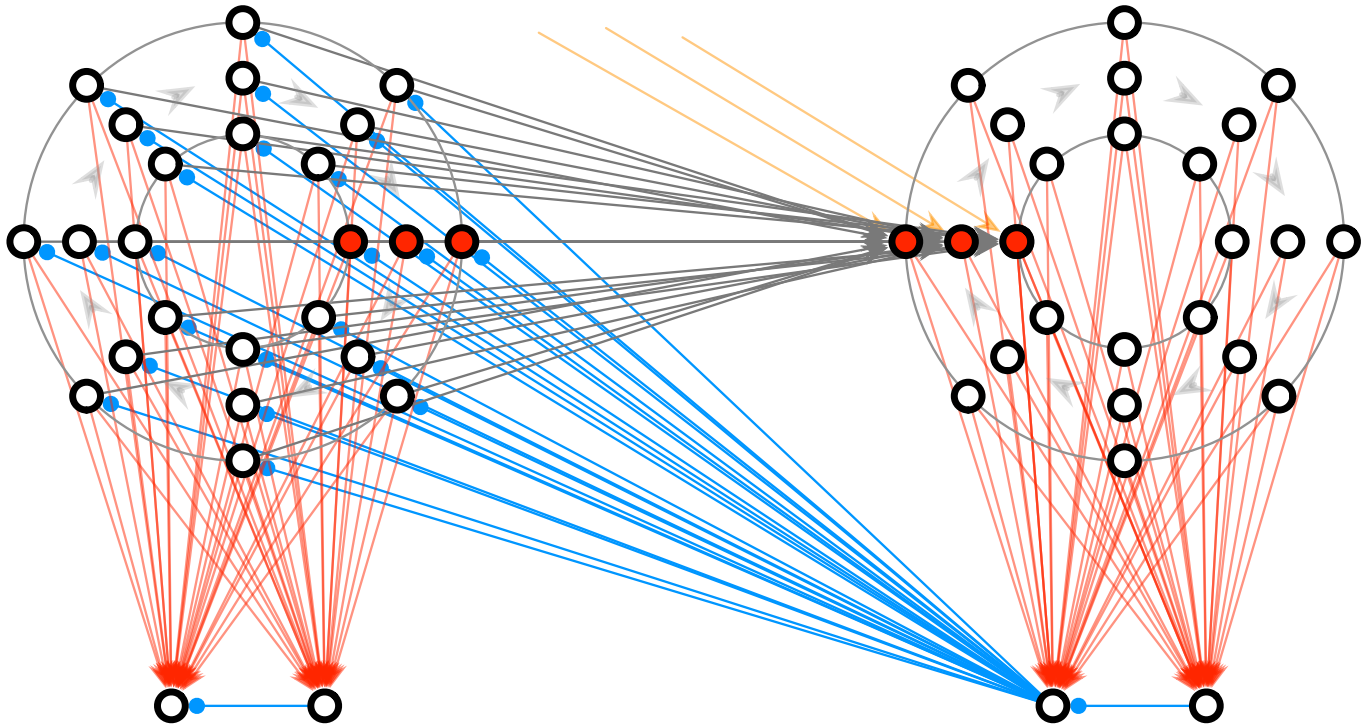
Fibres of connections & Inhibitory System



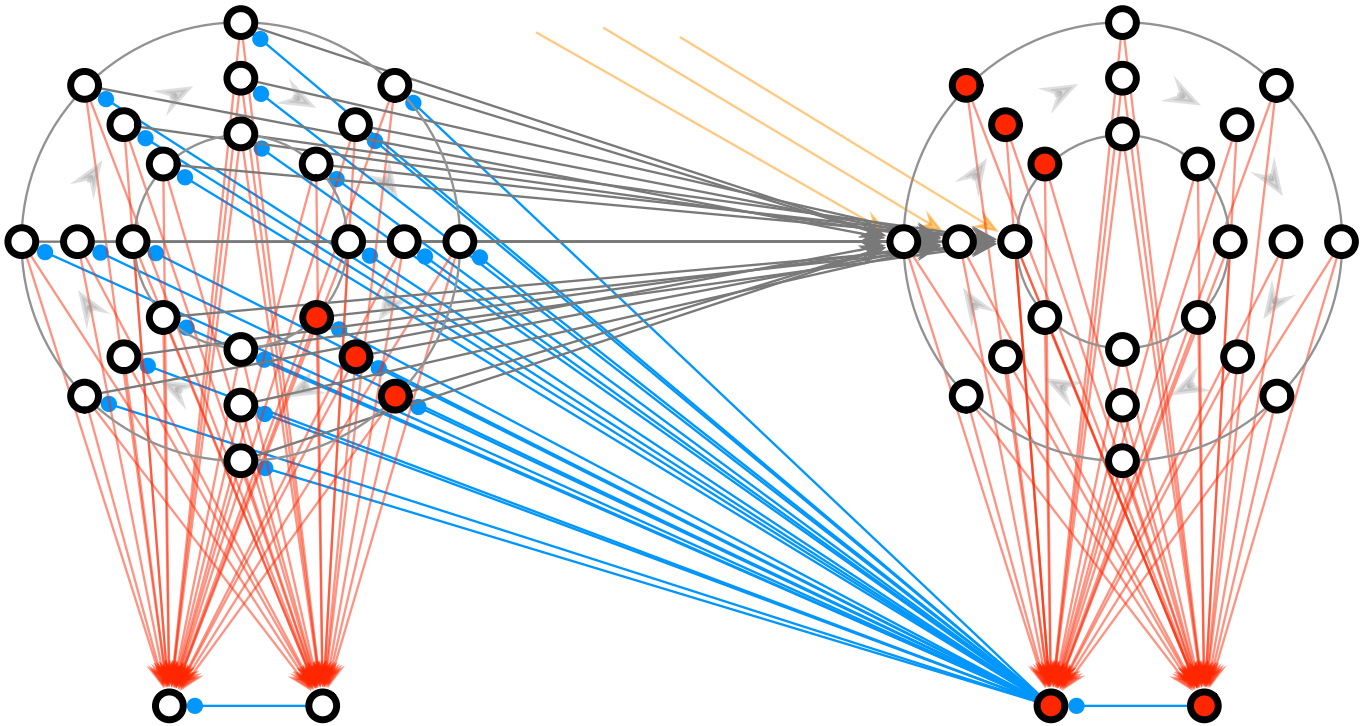
Fibres of connections & Inhibitory System



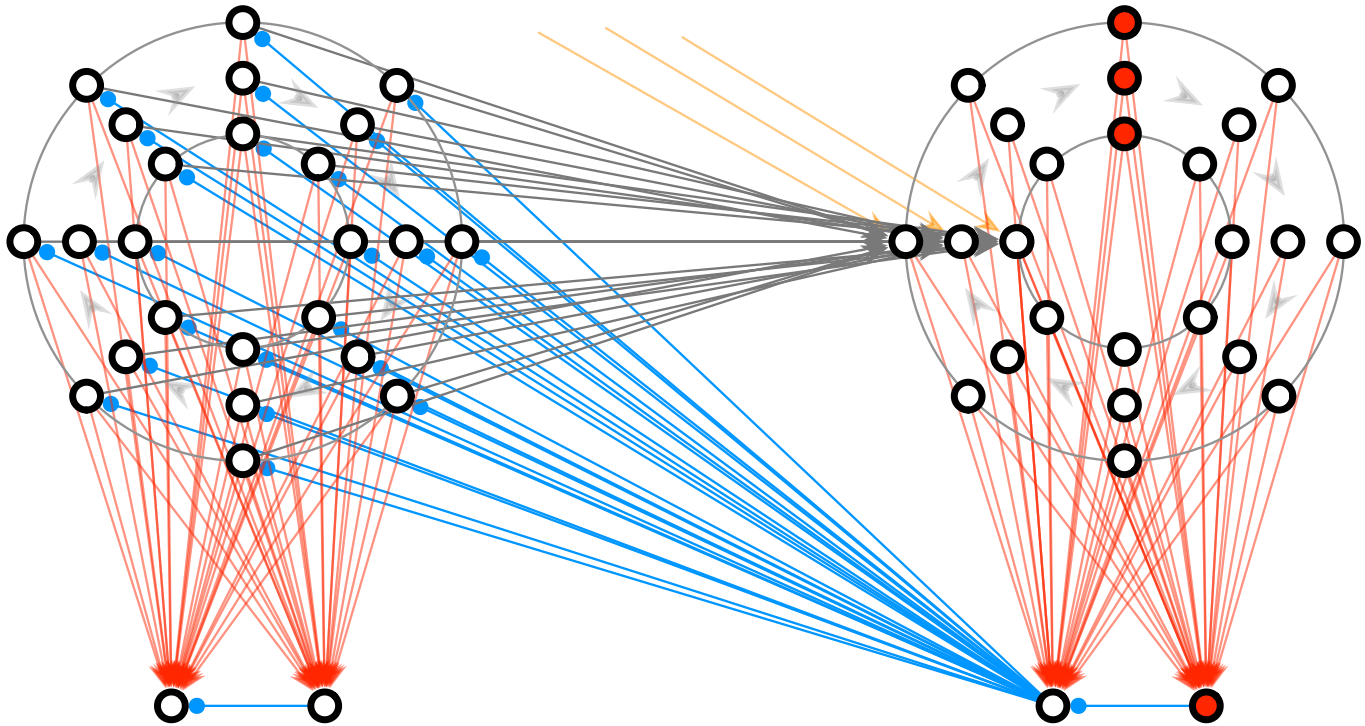
Fibres of connections & Inhibitory System



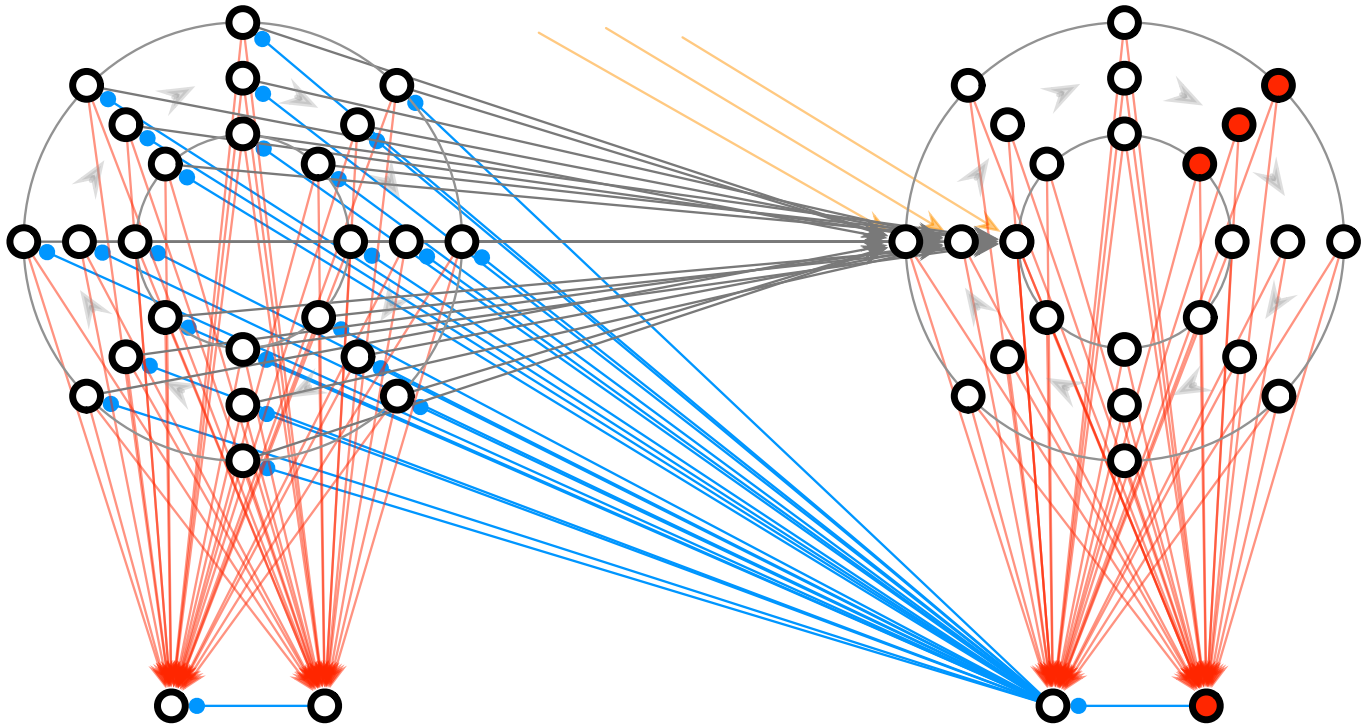
Fibres of connections & Inhibitory System



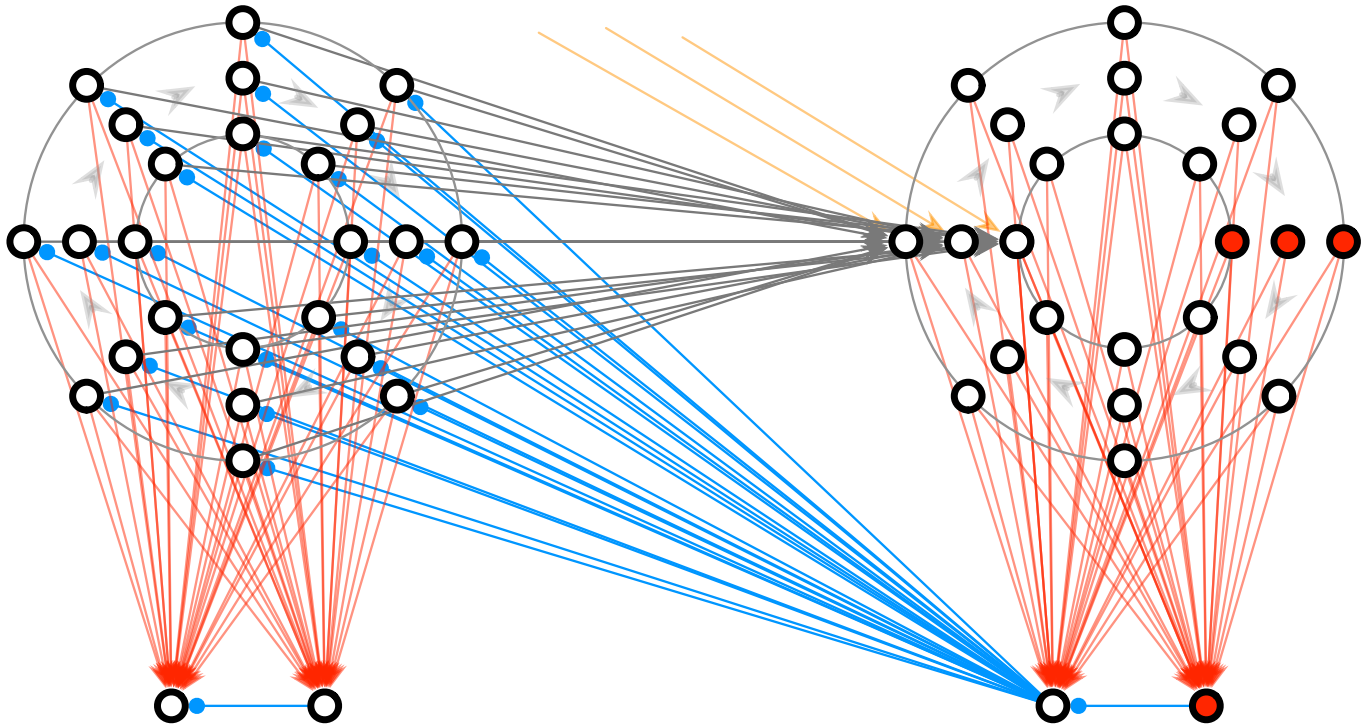
Fibres of connections & Inhibitory System



Fibres of connections & Inhibitory System



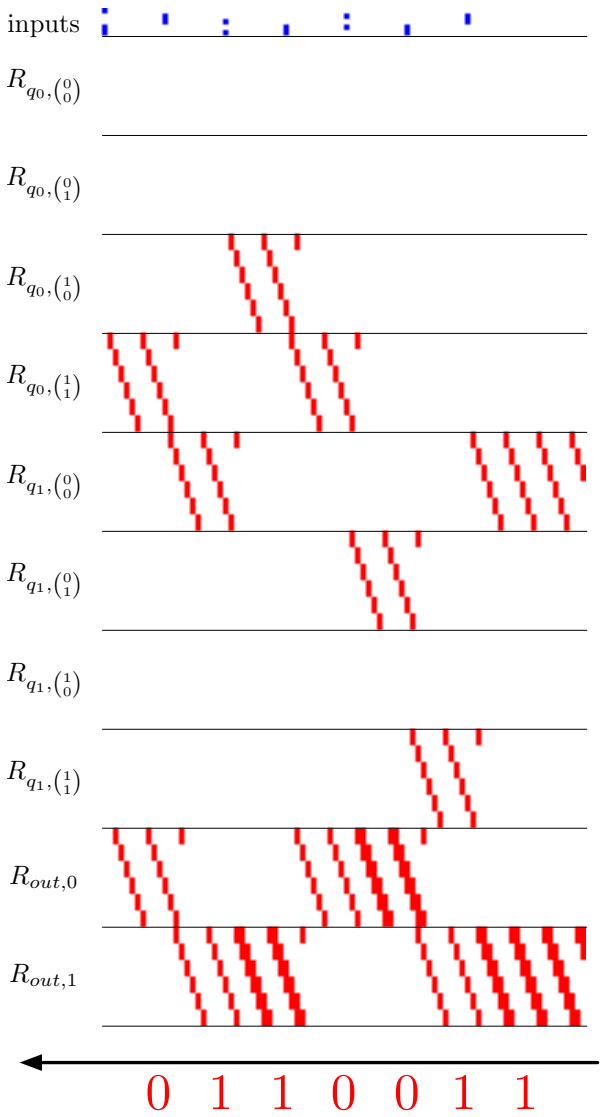
Fibres of connections & Inhibitory System



Simulation

Play movie...

Simulation


inputs
$R_{q_0,\binom{0}{0}}$
$R_{q_0,\binom{0}{1}}$
$R_{q_0,\binom{1}{0}}$
$R_{q_0,\binom{1}{1}}$
$R_{q_1,\binom{0}{0}}$
$R_{q_1,\binom{0}{1}}$
$R_{q_1,\binom{1}{0}}$
$R_{q_1,\binom{1}{1}}$
$R_{out,0}$
$R_{out,1}$
0 1 1 0 0 1 1

Automata & Boolean RNNs with Synfire Rings

Since the construction is generic, one has the following result:

Theorem

Any finite state automaton can be simulated by some Boolean neural network composed of synfire rings.

Izhikevich RNNs

- We now simulate finite state automata with Izhikevich-based recurrent neural networks composed of synfire rings.

Izhikevich Spiking Neural Networks

This construction can be extended to the case of Izhikevich Spiking neurons.

The Izhikevich differential equations are:

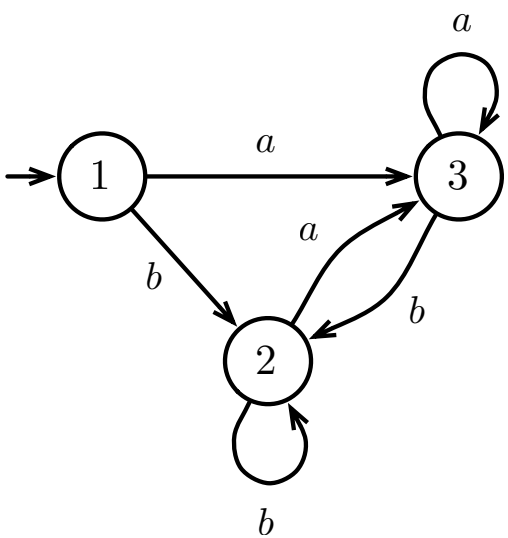
$$\begin{cases} v' &= 0.04v^2 + 5v + 140 - u + I \\ u' &= a(bv - u) \end{cases}$$

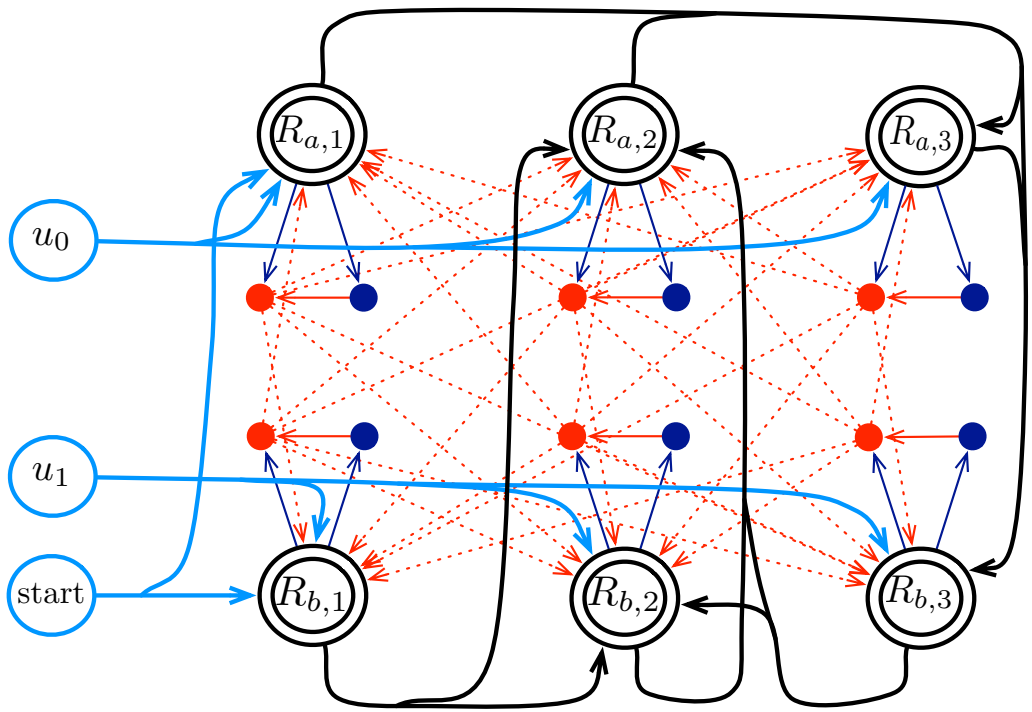
with the auxiliary after-spike reseting:

$$\text{if } v \geq 30 \text{ mV, then } v \leftarrow c \text{ and } u \leftarrow u + d$$

- v: membrane potential
- u: membrane recovery variable
- I: synaptic currents
- a, b, c, d: dimensionless parameters

Automata & Izhikevich RNNs with Synfire Rings





Simulation: without synaptic noise

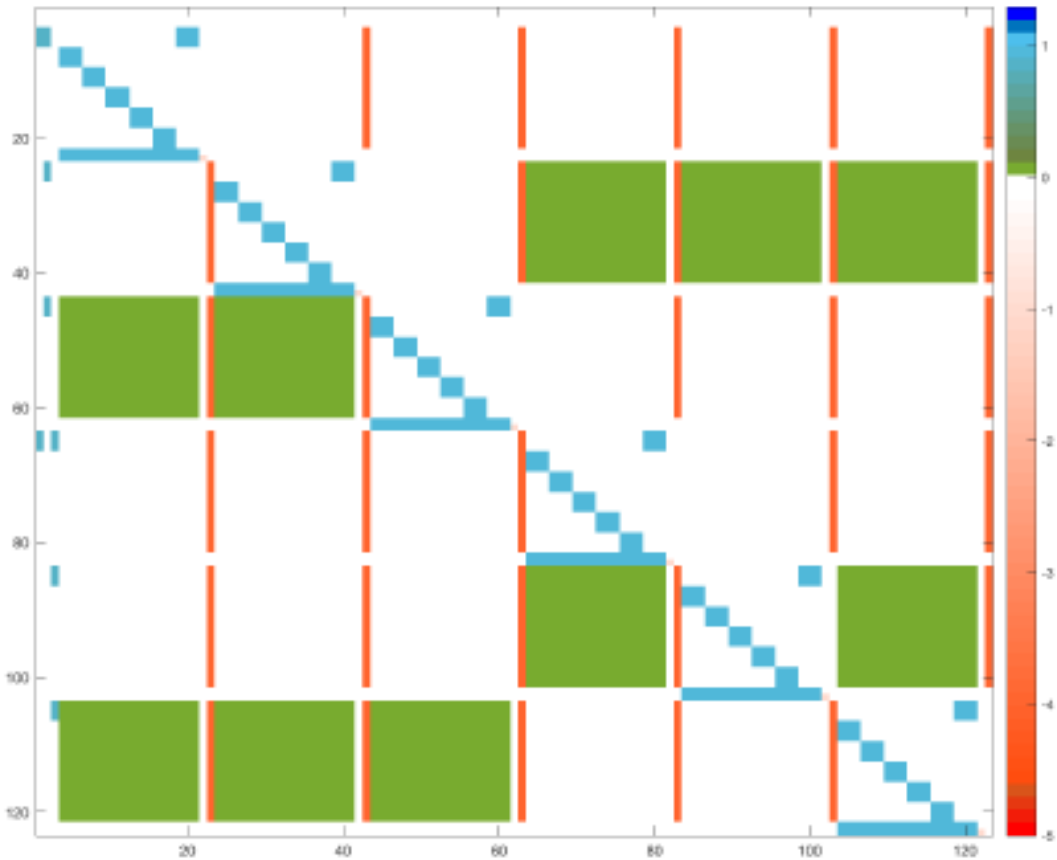


Figure: Connectivity matrix of the network

Simulation: without synaptic noise

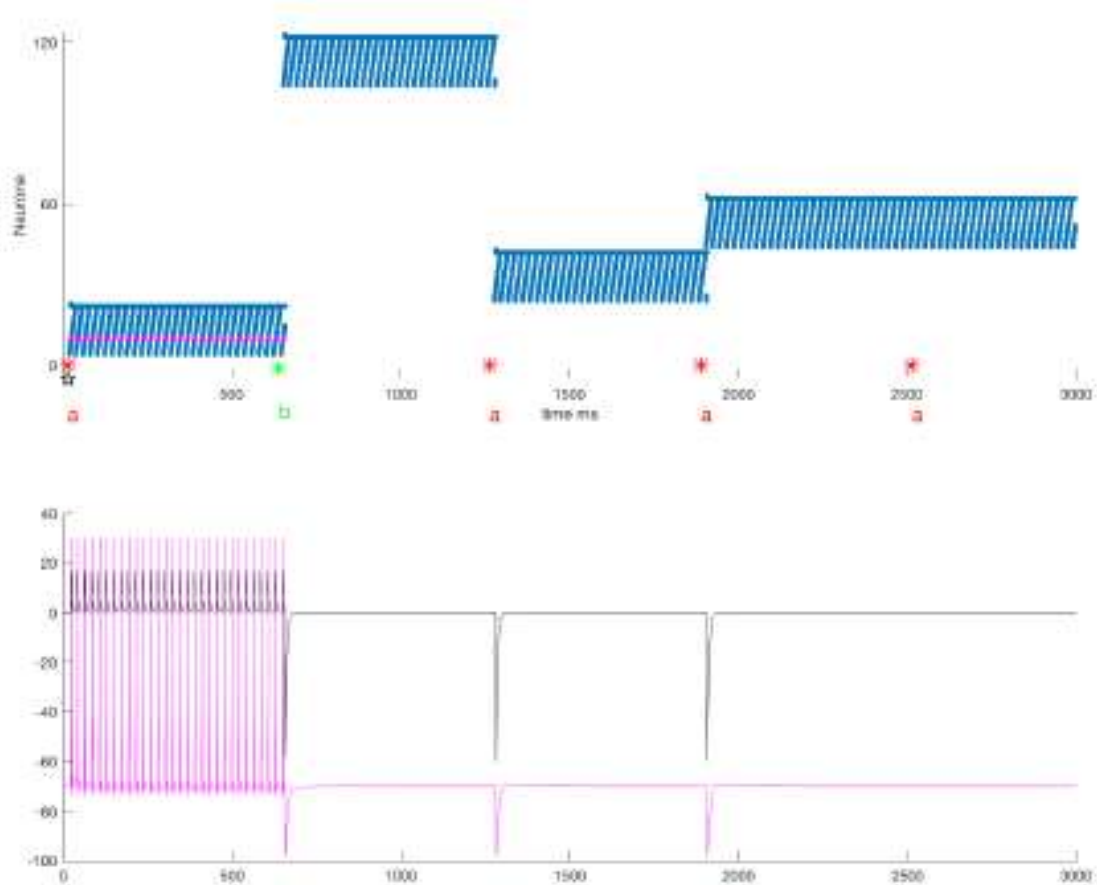


Figure: Raster plot of the simulation and activity of one spiking neuron

Simulation: with synaptic noise

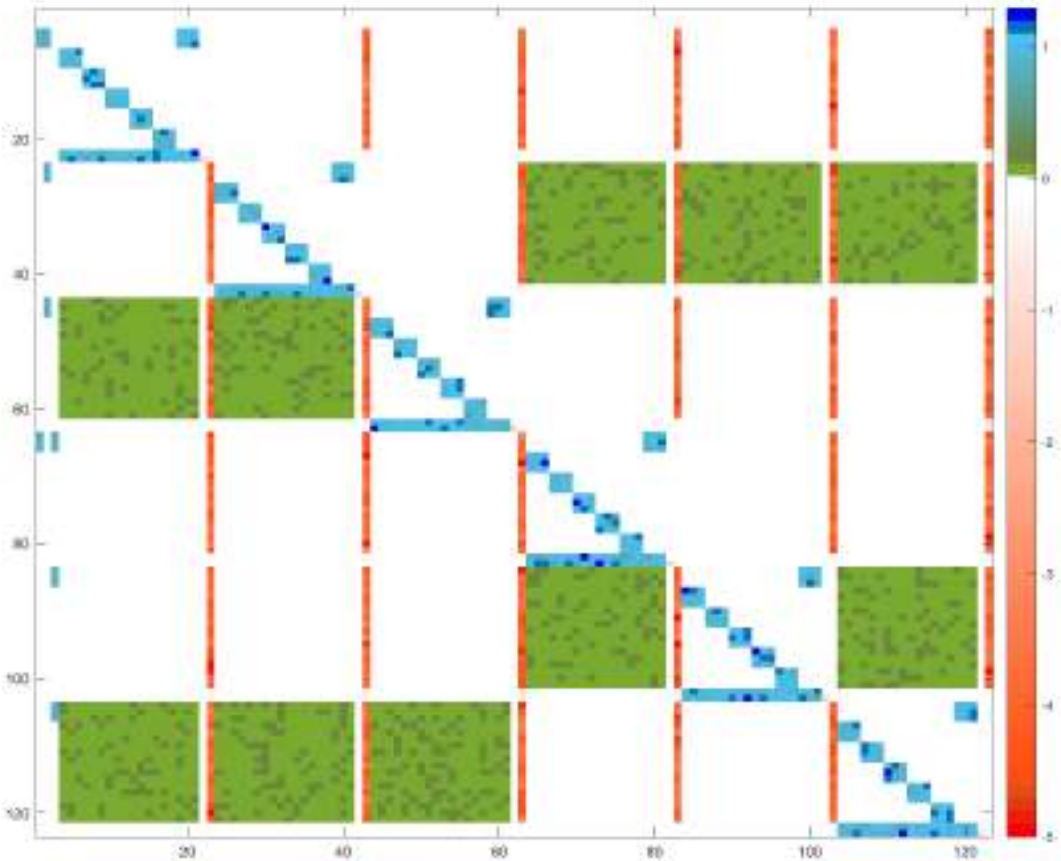
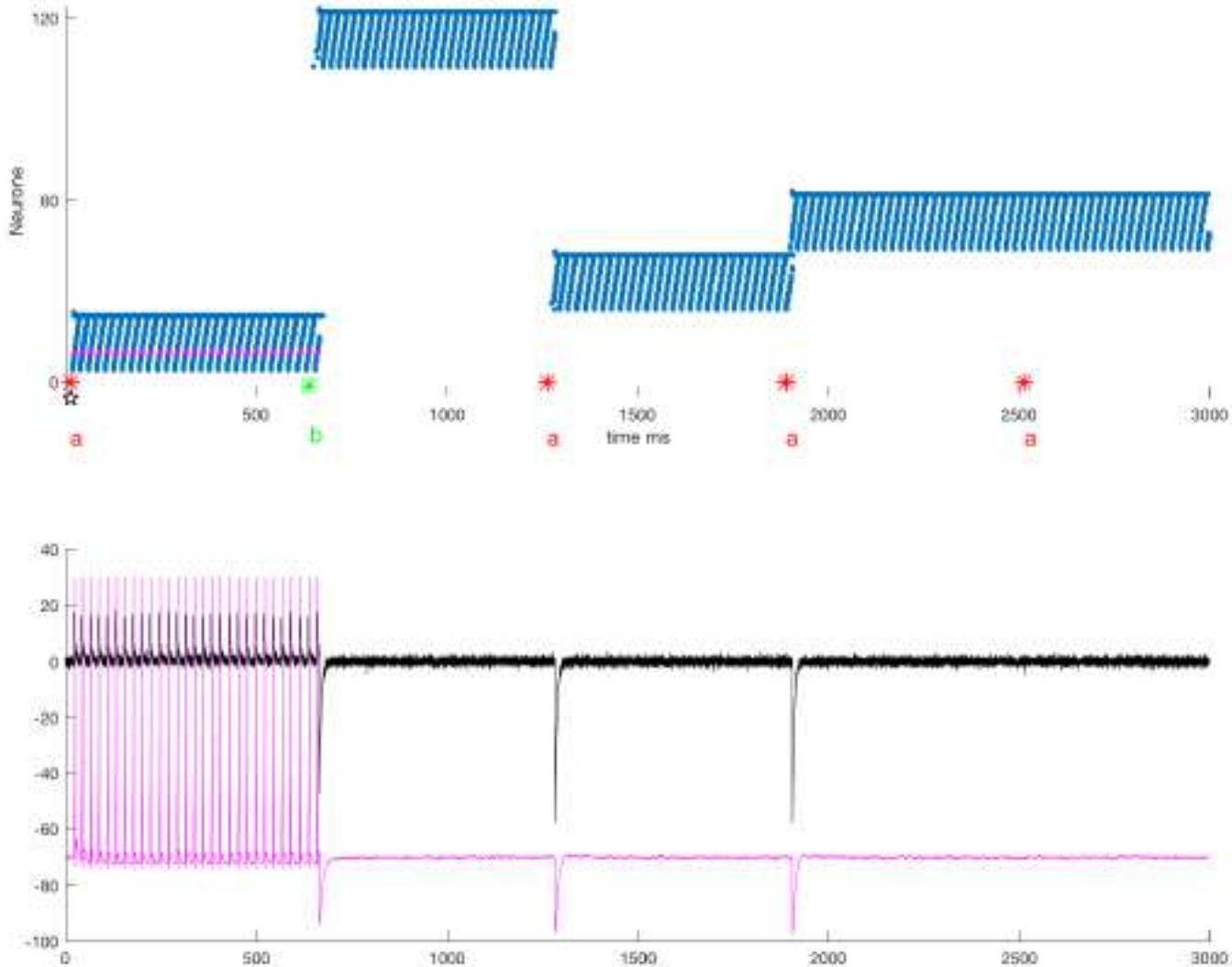


Figure: Connectivity matrix of the network

Simulation: with synaptic noise



Automata & Izhikevich RNNs with Synfire Rings

Since the construction is generic, one has the following result:

Theorem

Any finite state automaton can be simulated by some (noisy) Izhikevich spiking neural network composed of synfire rings.

Hodgkin-Huxley RNNs

- We finally simulate finite state automata with Hodgkin-Huxley-based recurrent neural networks composed of synfire rings.

Hodgkin-Huxley Neurons (software demo)

$$\alpha_n(V_m) = \frac{0.01(10 - V_m)}{\exp(\frac{10-V_m}{10}) - 1} \qquad \beta_n(V_m) = 0.125 \exp(\frac{-V_m}{80})$$

$$\alpha_m(V_m) = \frac{0.1(25 - V_m)}{\exp(\frac{25-V_m}{10}) - 1} \qquad \beta_m(V_m) = 4 \exp(\frac{-V_m}{18})$$

$$\alpha_h(V_m) = 0.07 \exp(\frac{-V_m}{20}) \qquad \beta_h(V_m) = \frac{1}{\exp(\frac{30-V_m}{10}) + 1}$$

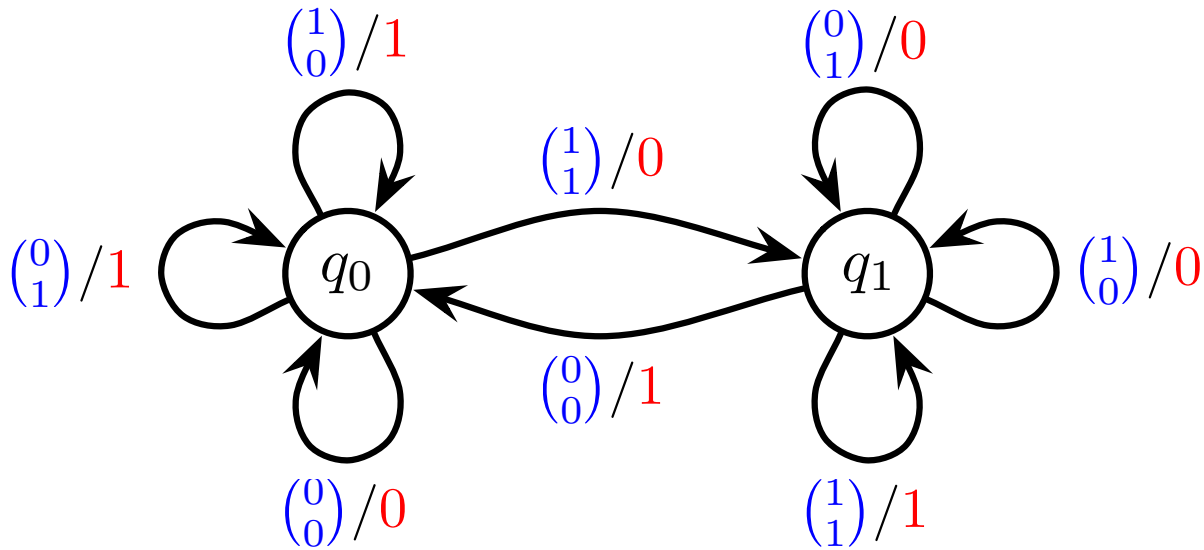
$$\frac{dn}{dt} = \alpha_n(V_m)(1 - n) - \beta_n(V_m)n$$

$$\frac{dm}{dt} = \alpha_m(V_m)(1 - m) - \beta_m(V_m)m$$

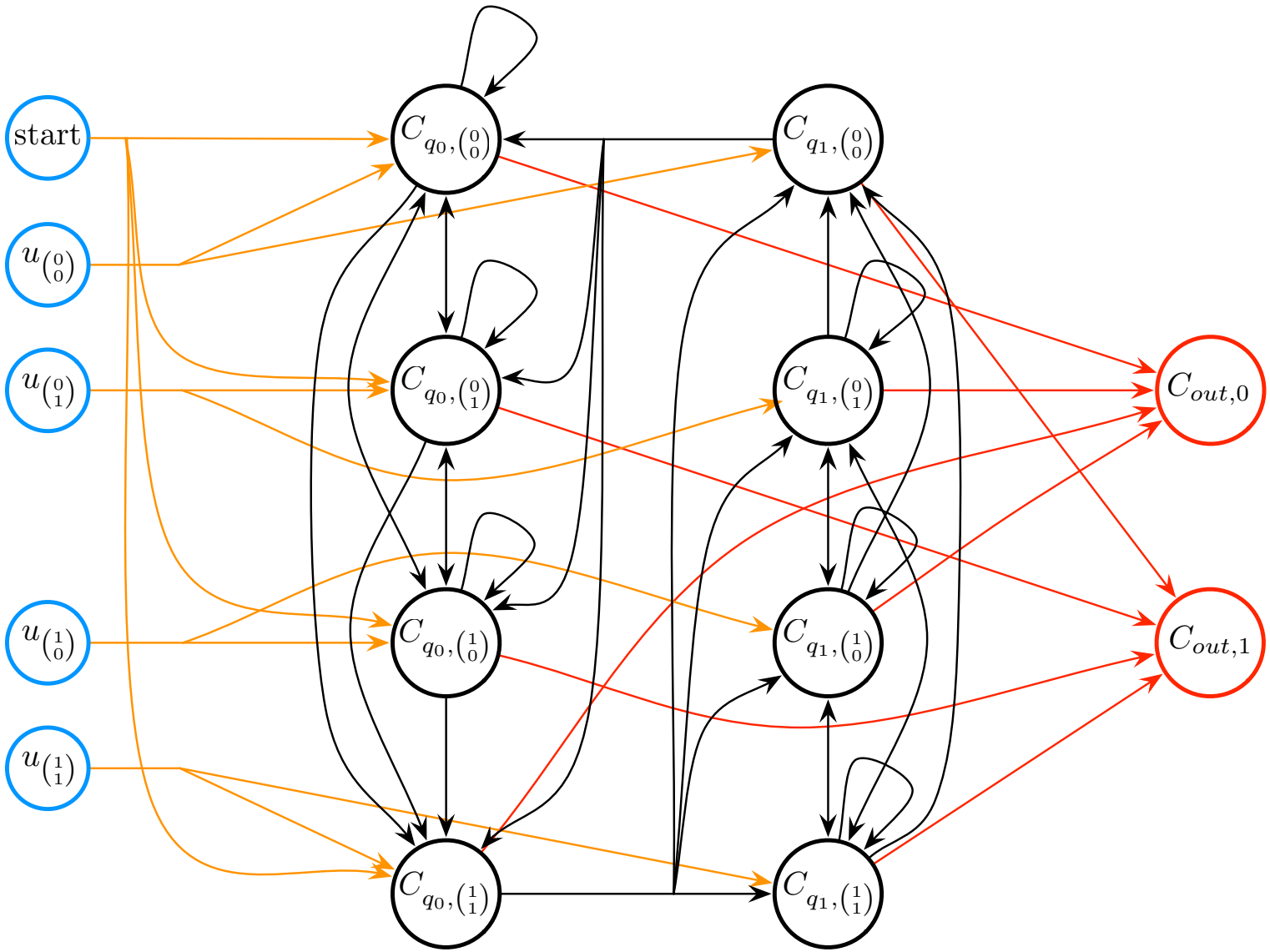
$$\frac{dh}{dt} = \alpha_h(V_m)(1 - h) - \beta_h(V_m)h$$

$$C_m \frac{dV_m}{dt} = I - \bar{g}_{\mathsf{K}} n^4 (V_m - V_K) - \bar{g}_{\mathsf{Na}} m^3 h (V_m - V_{Na}) - \bar{g}_l (V_m - V_l)$$

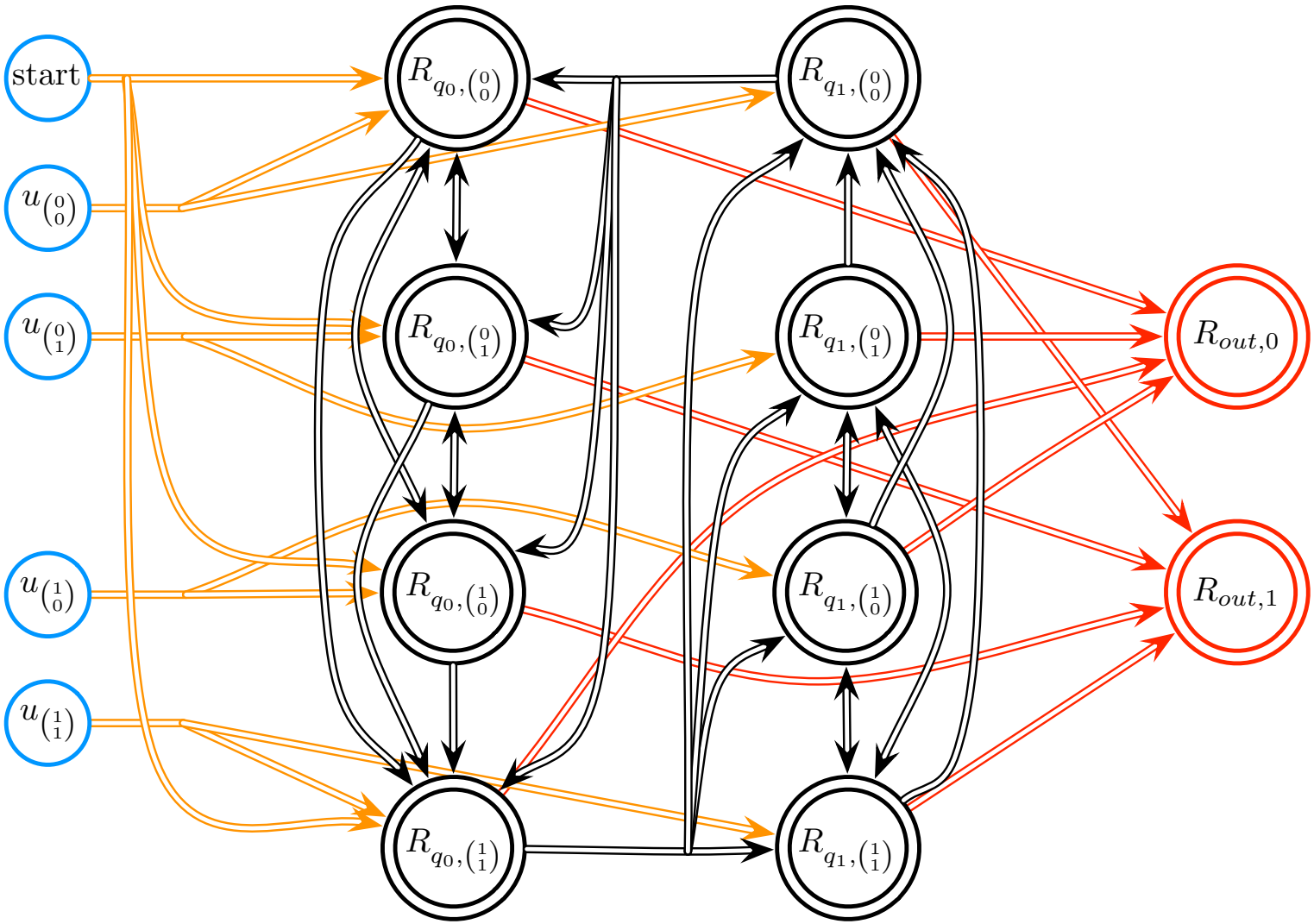
General Construction



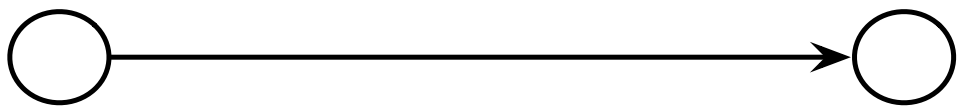
General Construction



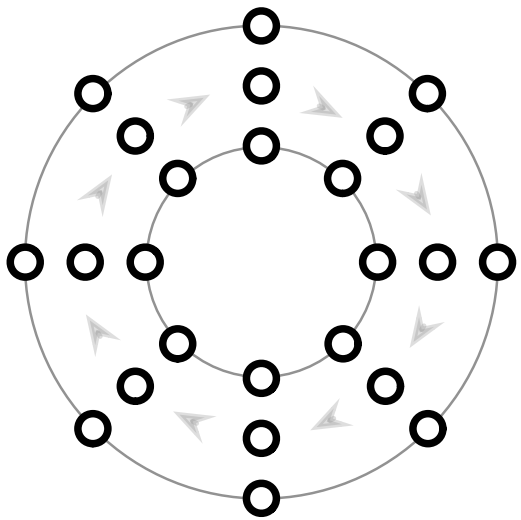
General Construction

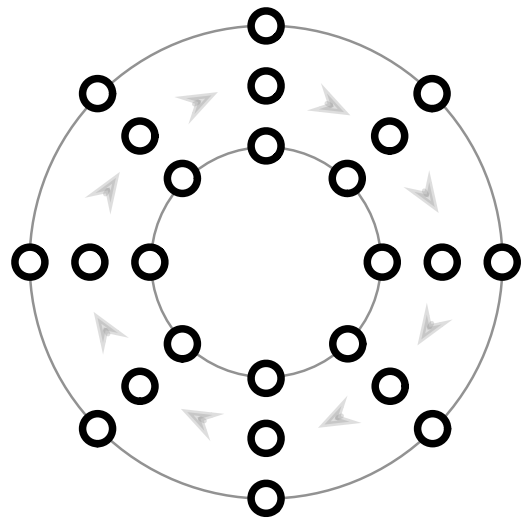


Fibres of connections & Inhibitory System

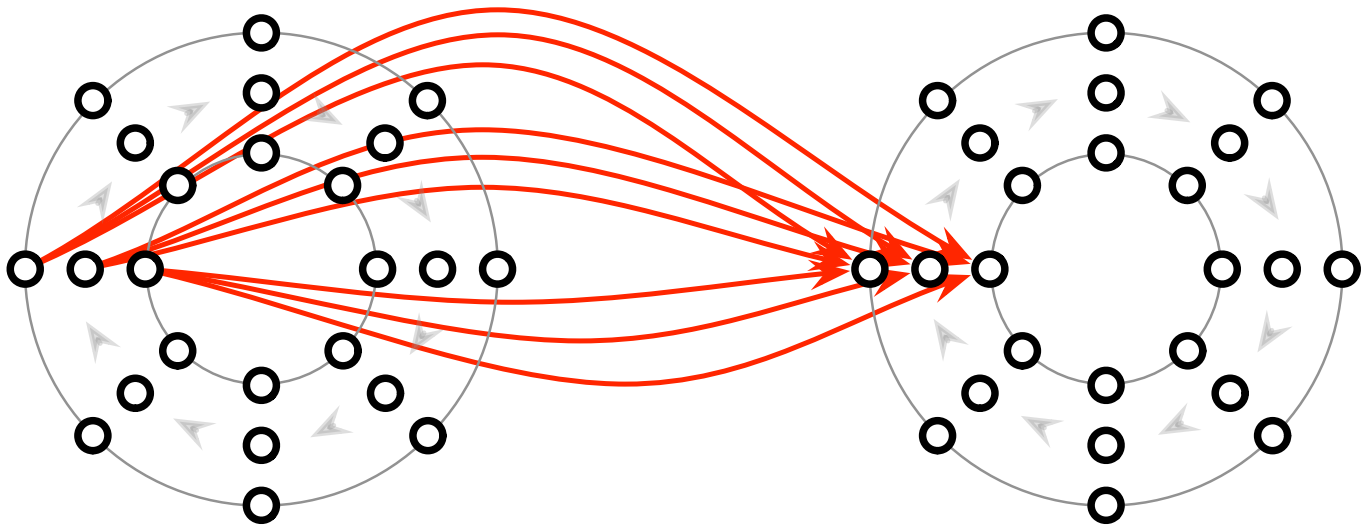


Fibres of connections & Inhibitory System

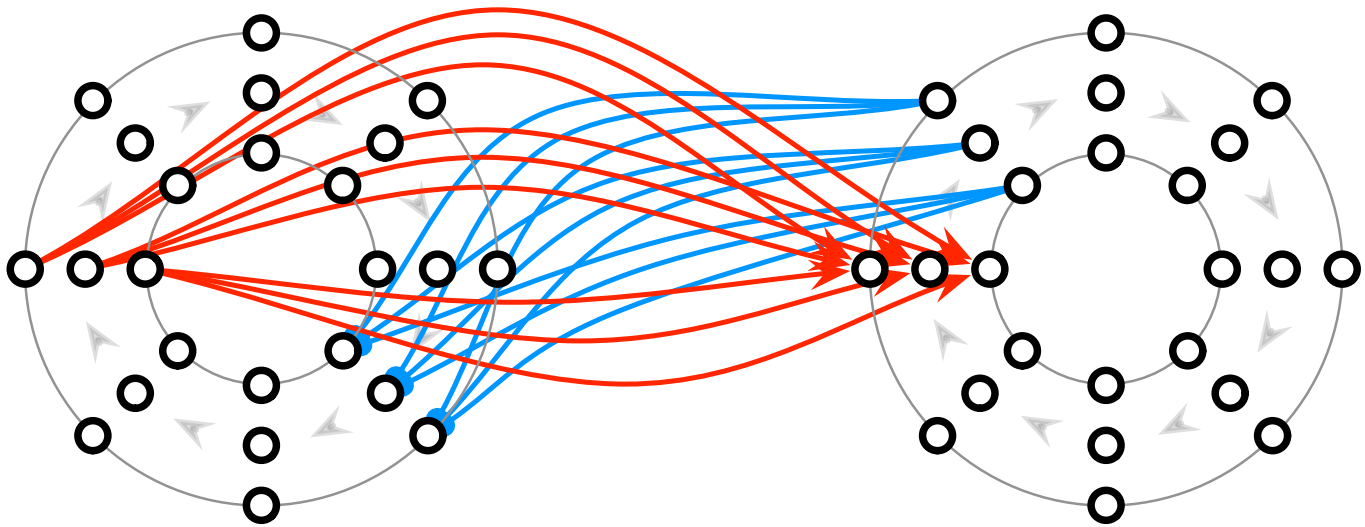




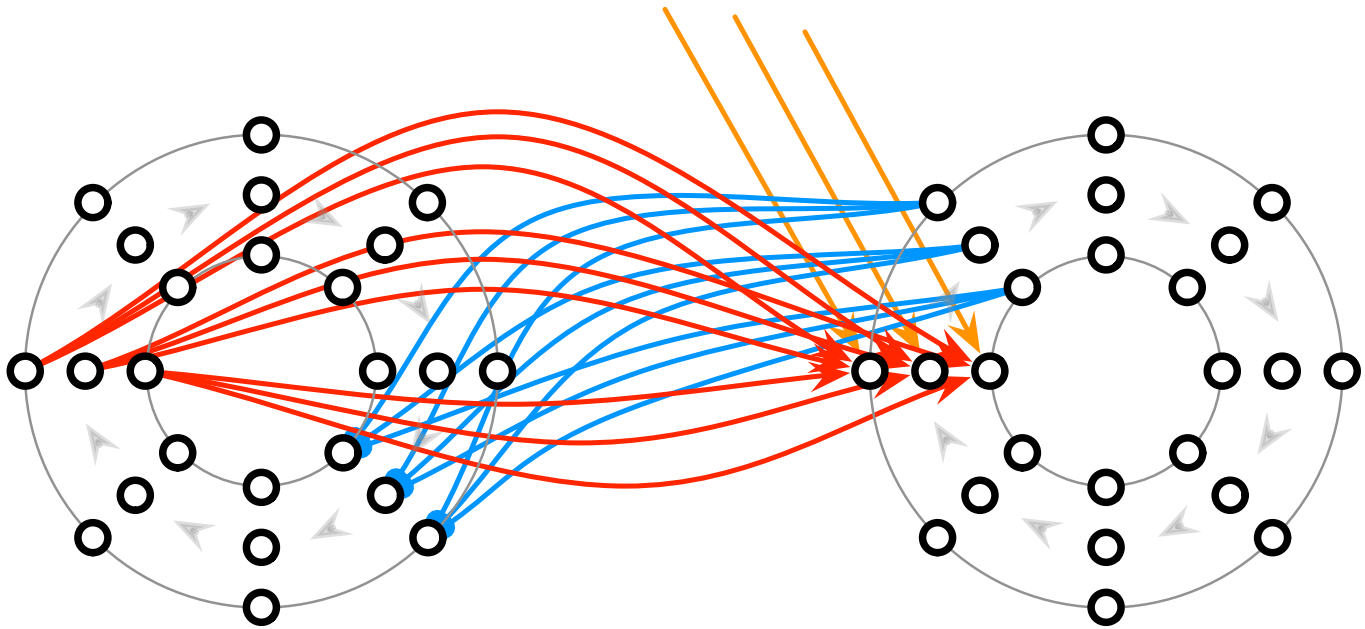
Fibres of connections & Inhibitory System



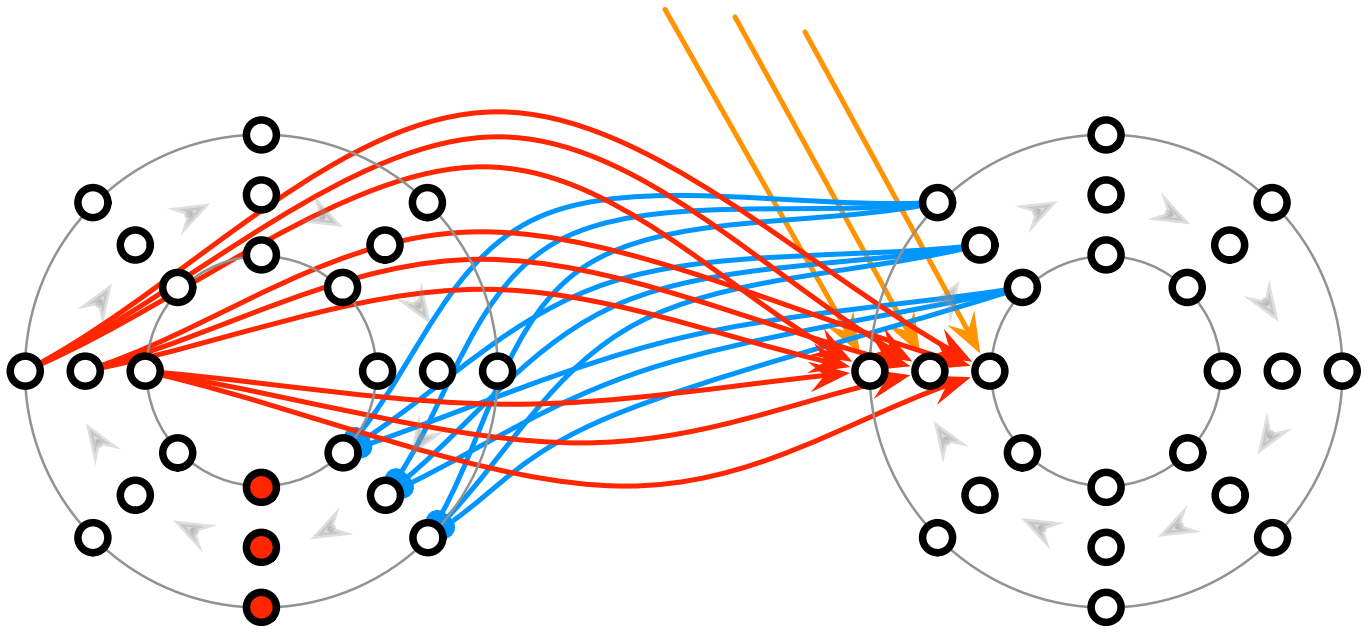
Fibres of connections & Inhibitory System



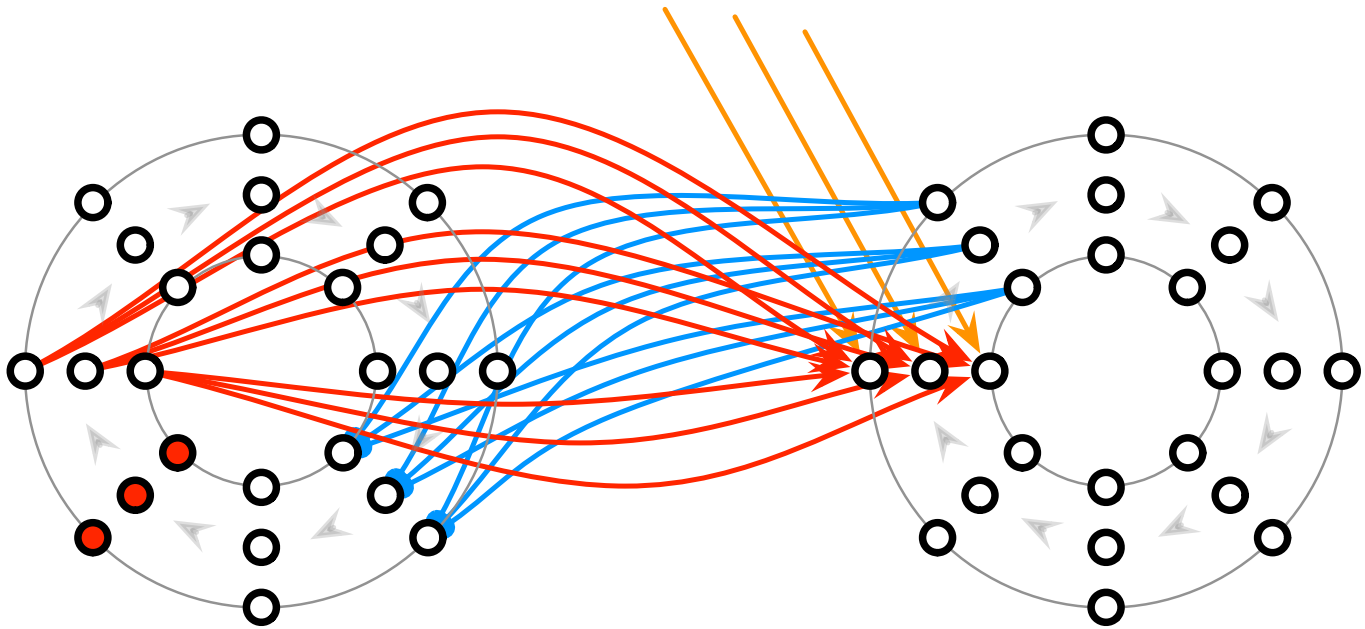
Fibres of connections & Inhibitory System



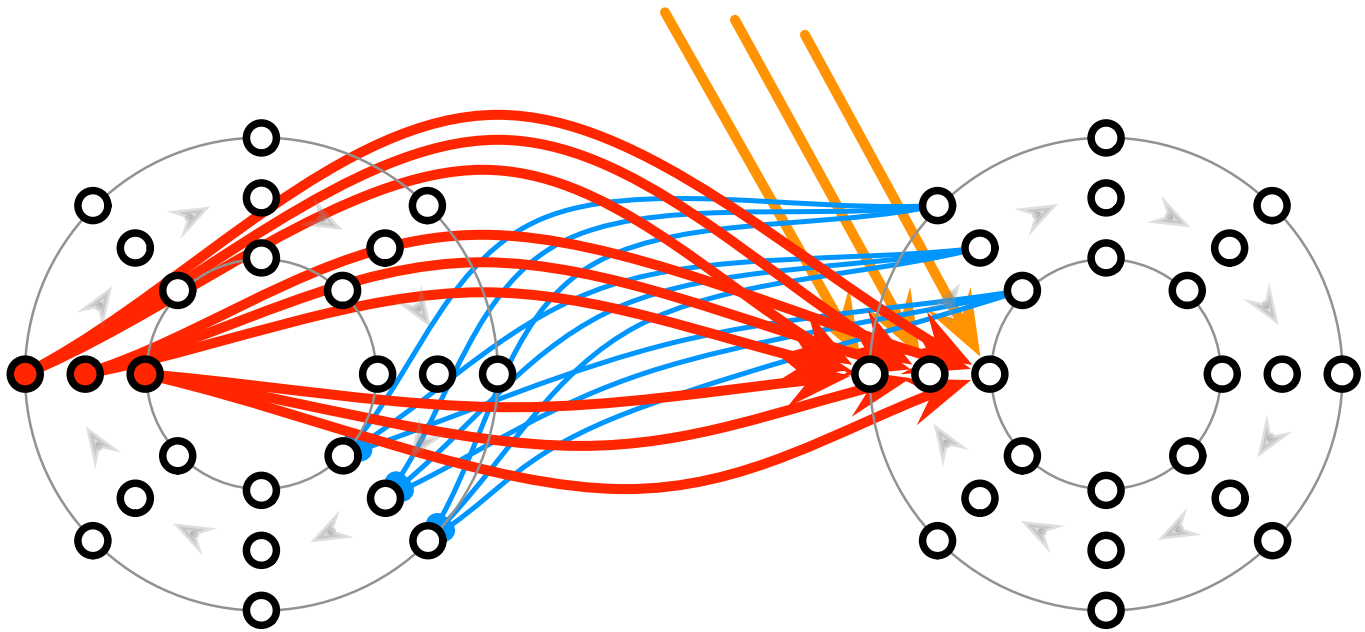
Fibres of connections & Inhibitory System



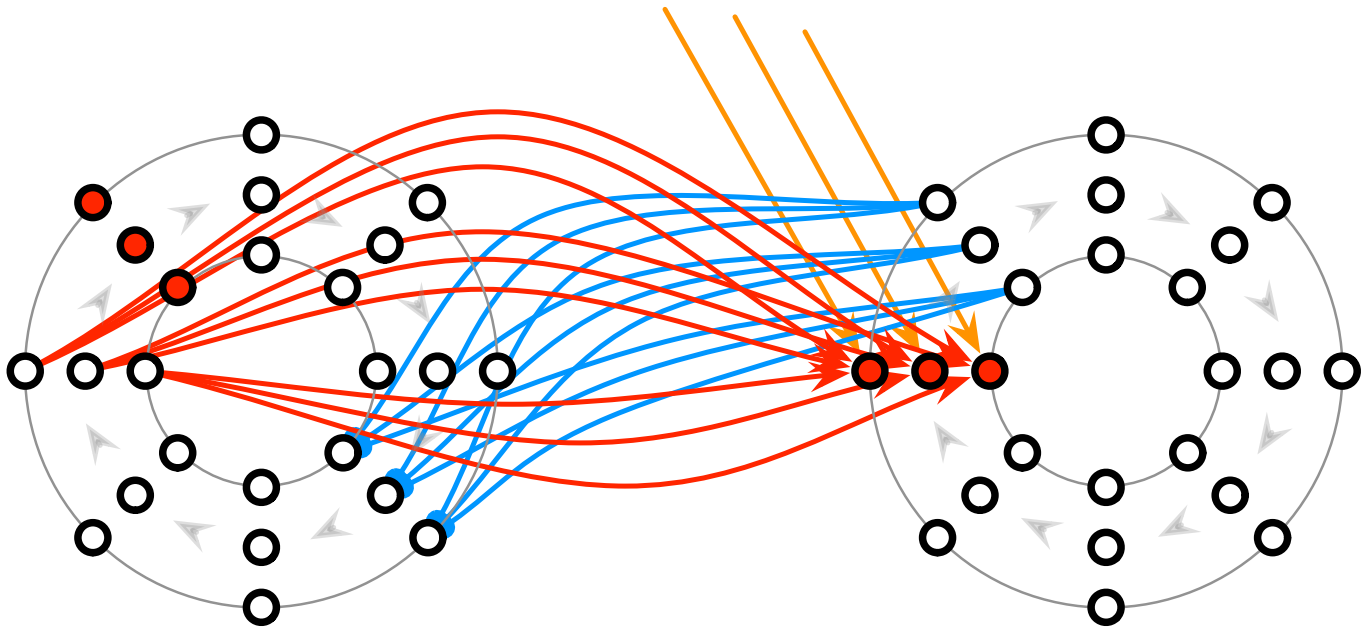
Fibres of connections & Inhibitory System



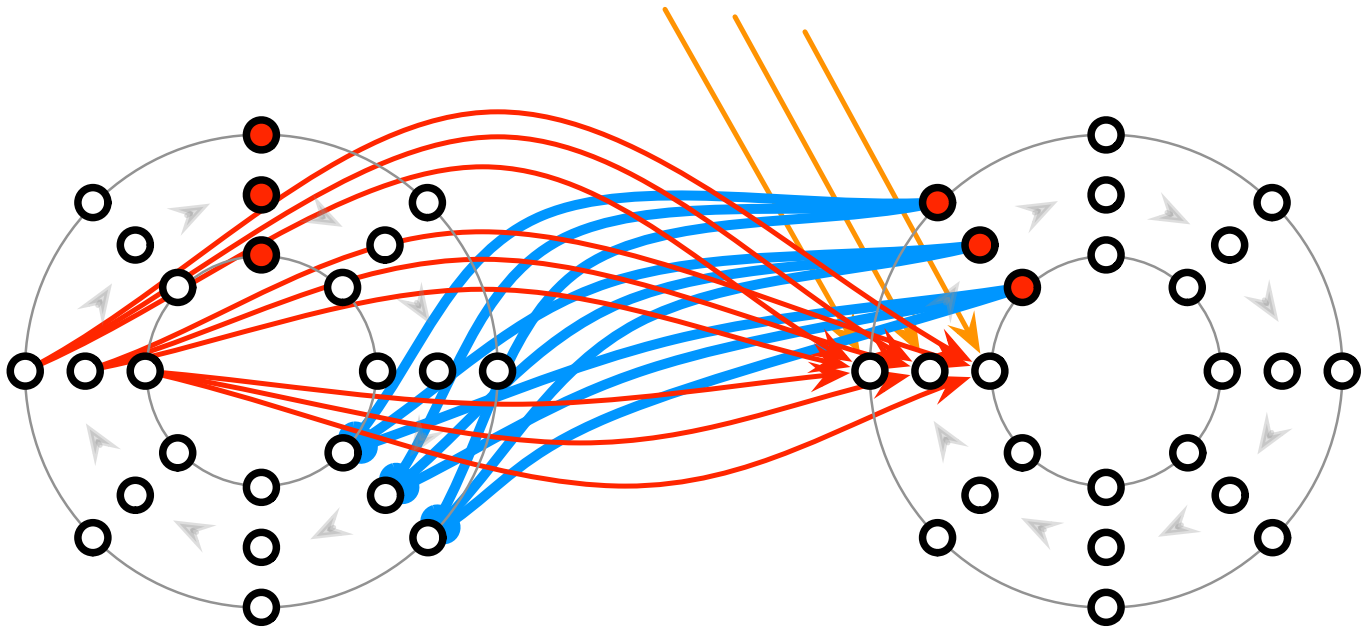
Fibres of connections & Inhibitory System



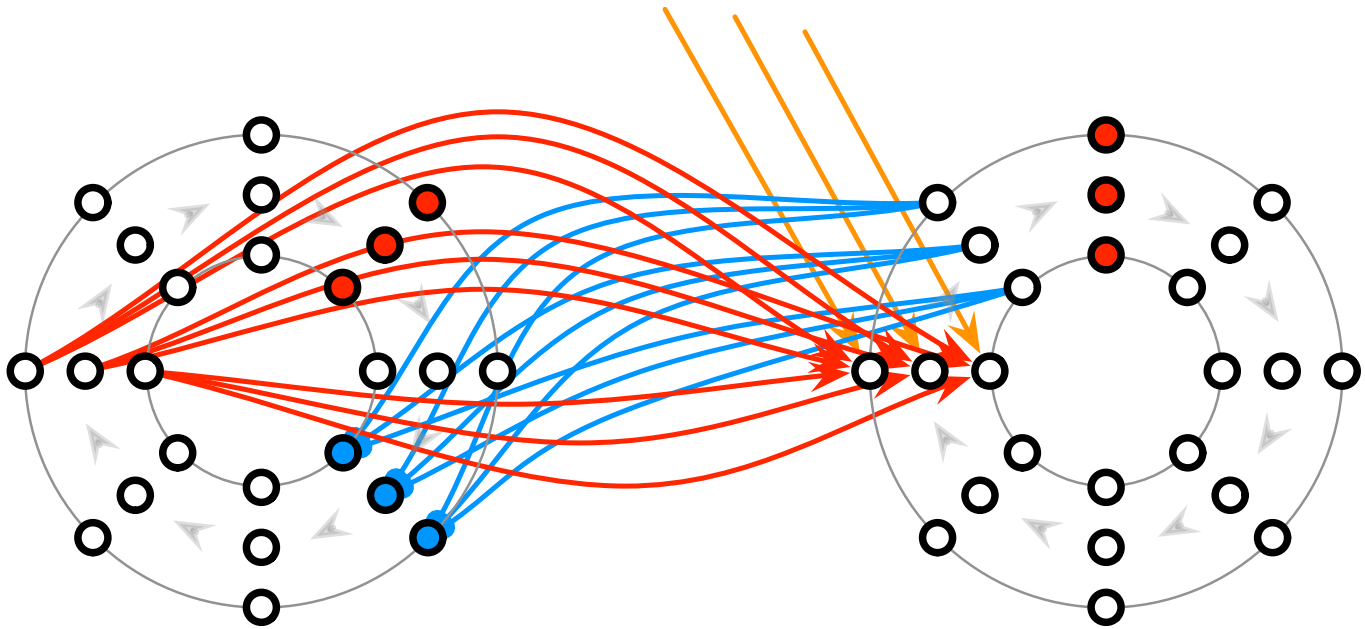
Fibres of connections & Inhibitory System



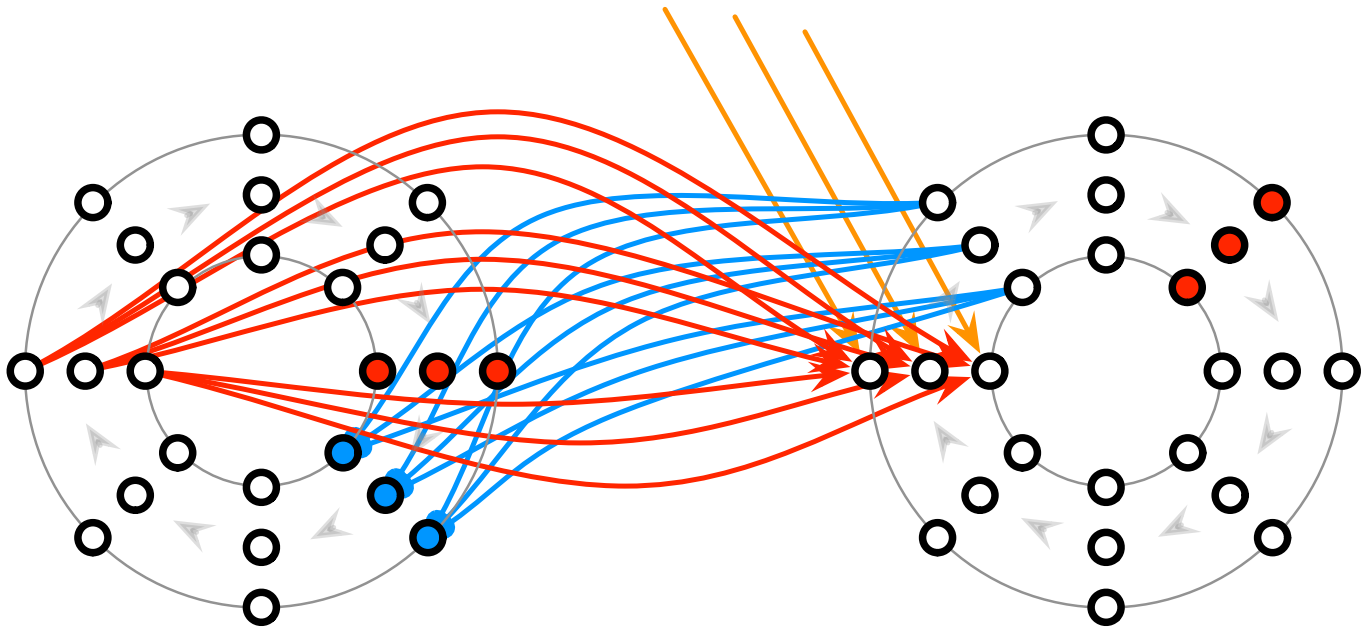
Fibres of connections & Inhibitory System



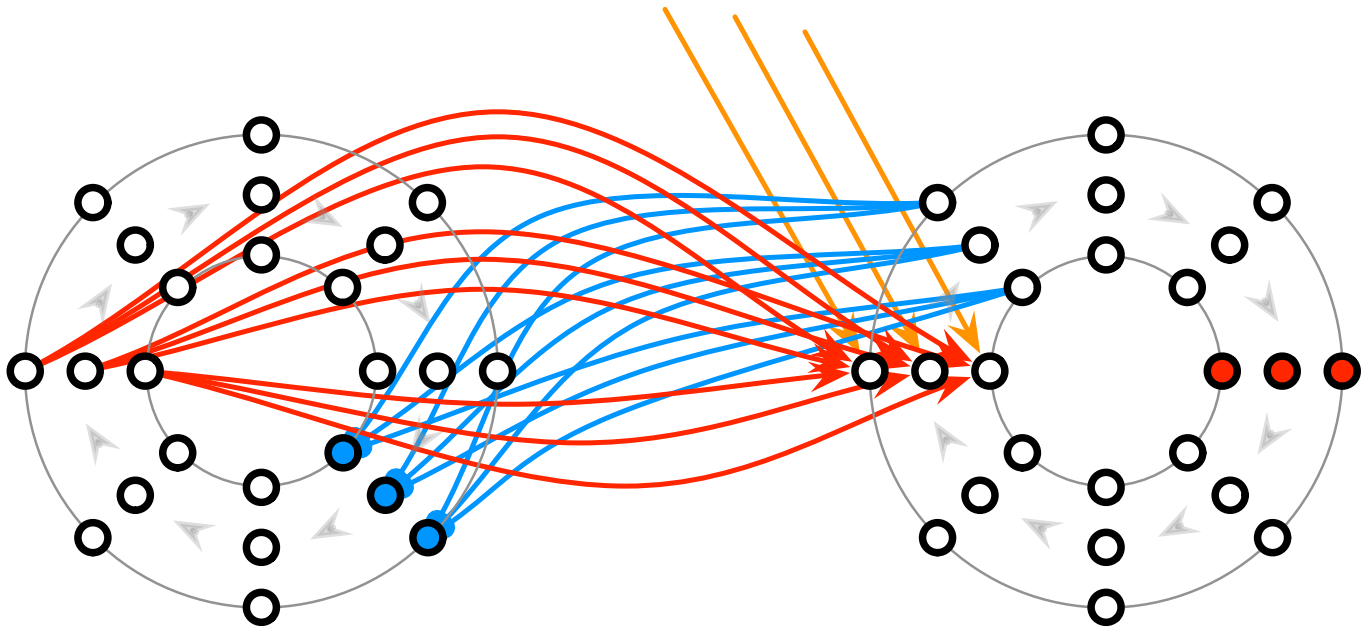
Fibres of connections & Inhibitory System



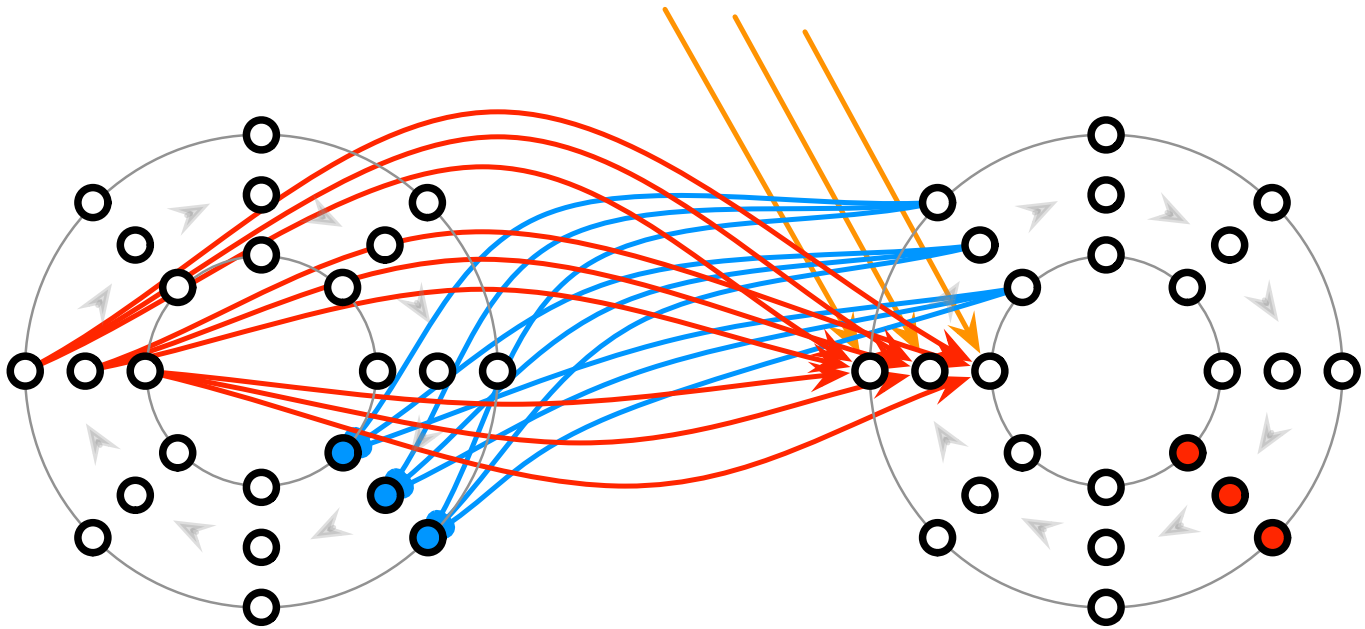
Fibres of connections & Inhibitory System



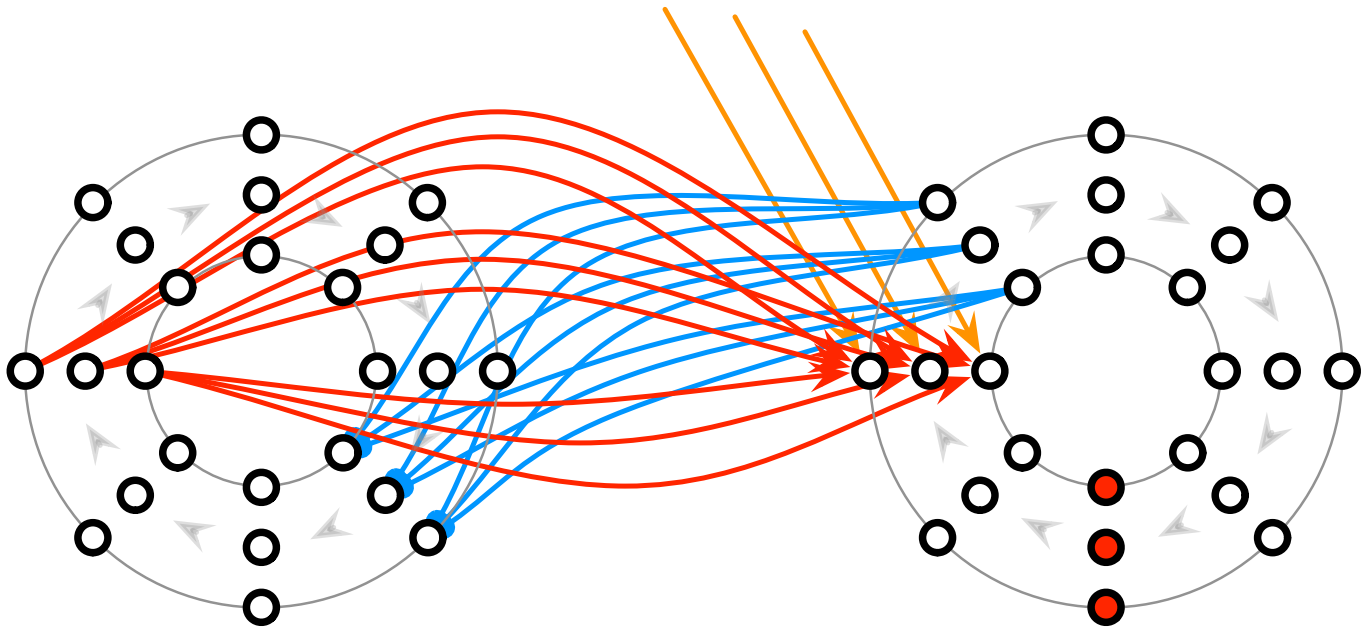
Fibres of connections & Inhibitory System



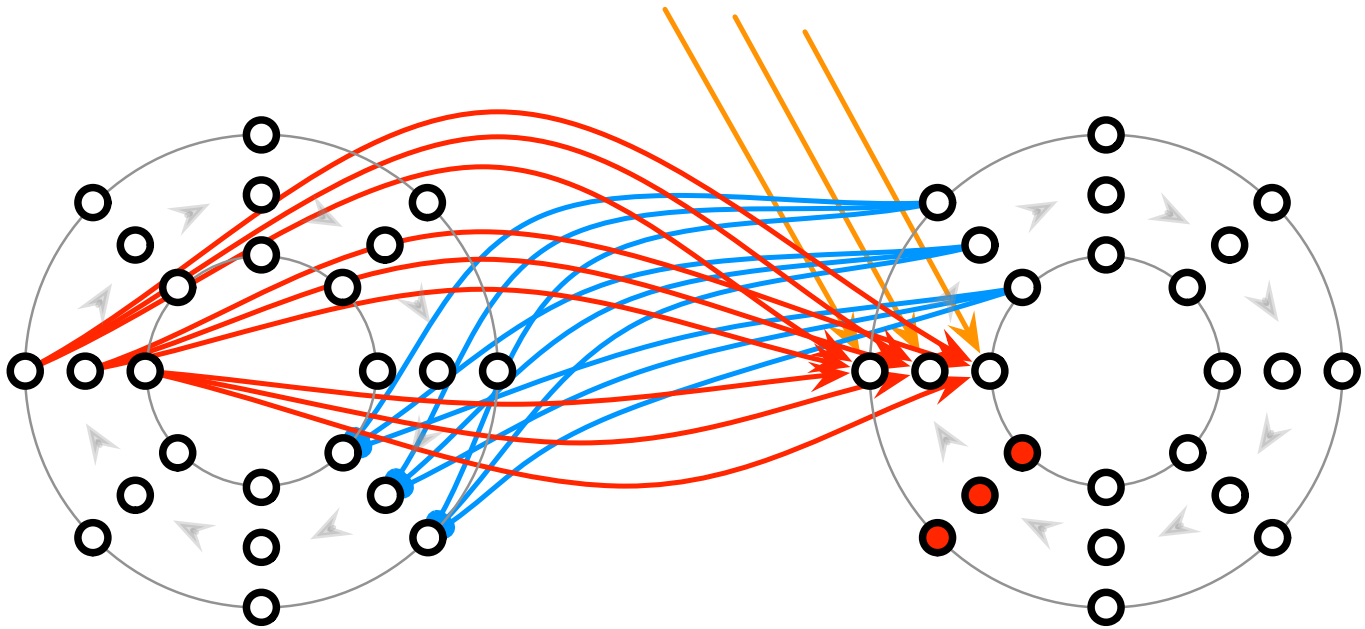
Fibres of connections & Inhibitory System



Fibres of connections & Inhibitory System



Fibres of connections & Inhibitory System



Simulation

Play movie...

Automata & Hodgkin-Huxley RNNs with Synfire Rings

Since the construction is generic, one has the following result:

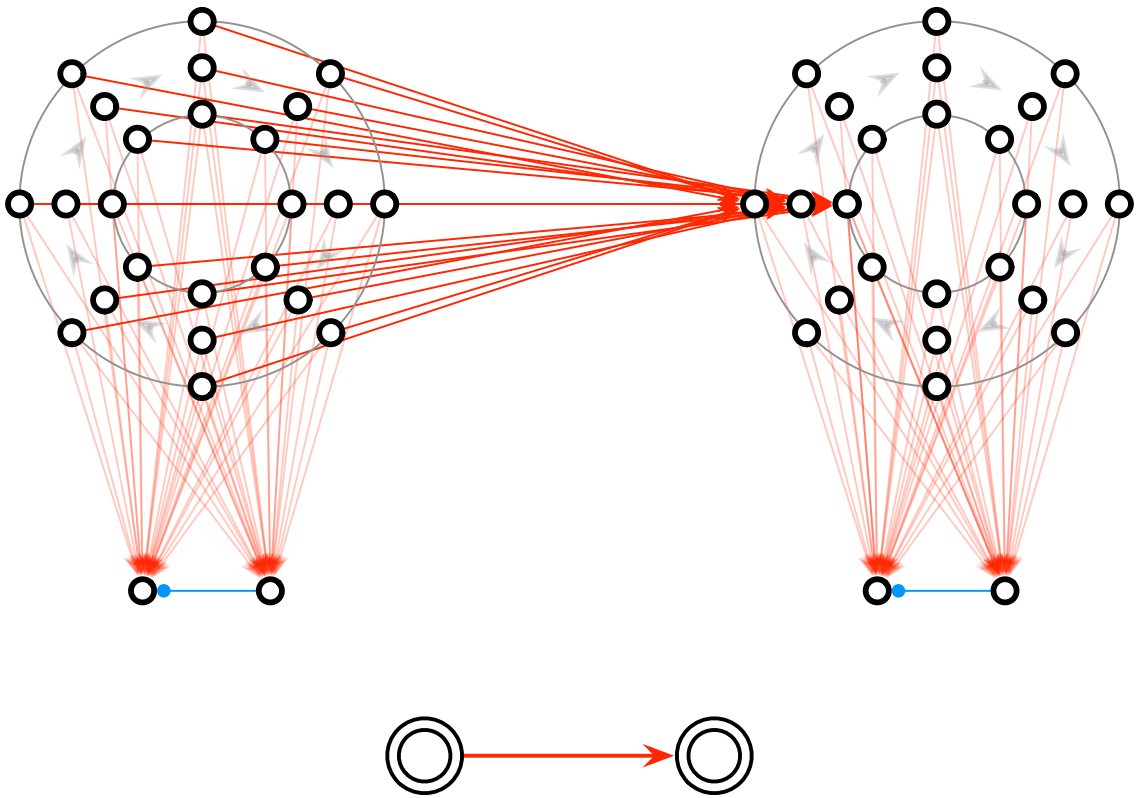
Theorem

Any finite state automaton can be simulated by some Hodgkin-Huxley based neural network composed of synfire rings.

Fibres of connections & Inhibitory System

We consider the following kind of (fibres of) connections.

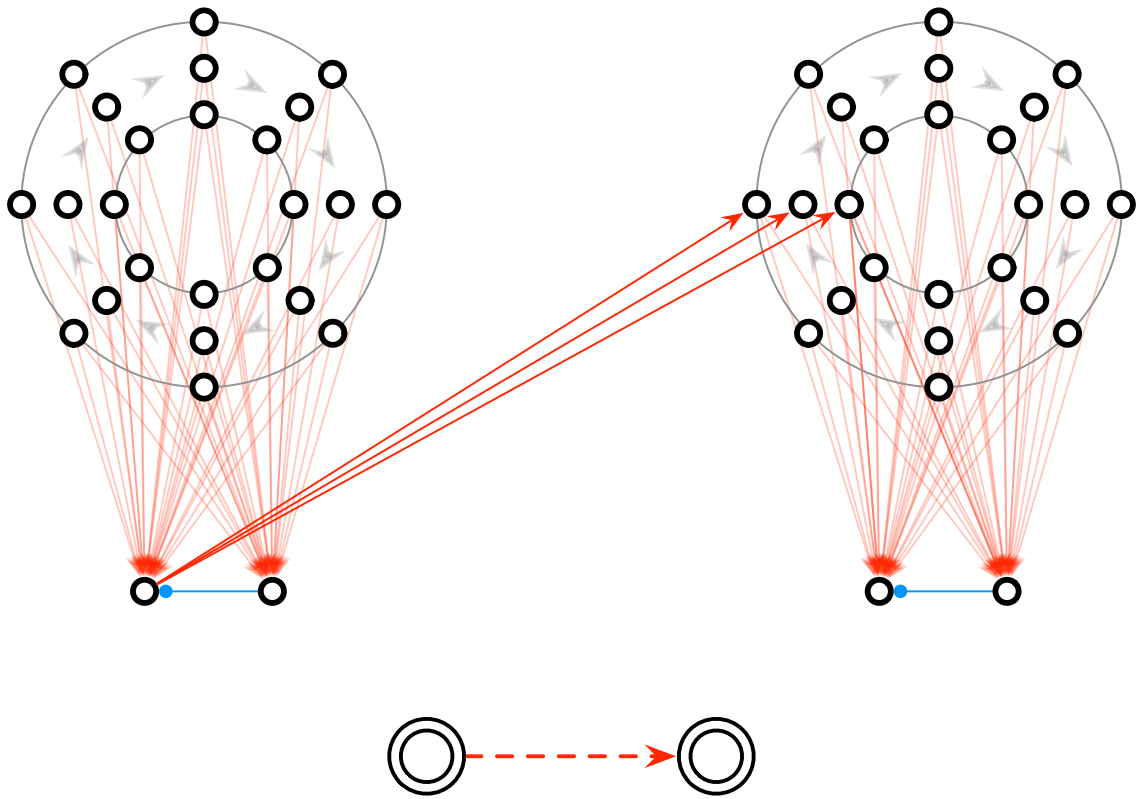
- excitatory



Fibres of connections & Inhibitory System

We consider the following kind of (fibres of) connections.

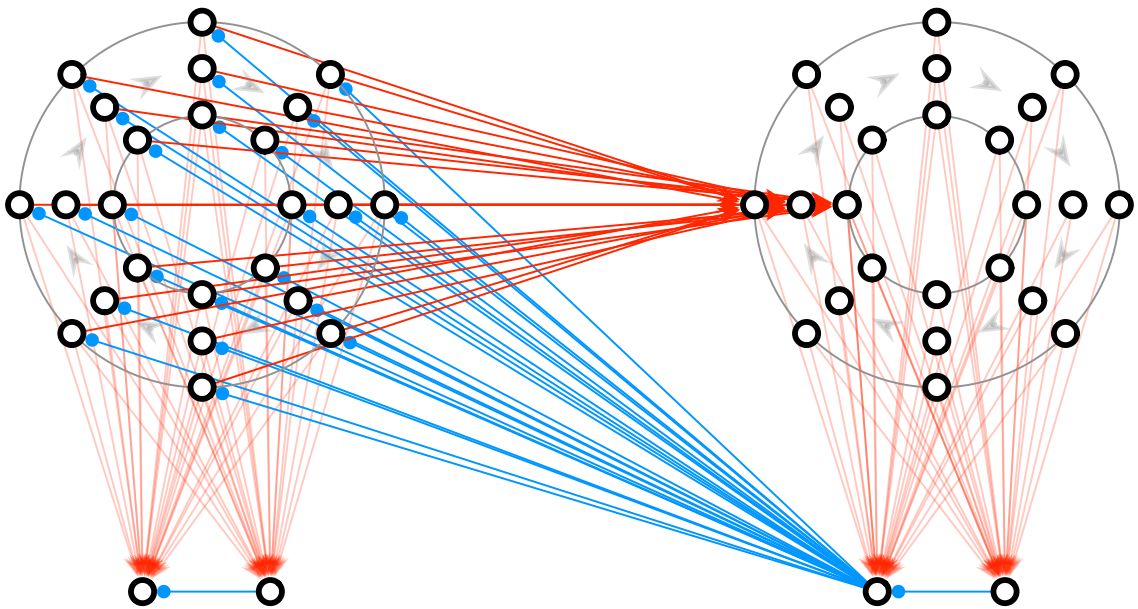
- one-shot excitatory



Fibres of connections & Inhibitory System

We consider the following kind of (fibres of) connections.

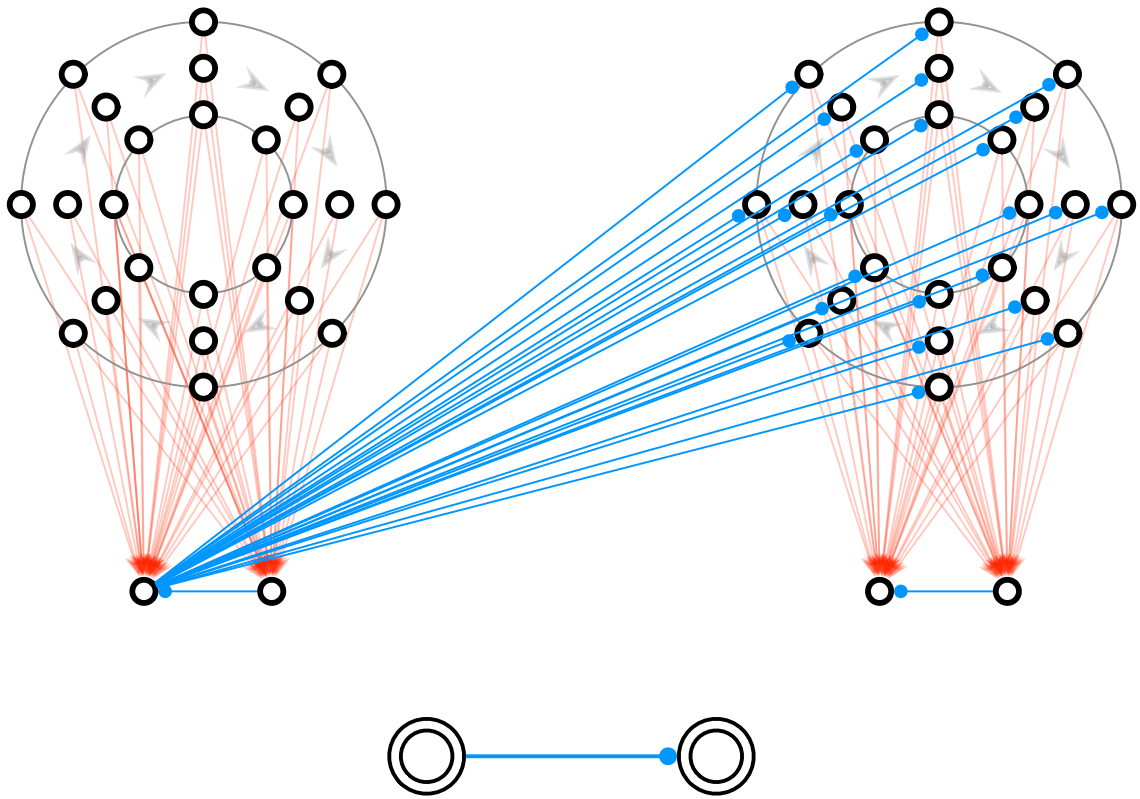
- excitatory / inhibitory



Fibres of connections & Inhibitory System

We consider the following kind of (fibres of) connections.

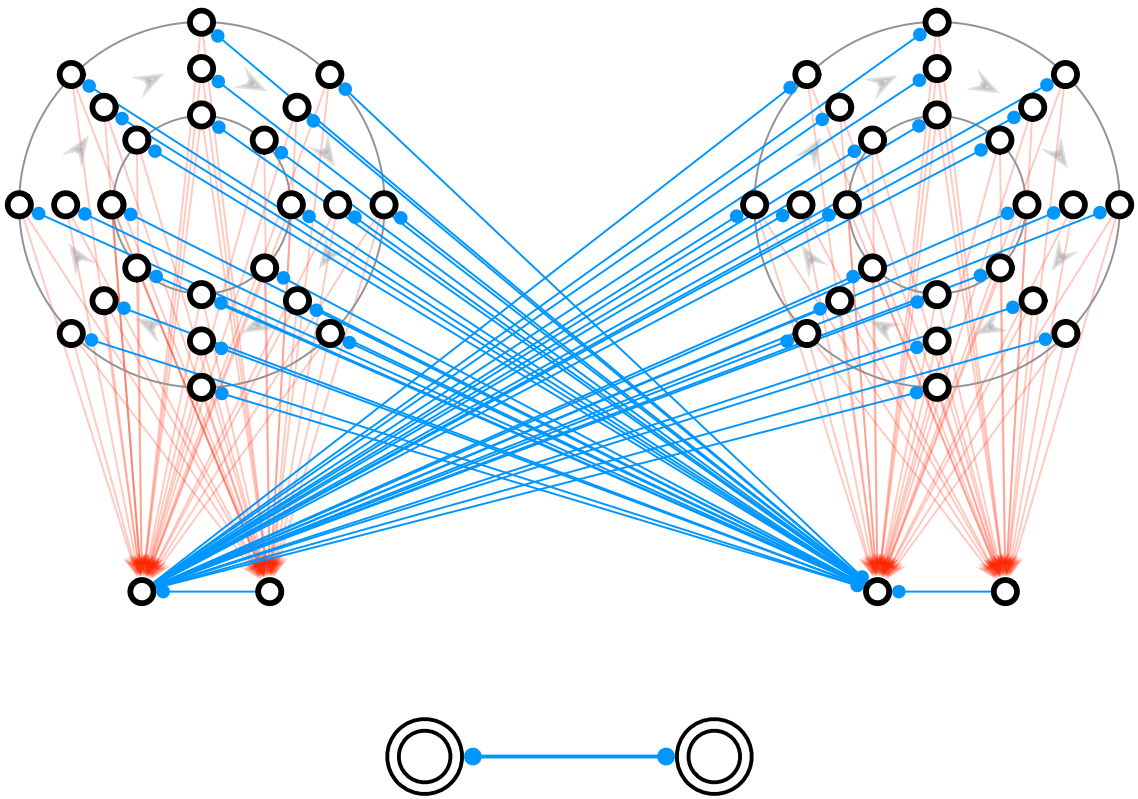
- one-shot inhibitory



Fibres of connections & Inhibitory System

We consider the following kind of (fibres of) connections.

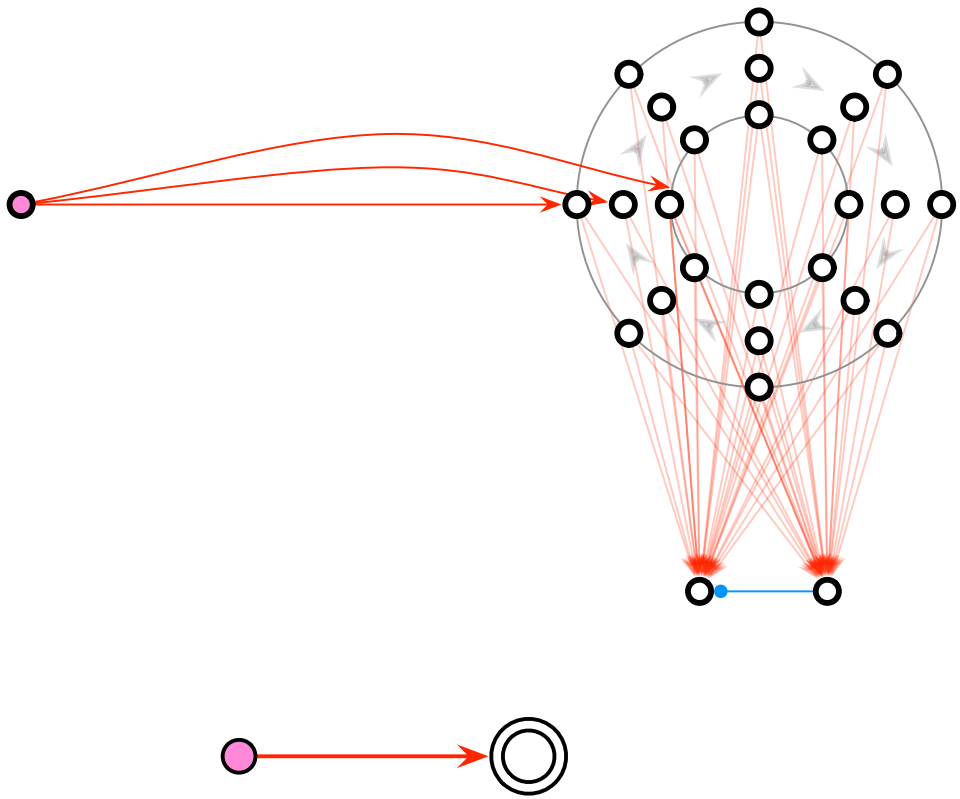
- one-shot inhibitory / one-shot inhibitory



Fibres of connections & Inhibitory System

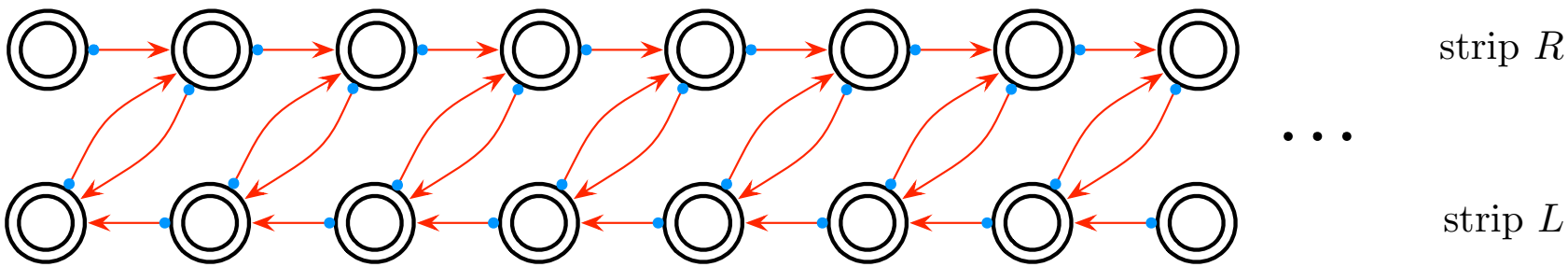
We consider the following kind of (fibres of) connections.

- cell to ring excitatory



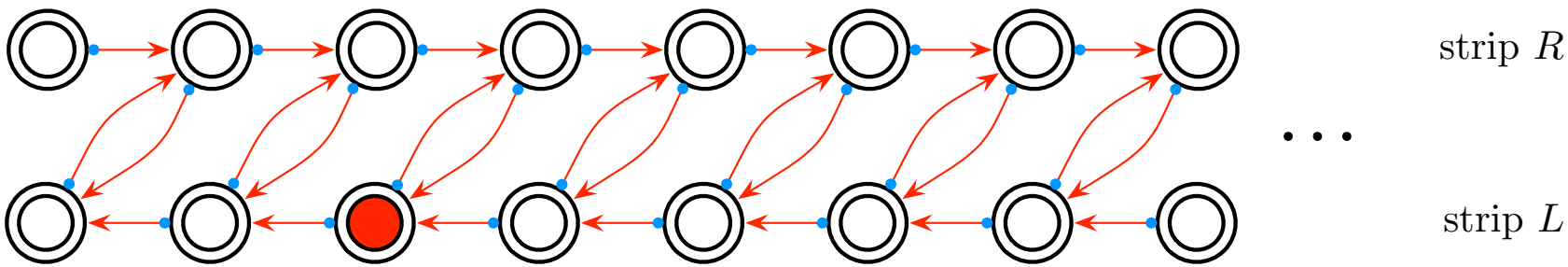
Position Tape

- Used to store the current position of the head.
- Composed of a "left" and a "right" strip.
- excitatory / inhibitory connections.



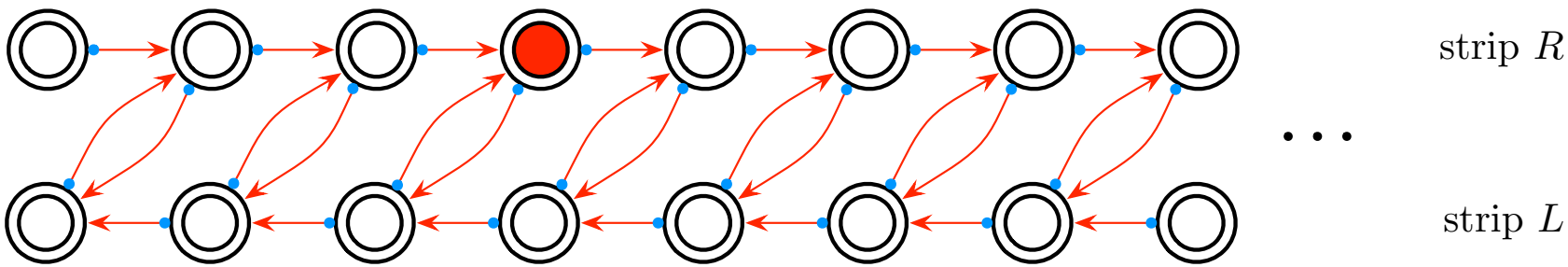
Position Tape

- Used to store the current position of the head.
- Composed of a "left" and a "right" strip.
- excitatory / inhibitory connections.



Position Tape

- Used to store the current position of the head.
- Composed of a "left" and a "right" strip.
- excitatory / inhibitory connections.



Position Tape

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- Composed of a "left" and a "right" strip.
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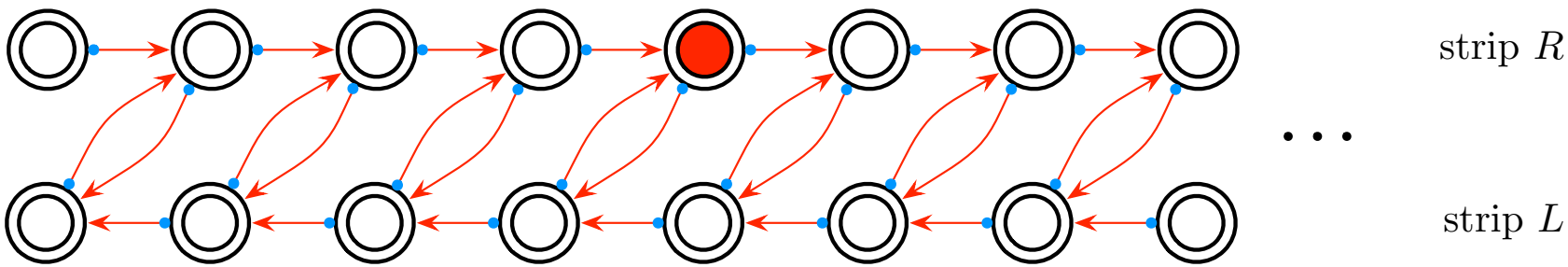

Position Tape

- Used to store the current position of the head.
- Composed of a "left" and a "right" strip.
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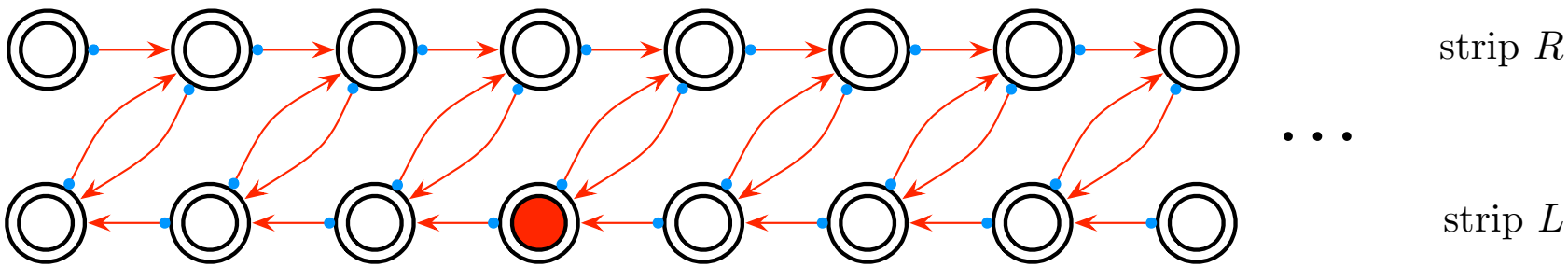
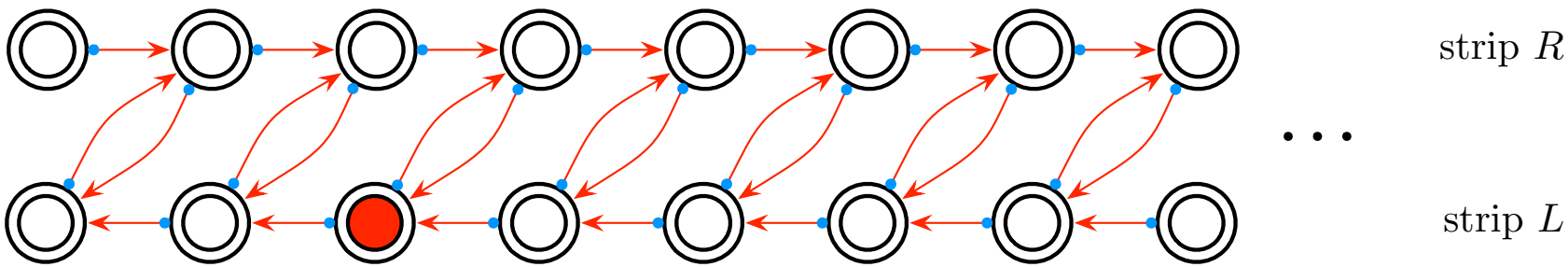


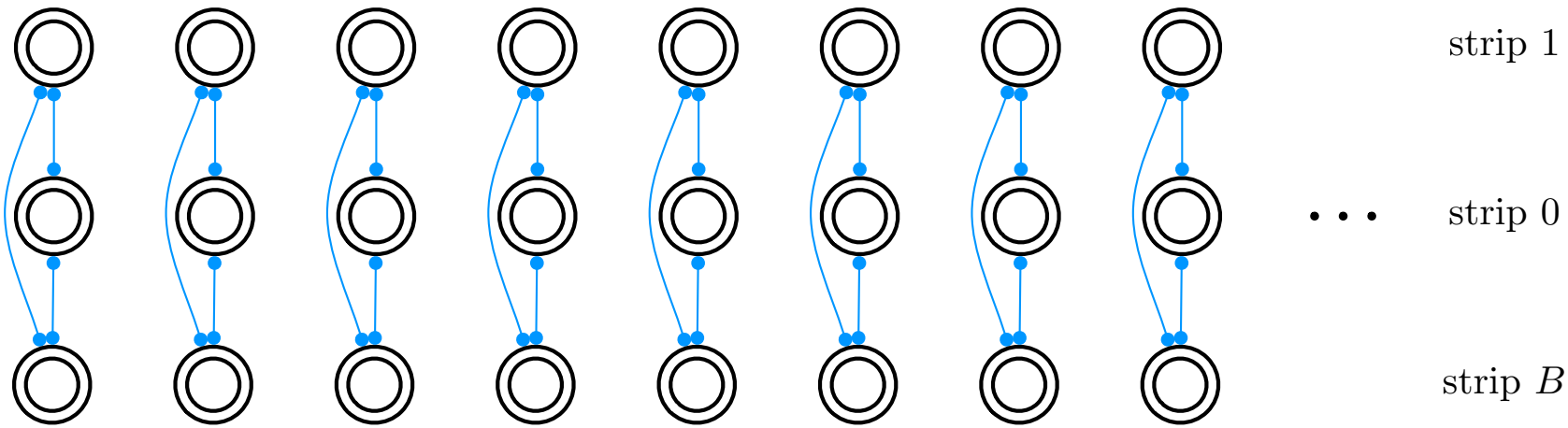
Position Tape

- Used to store the current position of the head.
- Composed of a "left" and a "right" strip.
- excitatory / inhibitory connections.



Symbol Tape

- Used to store the symbols written on the tape.
- Composed of a "blank", a "0" and a "1" strip.
- one-shot inhibitory / one-shot inhibitory connections.



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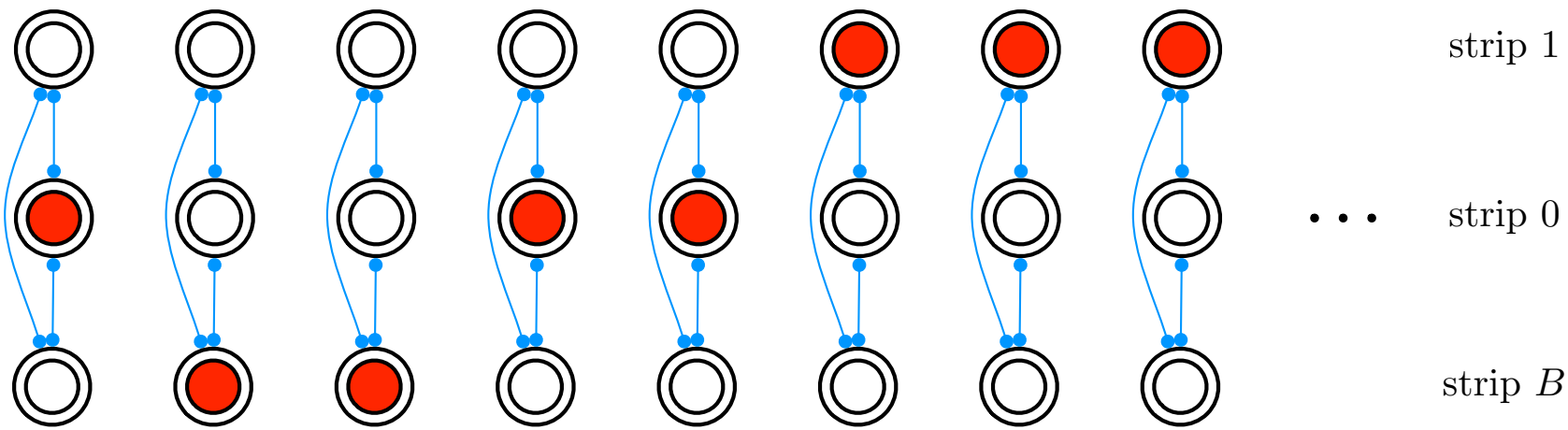

Cache Tape

- Used to store the symbol under the current head's position.
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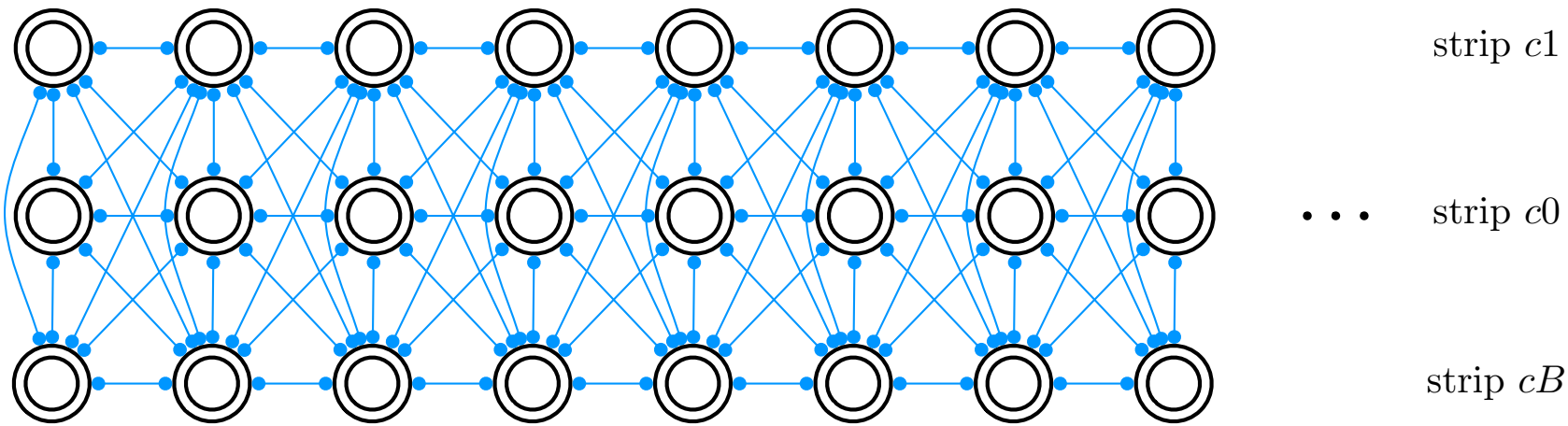

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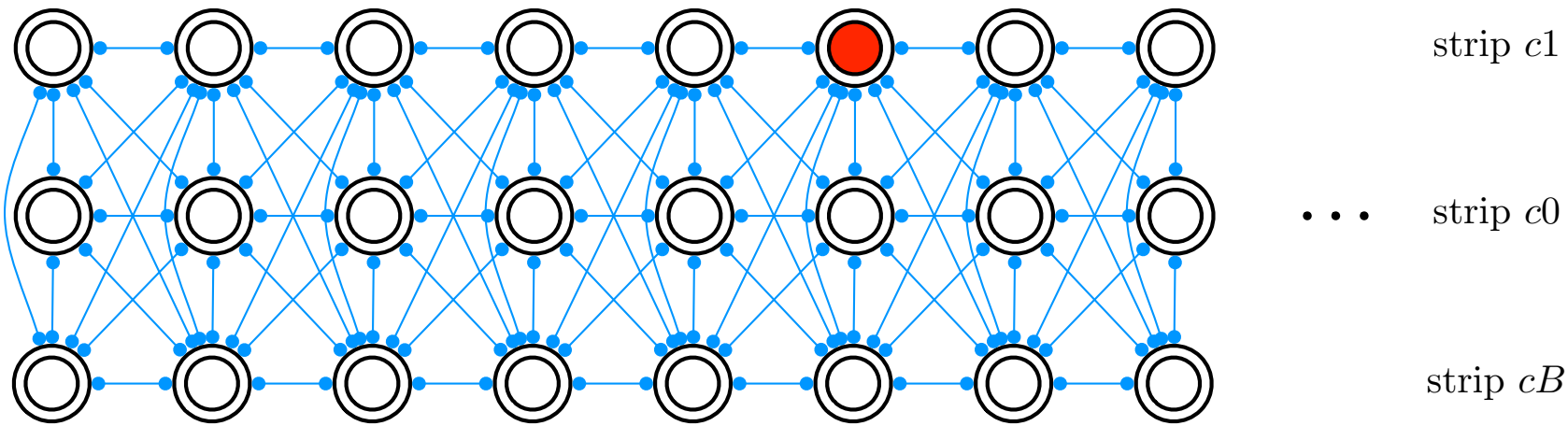
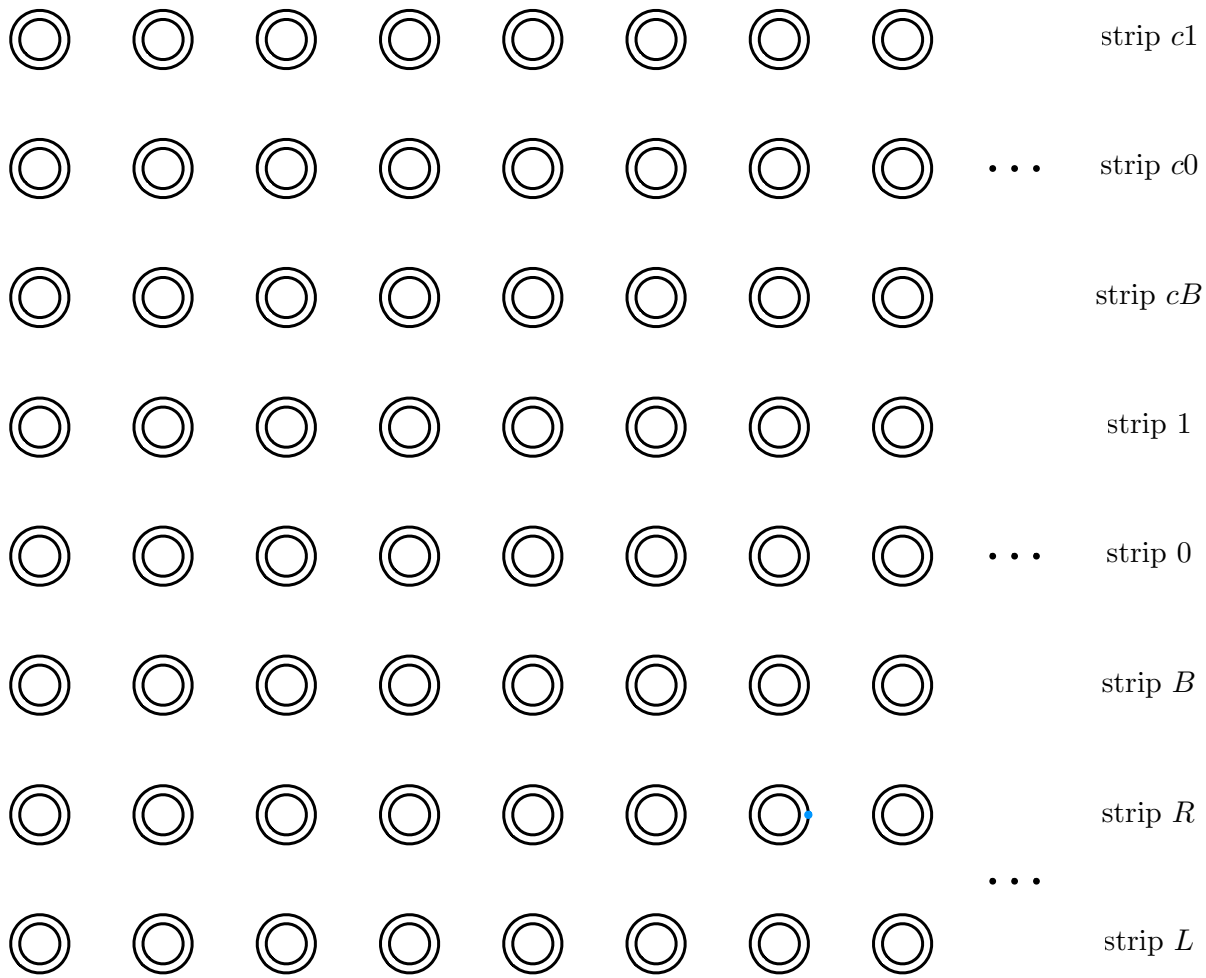


Position-Symbol-Cache tape Connections



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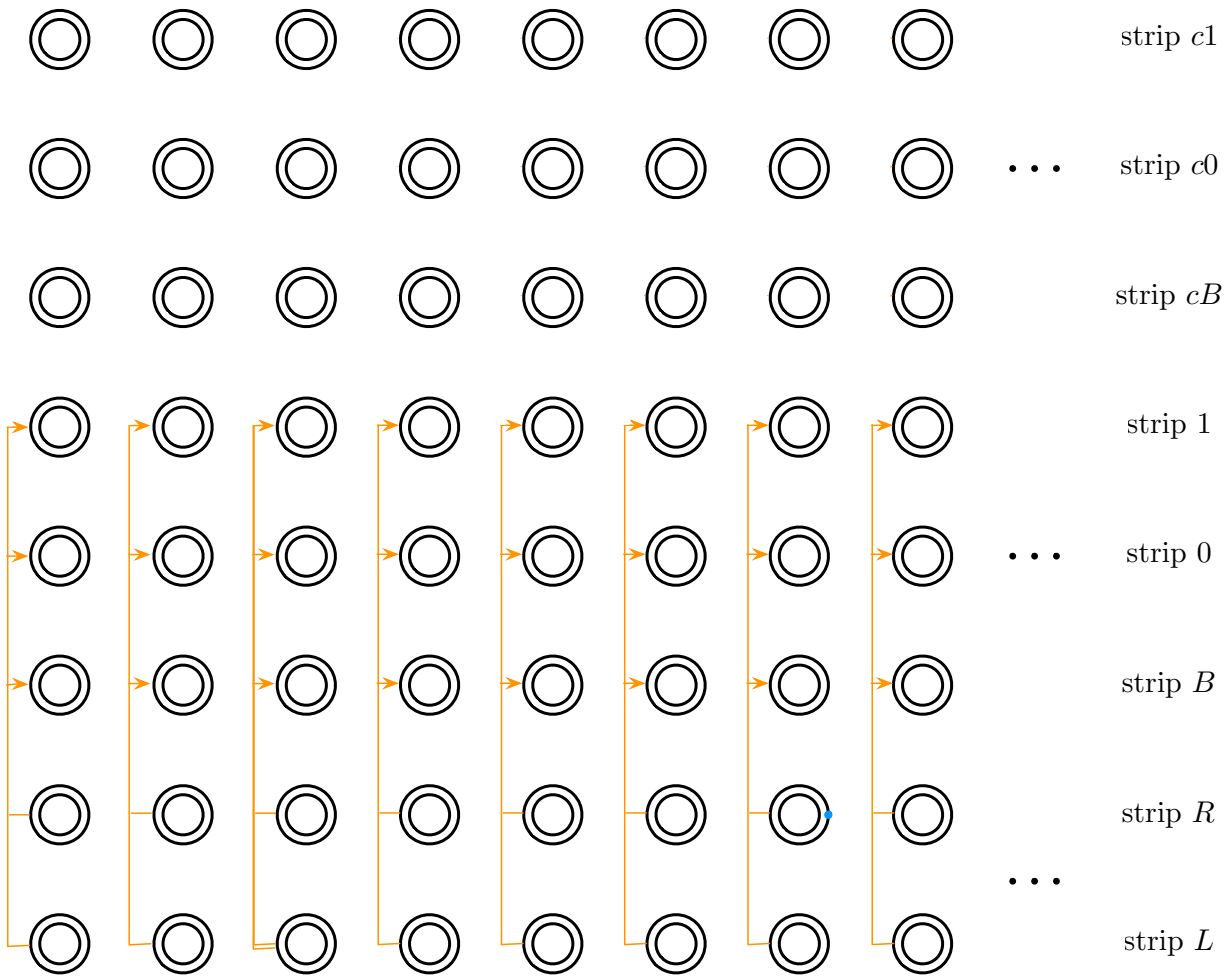

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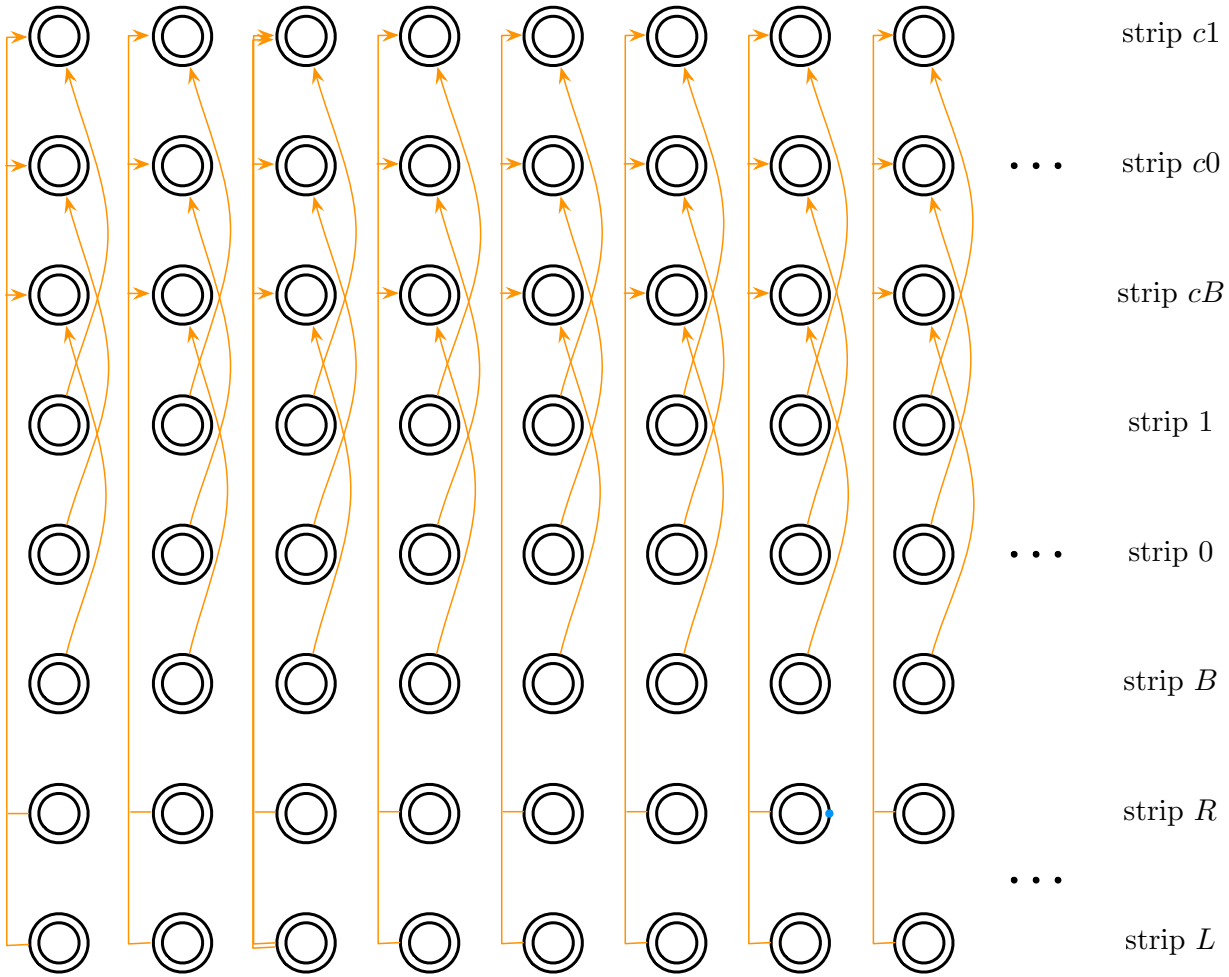

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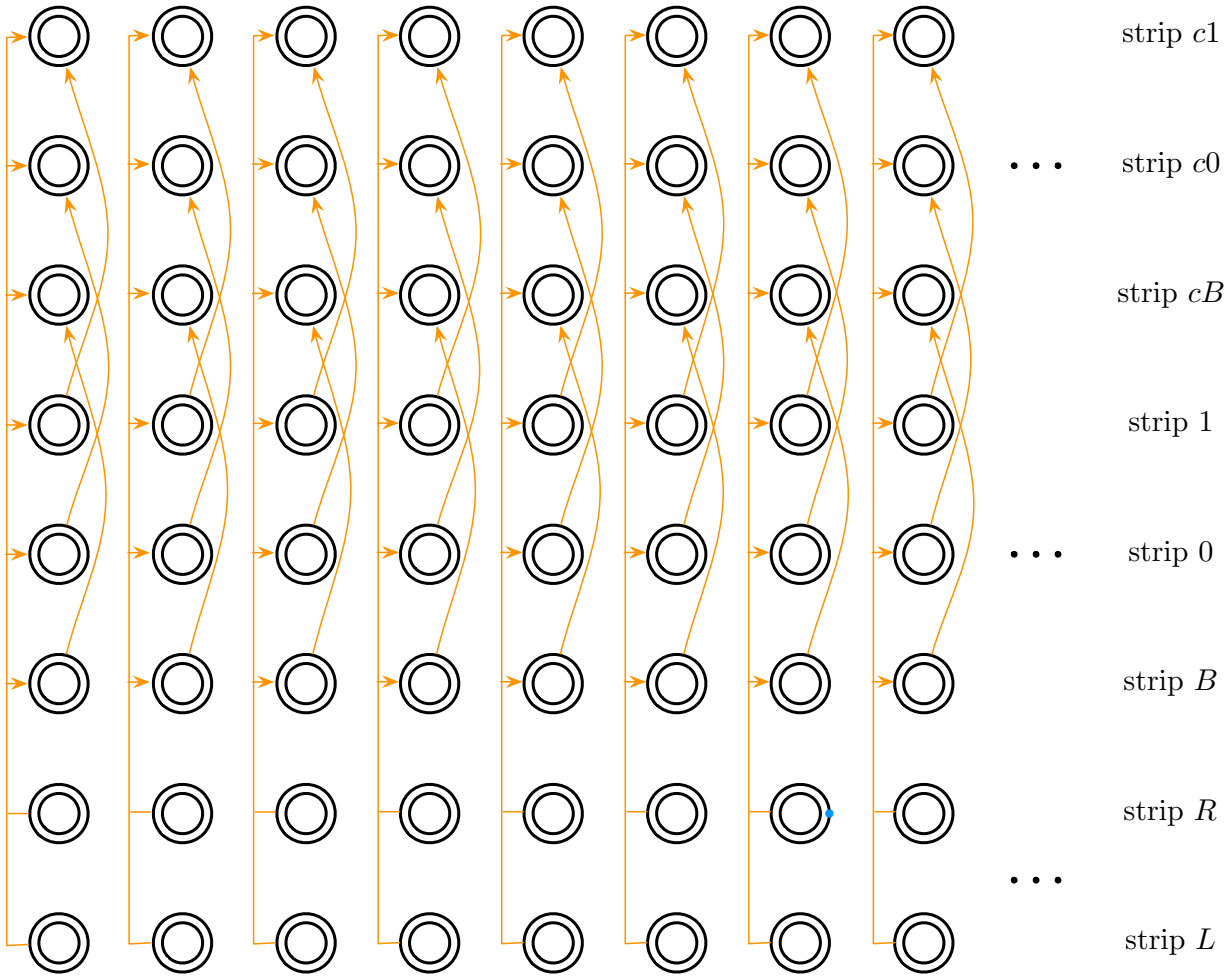

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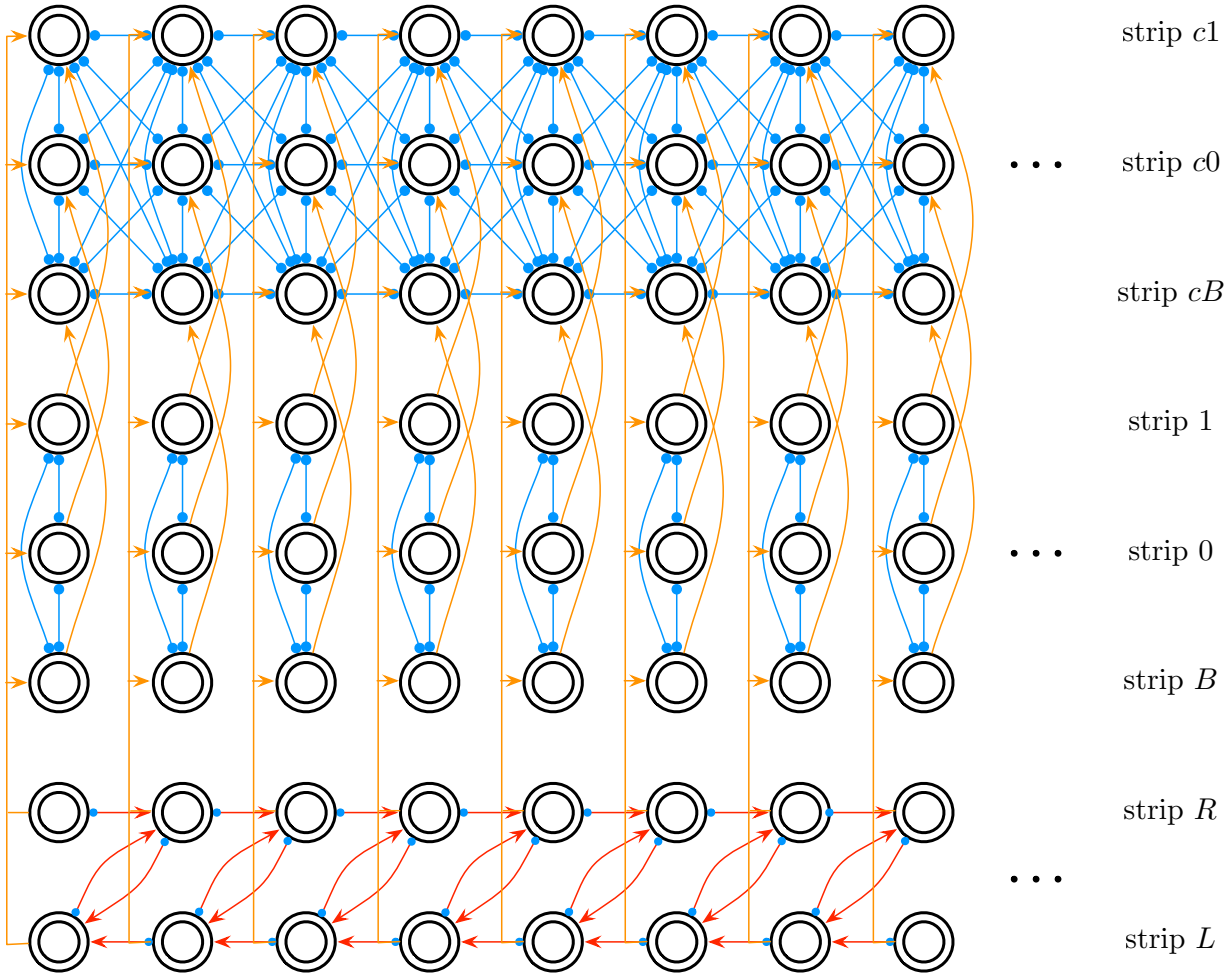

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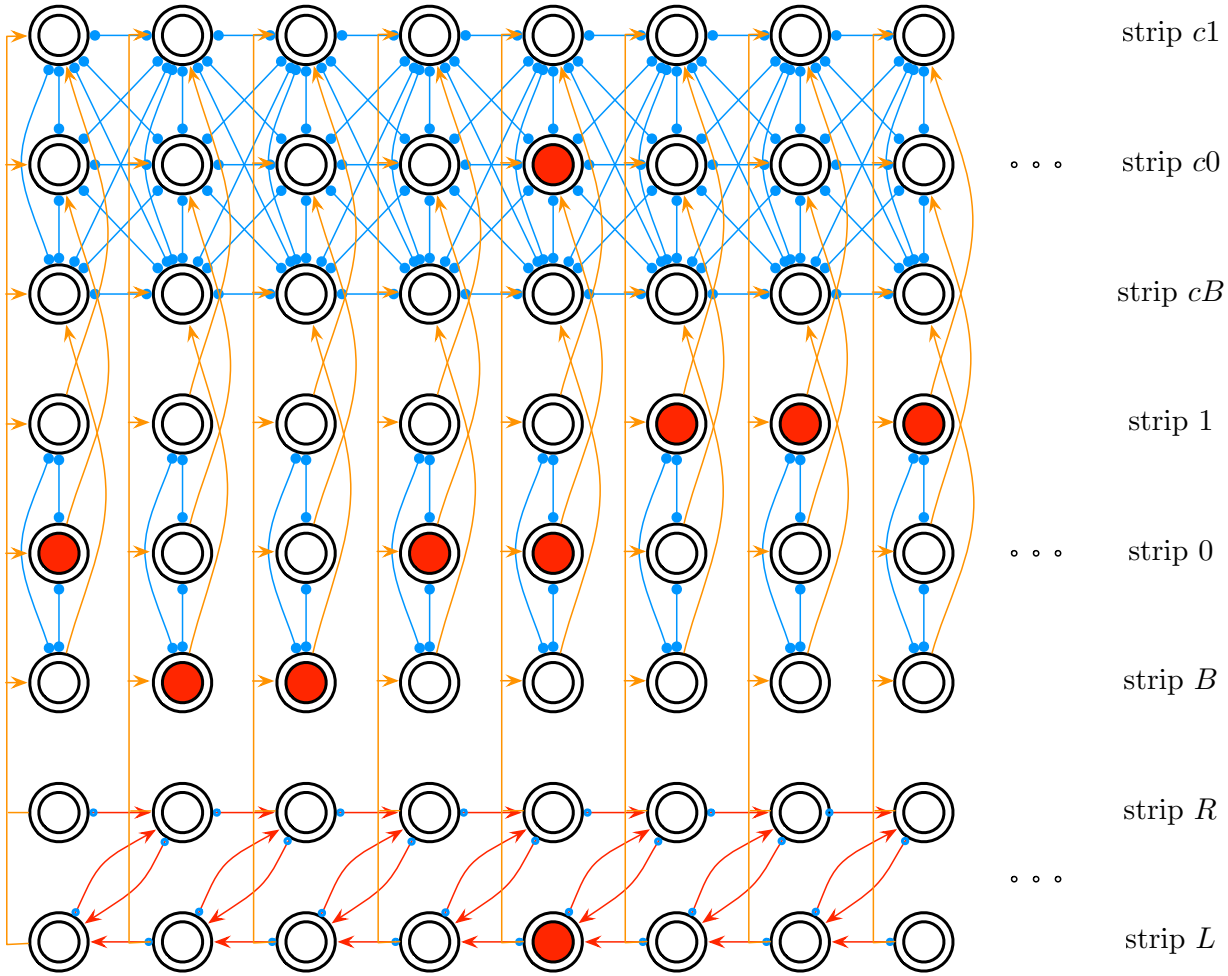

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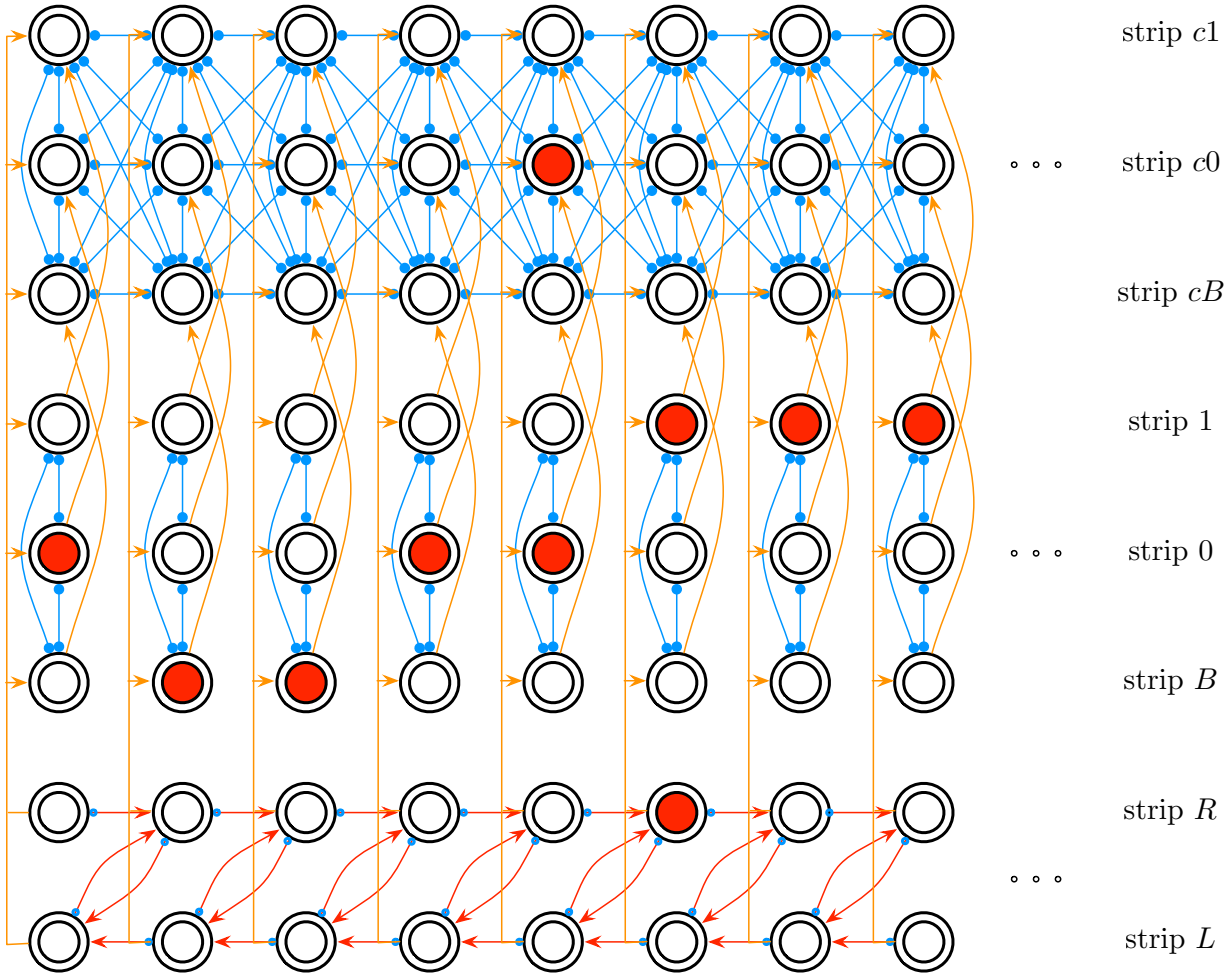

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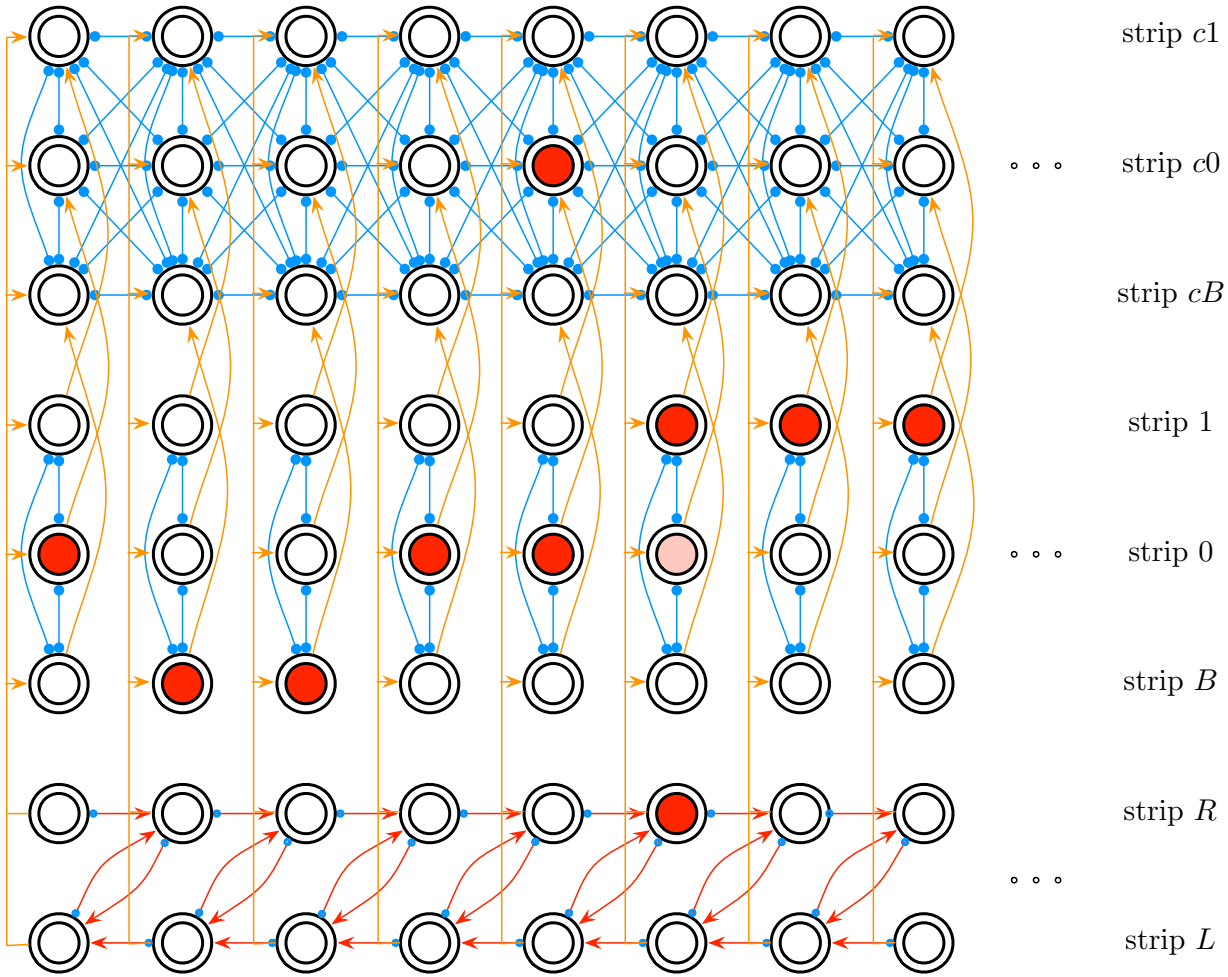

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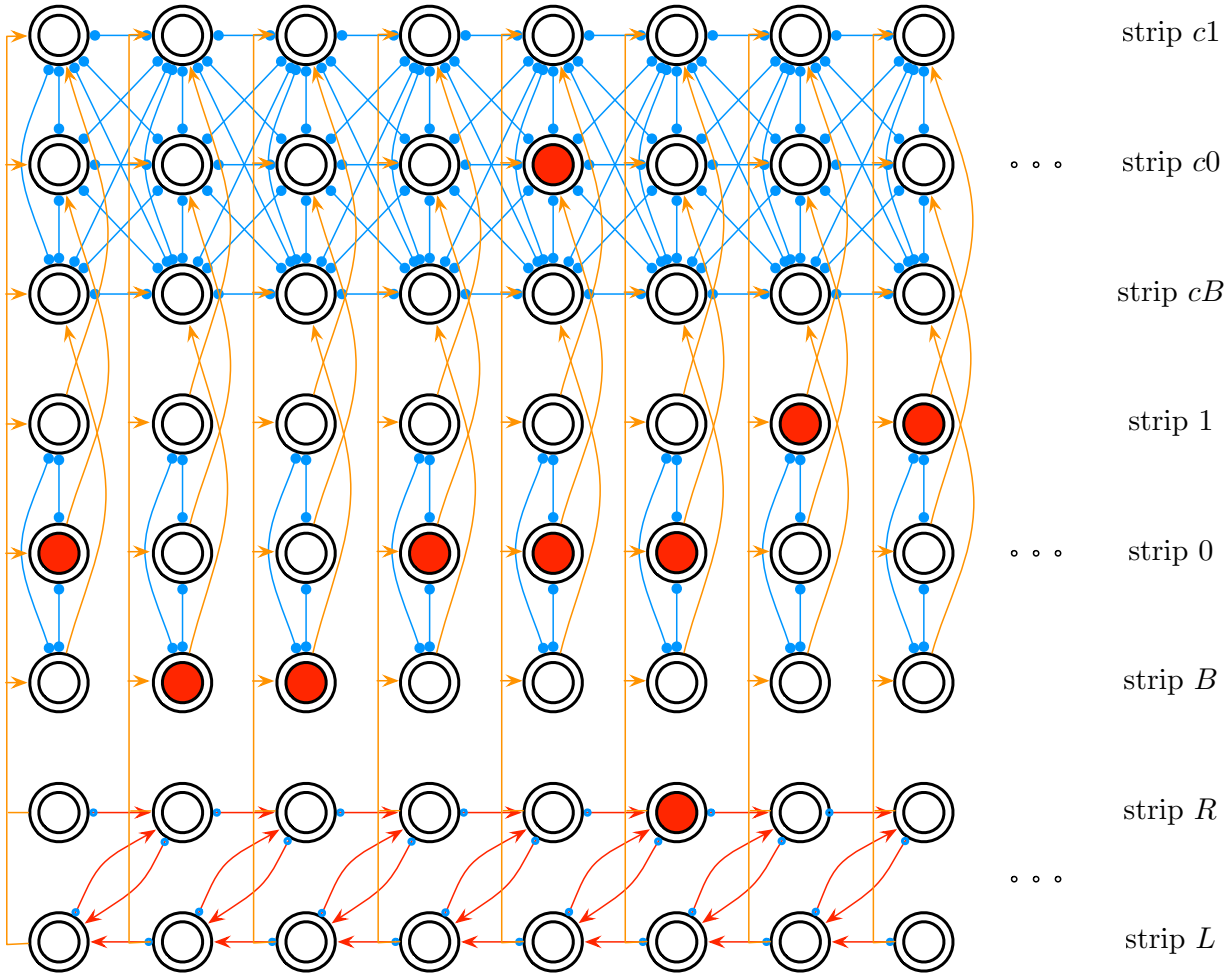

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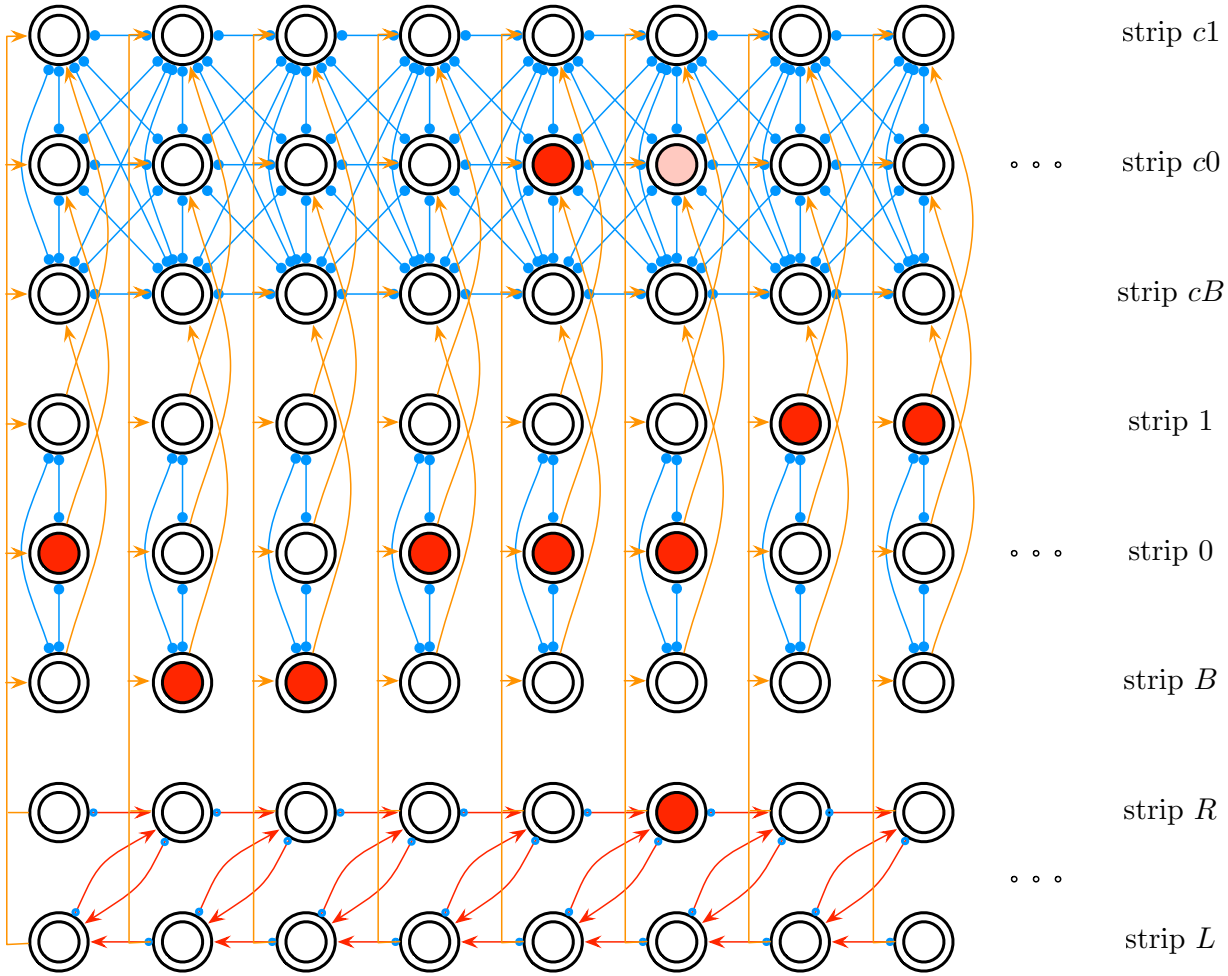

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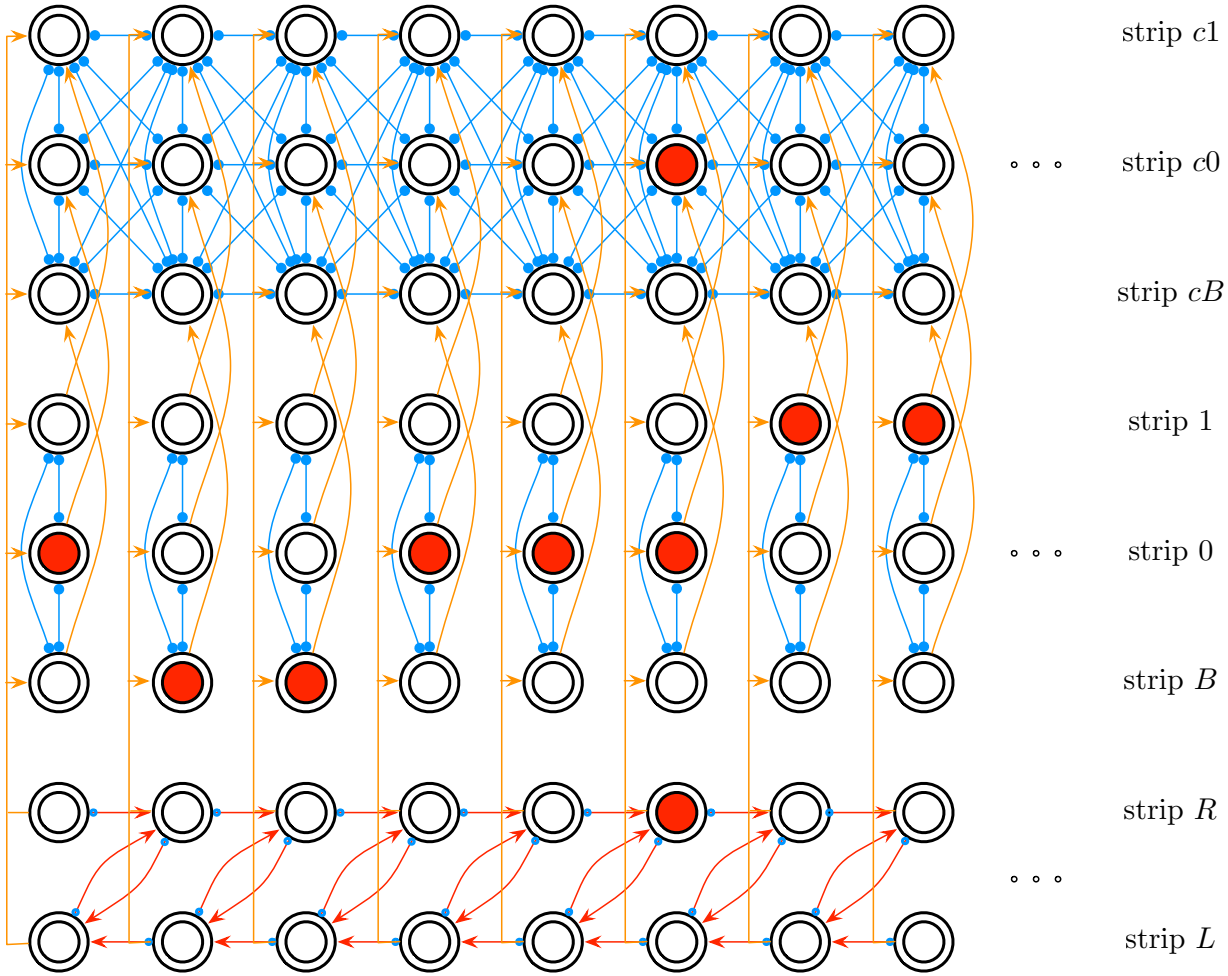

TIC_0-TIC-TAC PROCEDURE

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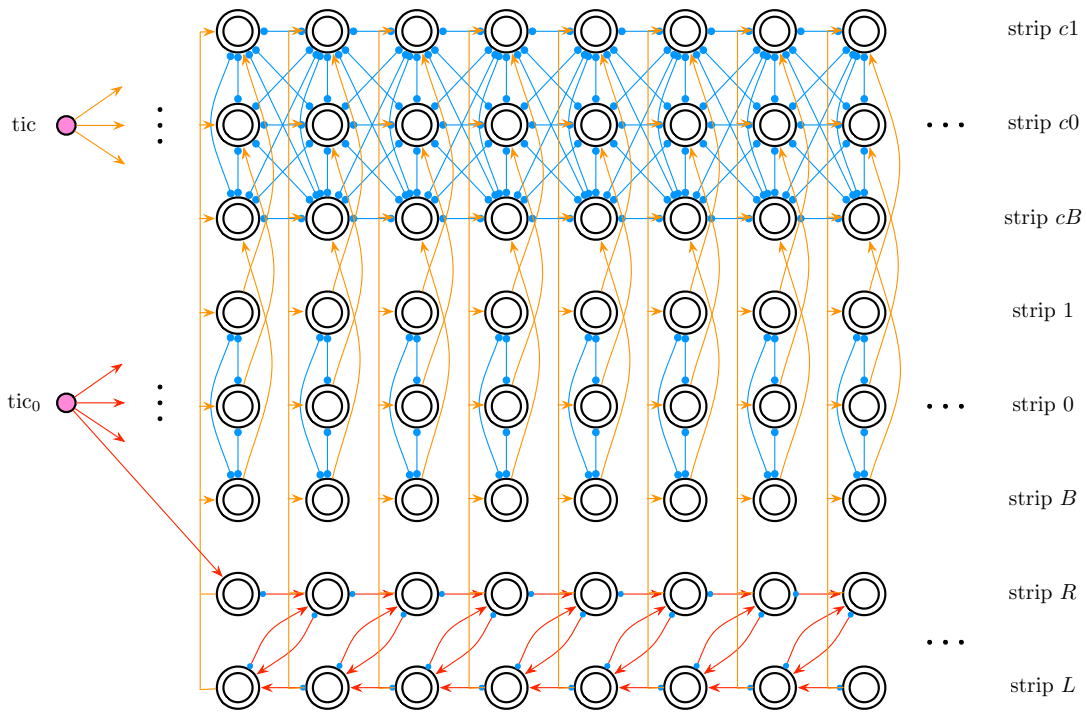
- "tac" cell triggers the writing of a new symbol and moving of the head (cf. next slide).



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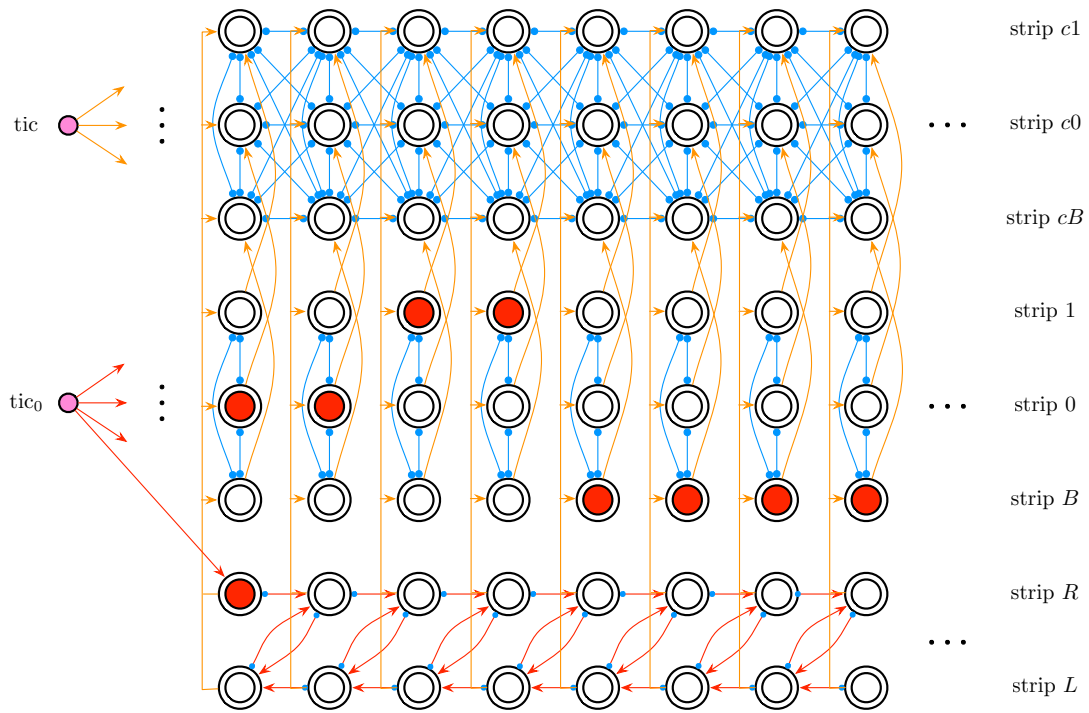

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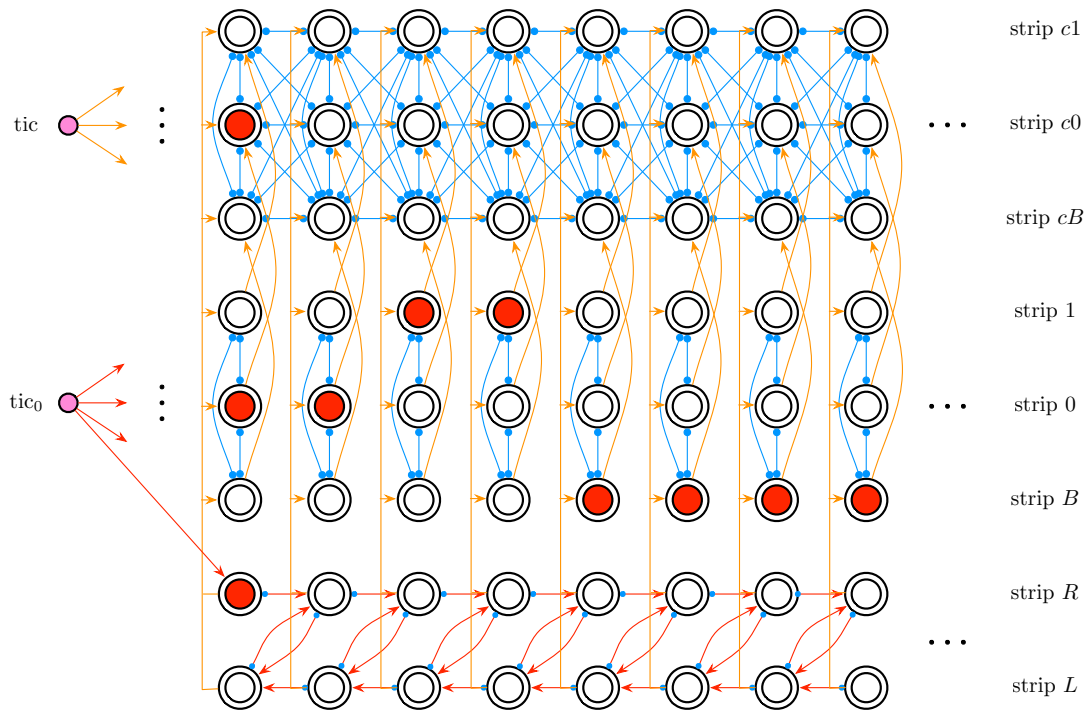

Program Rings

- One ring per event "state q_i & current symbols $a_1, \ldots, a_k$".
- Program to program (excitatory): to switch between rings.
- Cache to program (excitatory): to detect current symbols.
- Program to symbol (one-shot excitatory): to write symbols.
- Program to position (one-shot excitatory): to move heads.
- "tac" cell to program (excitatory): triggers the *writing* and *moving* procedures.

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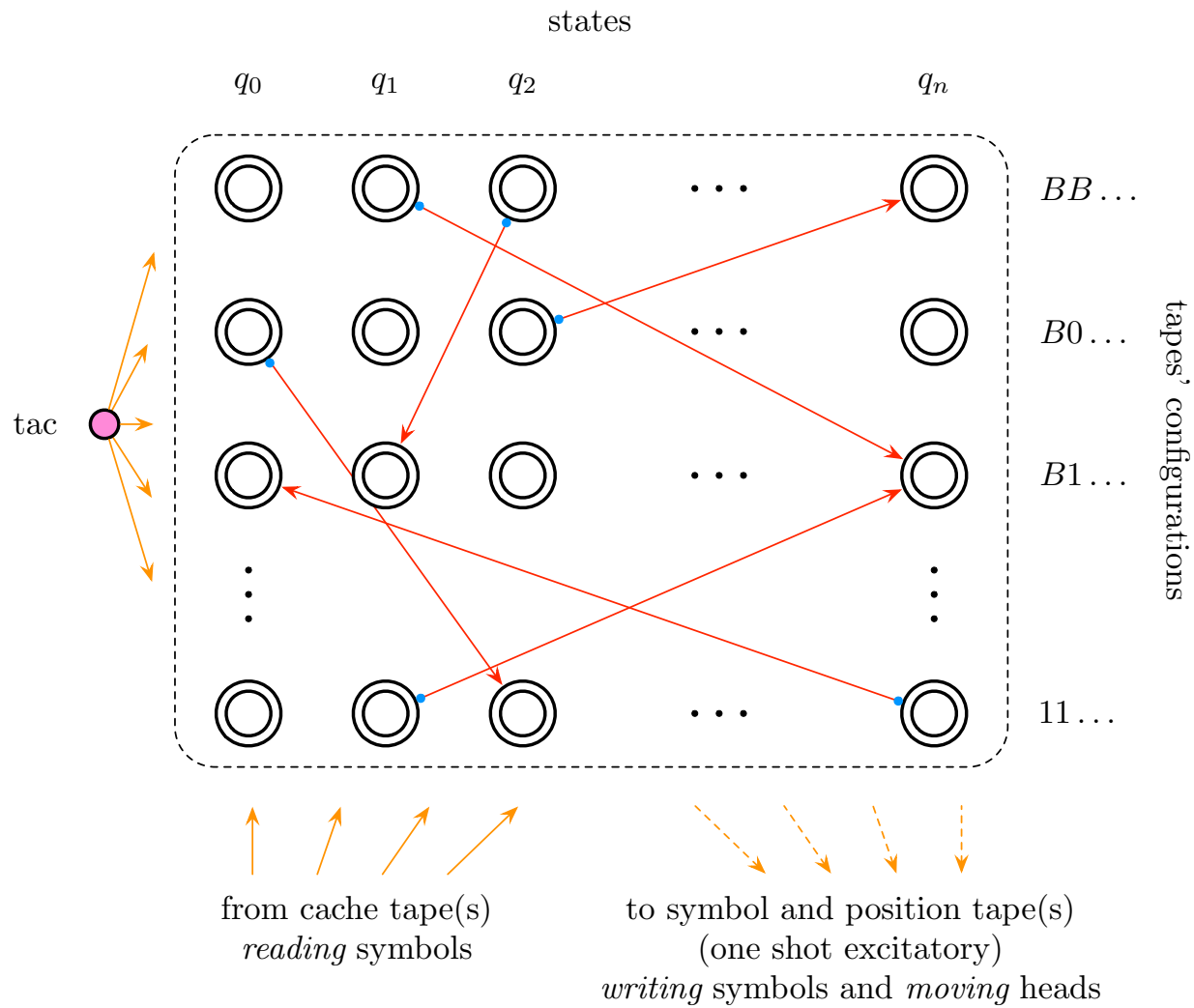
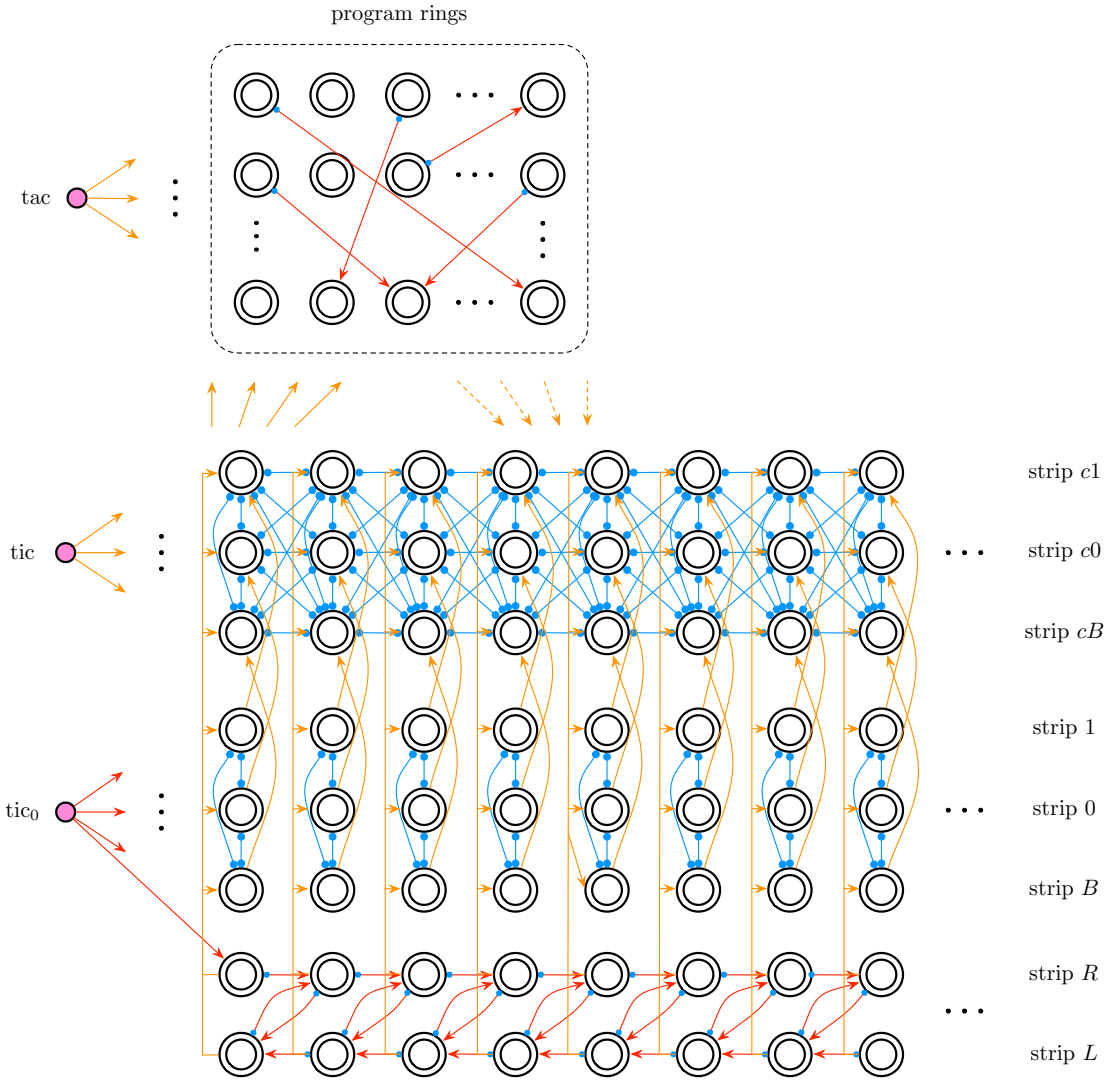


Putting everything together...



Turing machines & Boolean RNNs with Synfire Rings

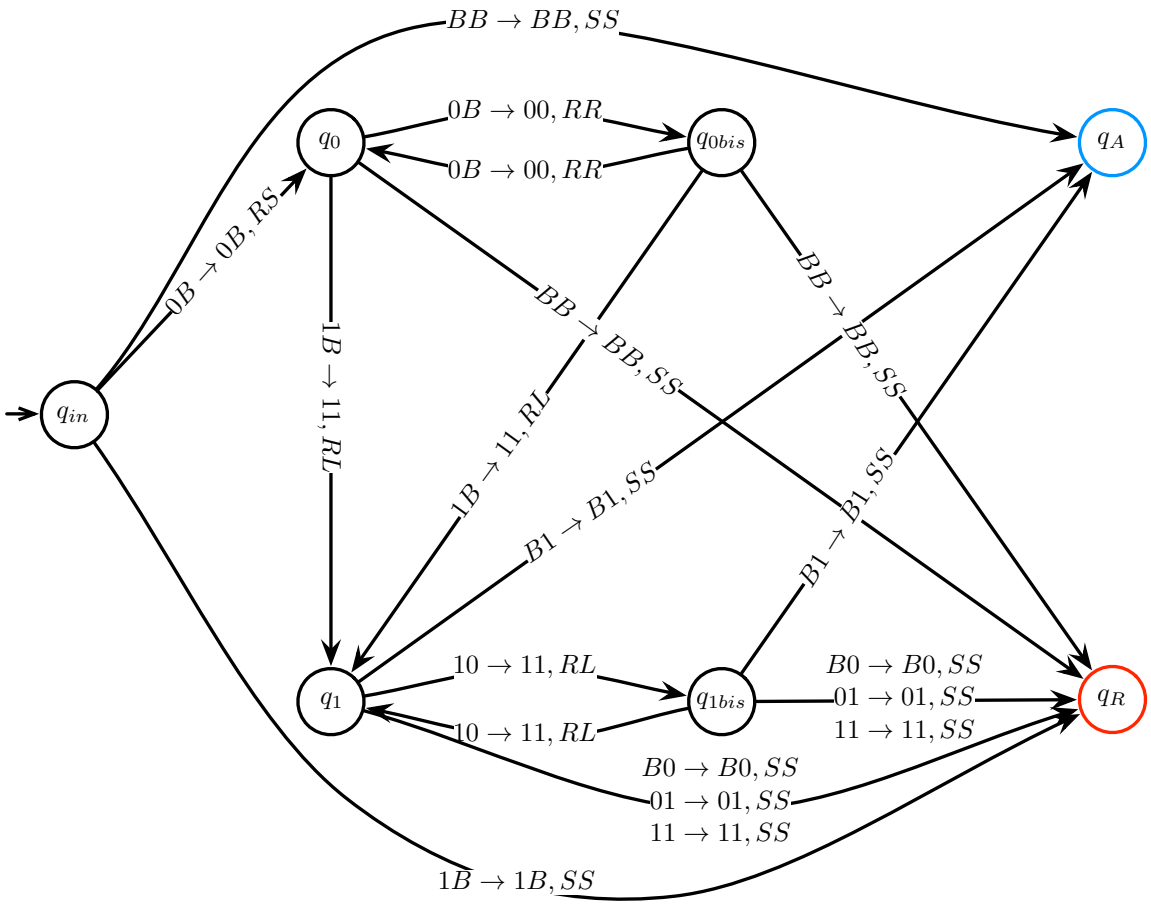
Since the construction is generic, one has the following result:

Theorem

Any Turing machine can be simulated by some Boolean neural network composed of synfire rings (up to assuming that the number of rings can be extended in an unbounded manner).

Turing Machine Recognizing $L = \{0^n 1^n : n \geq 0\}$

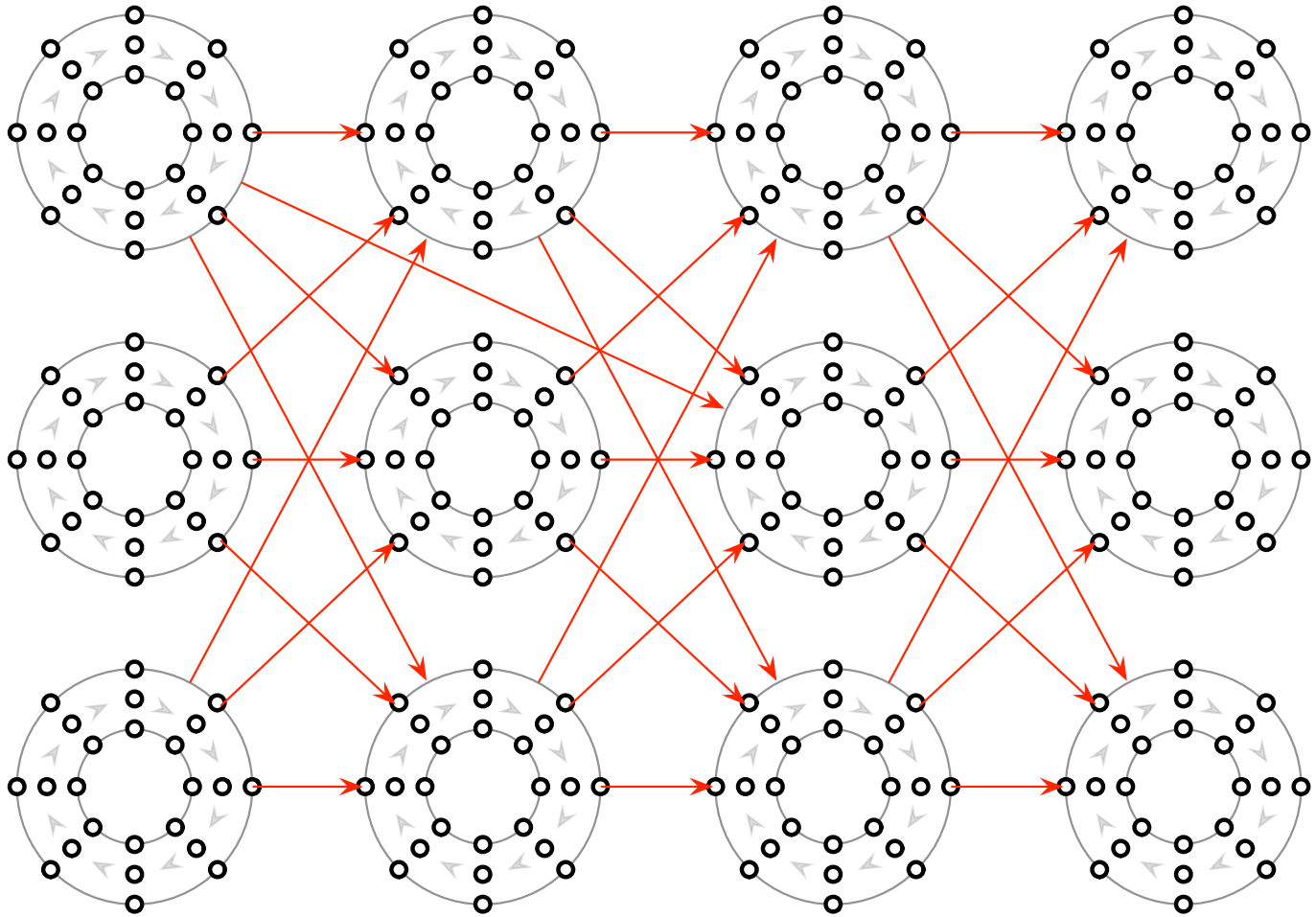
- TM modified such that it never stagnates on a particular state. . .



Simulation

Play two movies...

Future Work: Learning within the Synfire Ring Architecture



Conclusions

- Recurrent neural networks represent a natural models for oracle-based computation, beyond the Turing limits.
- We introduced a new paradigm of neural computation based on the concept of synfire rings.
- We intend to study the issue of *learning* within the synfire ring architecture.
- Towards *neuronal computers*... By growing cultures of neurons according to the synfire ring architecture, one could in principle simulate finite state machines with biological neural networks.

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